Mutual versus Stock Insurers: Fair Premium, Capital, and Solvency[1]

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Mutual and stock insurers are distinct corporate forms for organizing risk sharing when total claims are uncertain. Owners and policyholders are separated in a stock insurer, while they coincide for a mutual insurer. Based on this distinction, we show that (i) up-front capital is necessary to offer insurance at a fair premium for a stock insurer, but not for a mutual. (ii) For a mutual insurer, the probability of financial distress decreases in the number of policyholders, while it increases for a stock insurer, which offers insurance at a fair premium. (iii) Policyholders in a stock insurer with low level of capital benefit from premium loading. Furthermore, premium loading and a mutual insurer overcome the free rider problem, which arises, when it is collectively beneficial for policyholders to provide (additional) costly capital to improve risk sharing.

Keywords: risk transfer, mutual insurer, stock insurer, fair premium, capital, financial distress

1 Introduction

An insurance corporation "organizes" risk sharing between individuals. In principle, there are two different ways to organize risk sharing. First, risks can be shared within the pool of policyholders. A prominent example is a mutual insurer where policyholders are also the owners of the insurance corporation. In this case, policyholders have participating contracts by construction where all participate in each others' losses. Second, risks can be transferred from policyholders to another group of individuals (investors). A prominent example is a stock insurer, which transfers risks from policyholders to shareholders (i.e., the capital market). This transfer is achieved through the separation of rights to profits and rights to indemnity claims, i.e., the separation of owners and customers (policyholders).

A shortcut in much of the insurance literature is to assume that insurers hold a diversified portfolio of insurance policies such that the level of aggregate future claims payment is certain. In this case, a fair premium equals the expected indemnity payment and no capital beyond premiums is needed. Moreover, there is no insolvency and, with full insurance, the fair premium equals the expected loss. This implies that there is no difference between a mutual insurer and a stock insurer with respect to the required capital, the fair premium, and the distribution of risk. In this context, it is irrelevant whether rights to profits and rights to indemnity claims, i.e. owners and customers, are separated or not.

A different picture emerges if the level of total claims payment is uncertain. Then, the average policyholders' claim can be higher or lower than the insurance premium. In the latter case, the excess funds accrue to the owners of the insurer. For a mutual, an individual who buys an insurance policy also receives an ownership right in the insurer and the premium reflects the value of both the claim to the indemnity payment and the ownership right. The premium net of the expected value of the ownership right equals the expected claims payment. Hence, the implied insurance premium is always fair, independent of the level of the premium paid by policyholders. Moreover, no up front capital is required to offer insurance at a fair premium. This is not true for a stock insurer. Capital provided by shareholders is needed to offer insurance at a fair premium. To gain some intuition, imagine that no capital is available and that policyholders pay some positive premium. If the average claim is lower than the premium, the stock insurer is solvent and excess funds accrue to shareholders. In contrast, if total claims exceed collected premiums, then the insurer is insolvent and the maximum amount paid to policyholders is the level of collected premiums. Hence, given limited liability of shareholders and zero capital, the expected payment to policyholders is lower than the premium, which therefore is not fair.[1] Capital is required to reallocate funds. By providing capital, shareholders pay for the expected surplus which they make from situations where claims are lower than collected premiums. Policyholders benefit from this paid-up capital in situations where total premiums are lower than total claims. Our analysis highlights an important role of capital in a competitive insurance market. Capital does not merely reduce the probability of financial distress. It is an essential ingredient for a stock insurer to offer a fair premium. We show that there is a unique fair premium for each level of capital, which is below the expected loss

[1] In this case, shareholders earn a strictly positive expected return at the expense of policyholders.

and increases in the amount of capital that shareholders provide.

A fair premium is often justified by competition in the insurance market. However, we show that if the level of capital is restricted (fixed), it can be optimal for risk averse, utility maximizing policyholders if the policy exhibits loading for all policyholders. Hence, a fair premium may not be optimal for policyholders. The reason is that by collectively paying a loading policyholders provide additional funds which can be used to pay their indemnity claims. As a higher fraction of the additional funds is returned to policyholders with high losses, these funds provide additional insurance which is beneficial for risk averse policyholders. To create this additional risk sharing, however, it is important that policyholders collectively provide additional funds. This highlights a free rider problem as it is individually rational not to provide additional capital and free-ride on the others' investment. Loading and mutual insurance are ways to tie raising capital to insurance policies to overcome this free rider problem which is inherent in raising capital through buying shares.

We also show that a mutual differs from a stock insurer in terms of the effect of increasing the number of policyholders on the likelihood of financial distress. As mentioned above, there is no difference if future claims payments are certain. However, claimants (customers) of insurance have to bear risk in case of financial distress. If a mutual insurer increases its customerbase at a fixed premium, additional capital is added and the average total funds available per policy remain constant. For a stock insurer, however, there is a separation between owners and policyholders. If a stock insurer offers insurance at a fair premium with restricted and fixed capital provided by shareholders, then increasing the number of policyholders reduces the average shareholders' capital per policy. We show that this difference implies that increasing the customerbase has a positive effect on the likelihood of financial distress under the mutual form but a negative effect under the stock form. If managerial incentives are aligned with continued operation, i.e., a low likelihood of financial distress, this implies opposite managerial incentives to expand the pool of policyholders under the two organizational forms.

In this paper, we thus provide a novel insight into the difference between a mutual and a stock insurer by investigating the relation between capital, premium and risk sharing. In a mutual insurer, policyholders have (by definition) participating contracts. If a stock insurer offers full coverage, policyholders are fully insured in case of solvency and bear some risk in case of insolvency. Shareholders enter as a second and–if the probability of financial distress is small–primary group of risk bearer. This difference in degree and structure of risk sharing has important implications for the role of capital and premium as well as the relation of the number of policyholders and financial distress.

Our paper contributes to understanding the differences between stock and mutual insurers. Mayers and Smith (1981) focus on governance issues and argue that different organizational forms have different strengths in dealing with different types of agency problems. The mutual form internalizes the owner-customer conflict, while the stock form is better suited to align managers' and owners' interests. The choice of organizational form depends on the trade-off between the two. Smith and Stutzer (1990, 1995) focus on the different contractual structures of insurance contracts offered by mutuals (participating contract) and stock insurers (indemnity payment with a flat fee). They argue that participating contracts are driven by undiversifiable risk and used to cope with adverse selection and moral hazard by the insured. We focus on the link between capital, insurance premium, and financial distress. External capital is a prerequisite for a stock insurer to offer insurance at a fair premium in a competitive insurance market. If the capital market is efficient and the ownership is dispersed, the benefit of external capital is to spread the risk in the capital market and thereby improve risk sharing. If raising capital is very costly, it may be optimal for policyholders to collectively provide additional funds to improve risk sharing. As providing funds is costly, e.g., reduced diversification, each individual policyholder benefits from not providing the funds. Tying the provision of funds and insurance, as in a mutual, is a way to overcome the free-rider problem. Moreover, participating contracts in a mutual may provide better risk sharing than a flat fee with indemnity contracts in a stock insurer if only a few investors are willing to invest capital.

Smith and Stutzer (1995) and Zanjani (2004) find that mutual insurers are used more often in times of financial crises. Zanjani argues that a reason is that new stock insurers are more capital-intensive than mutual firms and that the use of mutuals in the times of distressed capital markets may represent a substitution away from (external) capital in production. This finding is consistent with our prediction as capital in our paper is required to reallocate funds between policyholders and owners in different states of the world. These funds are not required for mutuals as policyholders are also owners. Moreover, Viswanathan and Cummins (2003) find that recent demutualizations were motivated by access to capital. Given a capital market where many investors invest in stock corporations, a stock insurer can have a competitive advantage over a mutual through improved risk sharing.

We also contribute to the literature that analyzes the role of financial distress for insurance markets. Doherty and Schlesinger (1990) examine the demand for insurance under financial distress and Mahul and Wright (2004a, 2004b) the optimal structure of insurance contracts for a mutual insurer with limited capital. In contrast to our paper, this literature does not analyze the relation between capital, premium and financial distress. The authors fix the level of pre-paid premiums either directly, i.e. exogenously, or indirectly by fixing the probability of financial distress. Cagle and Harrington (1995) and Cummins and Danzon (1997) analyze insurance supply with capacity constraints and endogenous insolvency risk. Their approach is quite different from ours as the authors focus on the effect of loss shocks on capitalization and premiums in insurance markets. Shareholders supply insurance based on maximizing the expected value of net cash flows under the assumption that capital is costly. The demand side is given by some exogenously specified demand curve which is assumed to be negatively related to the price of insurance. In contrast to this literature, our paper focuses on the trade-off between price and quality of insurance which is crucial for the distinction between a mutual and stock insurer in terms of the relation between capital, premium, and risk sharing.

The paper is structured as follows. In Section 2, we present the economic environment. In Section 3 and 4, we examine the role of capital and premium, and the effect increasing the number of policyholders on risk sharing under the two different organizational forms. In Section 5, we discuss the role of the corporate form in terms of managerial incentives to expand and policyholders' incentives to provide capital. We conclude in Section 6.

2 Economic Environment

There are n identical, risk-averse policyholders who maximize expected utility with respect to some increasing, concave utility function $u(\cdot)$. Each policyholder is endowed with some initial wealth w_0 and faces a loss of size X_i, $i = 1, ..., n$. We assume that losses are independent and identically distributed according to some continuous distribution function F^1 with density function f^1. The aggregate loss in the economy, $\sum_{i=1}^n X_i$, is then distributed according to the n-fold convolution $F^n = \left(F^1\right)^{*(n)}$ with density function f^n.

Suppose a stock insurance company with financial capital C offers full insurance at a premium P. The stock insurer is insolvent if and only if aggregate claims exceed total capital in the company, i.e., if

$$\sum_{i=1}^n X_i > nP + C,$$

and solvent otherwise. If the company is solvent, then policyholders are fully indemnified and the remaining funds, $nP + C - \sum_{i=1}^n X_i$, are split amongst the shareholders of the company. If the insurer becomes insolvent, then the remaining funds, $nP + C$, are split amongst policyholders according to some pre-specified bankruptcy rule, $I_i(X_1, ..., X_n)$, with $\sum_{i=1}^n I_i(X_1, ..., X_n) = nP+C$ and $E[I_i(X_1, ..., X_n)] = P+C/n$ for all $i = 1, ..., n$. A pro-rata rule, for example, would be defined by[2]

$$I_i(X_1, ..., X_n) = \frac{X_i}{\sum_{i=1}^n X_i}(nP + C).$$

A mutual insurance company is owned by its policyholders who therefore own the right to profits. Suppose that each policyholder pays a premium P for full coverage, i.e., the mutual has

[2] Such a sharing rule, where policyholders receive a share of the insurer's assets that is proportional to their claim, is assumed in much of the literature (see, e.g., Cummins and Danzon, 1997, who confirm that "this liquidation rule is consistent with the way insurance bankruptcies are handled in practice," footnote 22).

no capital other than the collected insurance premiums.[3] The mutual is insolvent if and only if

$$\sum\nolimits_{i=1}^{n} X_i > nP,$$

and solvent otherwise. If the insurer is solvent, then all claims are paid in full and the remaining funds are split equally between policyholders, such that each policyholder receives, $P - \frac{1}{n}\sum_{i=1}^{n} X_i$. If aggregate claims exceed collected premiums, then the funds are split amongst policyholders according to some pre-specified bankruptcy rule, $I_i(X_1, ..., X_n)$, with $\sum_{i=1}^{n} I_i(X_1, ..., X_n) = nP$ and $E[I_i(X_1, ..., X_n)] = P$ for all $i = 1, ..., n$.

Profits in excess of total claims payment accrue to the owners of the insurance organization. Firms that are owned by shareholders therefore have the property that ownership rights to the insurer's profits reside with investors and not policyholders (customers).[4] For a mutual insurer, owners and customers coincide, i.e., the rights to the insurer's profits reside with the policyholders. This distinction between the separation and non-separation of rights to profits and indemnity claims, i.e., between the separation and non-separation of owners and customers, is central to our discussion in this paper and allows us to obtain novel insights about the differences between stock and mutual insurers. In the following sections, we examine the differences of the role of premium, capital, and number of policyholders under those two organizational forms.

[3] We note that our characterization of a mutual insurer, where policyholders pay P up front and excess capital is returned if losses are lower than total premiums, is qualitatively equivalent to a setting where policyholders hold assessable policies and pay some up-front premium P_0 and are obliged to pay an assessment of up to $P_1 = P - P_0$ if losses exceed the initially collected premiums-if there is no counterparty-risk.

[4] Customers may also be shareholders, but they are usually only a small subgroup in the case of stock insurers. In our analysis, we assume a distinct separation between owners and customers in a stock corporation in order to clarify the difference to a mutual organization.

3 Premium and Capital

3.1 Actuarially Fair Premium

The actuarially fair premium, P_{fair}, is equal to the expected indemnity payment. If there is a positive probability that the company becomes insolvent, then the level of expected indemnity under full insurance is lower than the level of expected loss to the insured. The actuarially fair premium thus depends on the company's insolvency probability. On the other hand, the company's insolvency probability in turn depends on the level of the premium as collected premiums are available for claim payments. Other important determinants of the counterparty risk and level of actuarially fair premium are the amount of shareholders' capital in the company, C, and the number of policyholders, n. We now focus on the ability of a stock versus a mutual insurer to offer insurance at a fair premium.

To clarify the interconnection between the fair premium and insolvency risk in a stock corporation we examine the two extreme scenarios, zero and unlimited financial capital held by the company. Given some positive premium P, there is a strictly positive probability that the company remains solvent, that is the collected premiums exceed the aggregate claims by policyholders, even if the stock corporation has no capital, i.e. $C = 0$. In this case, however, the remaining funds are paid out to shareholders. This implies that the expected payout to policyholders is lower than the premium, which contradicts the definition of a fair premium. We thus conclude that a stock insurer cannot offer insurance at a fair premium without capital. That is, without capital the only fair premium is zero. With unlimited capital, i.e. $C = +\infty$, the company never goes bankrupt and policyholders are always fully indemnified. The expected indemnity payment thus equals the expected loss to the insured. This implies that the actuarially fair premium is given by the expected loss. In the following proposition, we formalize these arguments and show that the actuarially fair premium is increasing in the amount of capital provided by shareholders.

Proposition 1 *For a stock insurer, there exists a unique actuarially fair premium for each fixed* *level of capital provided by shareholders. Furthermore, the actuarially fair premium is strictly increasing in the amount of capital, from zero without capital to the level of the expected loss with unlimited capital.*

Proof. See Appendix A.1. ■

This proposition shows that capital provided by shareholders is needed for a stock insurer to provide insurance at an actuarially fair premium. Shareholders' capital effectively reallocates funds from states in which aggregate claims are lower than premiums collected to states in which aggregate claims exceed total premiums. Furthermore, for any fixed level of capital C there exists a unique fair premium $P_{fair}(C)$ which in turn determines the insolvency probability

$$prob\left(\sum_{i=1}^{n} X_i > nP_{fair}(C) + C\right) = 1 - F^n\left(nP_{fair}(C) + C\right).$$

Under a mutual organization, policyholders are also the owners of the firm and all premiums collected are redistributed. Each policyholder receives $X_i + P - \frac{1}{n}\sum_{i=1}^{n} X_i$ in case of solvency and $I_i(X_1, ..., X_n)$ in case of insolvency. The premium therefore comprises the expected indemnity payment and the value of the ownership right. This implies that the policyholder of a mutual insurer always pays a fair premium independent of the level of P and other capital C.[5]

Proposition 2 *For a mutual insurer, the implied insurance premium is always fair.*

Proposition 2 highlights an important distinction between stock and mutual insurers. A sufficiently high level of capital C is required to offer a substantial amount of insurance at a fair premium in the case of a stock insurer. This capital has to be provided by shareholders who benefit from states where total premiums exceed the aggregate loss. A fair premium implies that shareholders make zero expected profits from selling insurance policies. The role of capital is thus to reallocate funds from states where shareholders make a profit to states where total premiums are lower than the policyholder's losses. For a mutual insurer, the initial level of capital is less of

[5]Note that $E\left[X_i + P - \frac{1}{n}\sum_{i=1}^{n} X_i\right] = P$ and $E\left[I_i(X_1, ..., X_n)\right] = P$ for all $i = 1, ..., n$.

a concern as the implied premium is always fair for any level of P. Recall that for a stock insurer there is a unique actuarially fair premium for each level of capital C, which is zero for $C = 0$; for a mutual insurer the implied insurance premium is fair for any level of P and C.

3.2 Optimal Loading

We now investigate the optimality of the actuarially fair premium based on policyholders' preferences. For a stock insurer the actuarially fair premium and thereby the likelihood that the company stays solvent is increasing in the amount of capital provided by shareholders. Policyholders' level of expected utility is therefore also increasing in the amount of capital. If providing capital is costless, i.e. if capital markets are perfect, then it would it optimal to have unlimited capital available, i.e. $C = +\infty$. In this case, the insurer is always solvent and full insurance is achieved at a fair premium, i.e. $P_{fair}(\infty) = E[X_1]$, as shown in Proposition 1. If providing capital is infinitely costly, then no capital will be available and there is no insurance at a fair premium, i.e. $P_{fair}(0) = 0$.

Suppose that a stock insurer's capital is limited by some level $\bar{C}$, e.g., raising capital might be costless up to $\bar{C}$ but infinitely costly beyond $\bar{C}$. If the company is insolvent, then policyholders will not be fully indemnified. Their marginal utility is therefore higher in states in which the company is insolvent compared to states in which funds are sufficient to receive full coverage. As policyholders are risk-averse they wish to transfer money from solvency-states to insolvency-states and in particular to those insolvency-states with relatively high claims.

A mechanism that might allow policyholders to do so would be for policyholders to collectively pay a loading in excess of the fair premium.[6] By doing so, policyholders reduce their wealth in solvency-states to the benefit of shareholders. At the same time, more funds are available to be distributed to policyholders in insolvency-states. Reasonable bankruptcy rules may therefore create a form of coinsurance amongst policyholders if these additional funds accrue to those policyholders

[6] Alternatively, policyholders could raise capital by buying shares in the stock insurer. This mechanism, however, is susceptible to a free rider problem which we discuss in Section 5.2.

with relatively high claims.[7] Policyholders thus trade-off the benefit of traditional insurance through higher premiums against the possibility that these funds are not used to pay claims and accrue to shareholders. In addition to this trade-off, the loading also reduces the probability of insolvency. On the one hand, this is beneficial to policyholders as they are more likely to be fully indemnified. On the other hand, a reduction of the insolvency probability has a negative effect on the trade-off described above. It is now more likely that the additional funds accrue to shareholders.

Creating this form of coinsurance in insolvency states by paying a loading seems particularly beneficial if the funds available are relatively small otherwise, i.e., if little capital is provided by shareholders. In the extreme scenario, i.e., if $C = 0$, we have shown in Proposition 1 that no insurance can be offered at an actuarially fair premium. If increasing shareholders' capital is not an option, policyholders by paying a loading would in fact initiate insurance. In the following proposition, we show that it is optimal for policyholders to pay a loading on top of the fair premium if capital provided by shareholders is small. In the other extreme scenario with unlimited capital, i.e. if $C = +\infty$, policyholders are always fully indemnified and it is thus not optimal for them to pay a loading in excess of the fair premium.

Proposition 3 *For a stock insurer, if capital provided by shareholders is small, then it is optimal for policyholders to pay a loading in excess of the actuarially fair premium. If capital provided by shareholders is large, then it is not optimal for policyholders to pay a loading in excess of the actuarially fair premium. Furthermore, if policyholders' preferences exhibit constant absolute risk aversion, then the optimal loading is decreasing in the level of capital provided by shareholders.*

Proof. See Appendix A.2. ■

This proposition implies that under constant absolute risk aversion there exists a critical threshold of capital such that is optimal for policyholders to collectively pay a loading for all levels of capital below that threshold and not to pay a loading for all levels above that threshold.

[7] Mahul and Wright (2004b) show that the Pareto-optimal mutual risk sharing contract with limited capital includes a deductible which is adjusted *ex-post* depending on realized losses to meet the capital constraint.

4 The Number of Policyholders and Financial Distress

The number of policies underwritten is important for risk sharing as it affects the probability of financial distress and thereby claimants (customers) of insurance who have to bear the risk in case of financial distress. This is true for both organizational forms, a mutual and a stock insurer with limited capital. We show in this section, however, that the effect of increasing the number of policyholders on risk sharing crucially depends on whether owners and customers are separated or whether they coincide. In fact, we show that increasing the number of policyholders causes opposite effects for a mutual and a stock insurer with fixed capital C offering insurance at an actuarially fair rate.

In case of a mutual insurer, risks are spread among policyholders only since policyholders are also the owners of the insurer. To obtain sufficient risk sharing it is thus essential that the number of policyholders is sufficiently high and that the individual risks have low correlation. For a stock insurer, risks are in principle transferred to the shareholders of the company. What is important for sufficient risk sharing are diversified shareholders, i.e., the number of shareholders has to be sufficiently high. In the extreme scenario with unlimited capital, the insurer is never insolvent and policyholders obtain full insurance. Shareholders then bear the total risk and the number of policyholders or the correlation between the individual risks per se is irrelevant for risk sharing.[8] With limited capital $\bar{C}$, the number of policyholders and the correlation between their risks do play a role. Both affect the probability of financial distress and the implied fair premium, $P_{fair}\left(n, \bar{C}\right)$, for a given level of $\bar{C}$, which in turn determines the degree to which policyholders are fully insured.

The difference between a stock insurer and a mutual insurer in this context is that the level of capital provided by shareholders is fixed in the case of a stock corporation. Increasing the customerbase thus has a negative effect on the average capital per policy. In contrast, since policyholders are also owners in the case of a mutual, selling actuarially fair insurance to policyholders is tied to ownership rights. Policyholders in a mutual provide capital and pay for the indemnity

[8] In the mutual, the number of policyholders matters because it is inseparable from the number of owners.

payment at the same time. Selling insurance policies at a fixed premium thus also raises capital and the average total funds per policy remain constant.

In the following proposition we show that if a stock insurer with limited capital offers insurance at a fair rate expanding the pool of policyholders has the opposite effect on the probability of financial distress compared to a mutual insurer: it is decreasing in the number of policyholders for a mutual but increasing for a stock insurer. The negative effect on shareholders' capital per policy thus outweighs the positive "diversification" effect and the effect on the fair premium.

Proposition 4 *Suppose that the number of policyholders is large such that the Central Limit Theorem can be applied. For a stock insurer with a given level of capital $\bar{C}$ and implied fair premium $P_{fair}\left(n, \bar{C}\right)$, the probability of financial distress is increasing in the number of policyholders. For a mutual insurer with premium $P > E[X_1]$ and $C = 0$, the probability of financial distress is decreasing in the number of policyholders.*

Proof. See Appendix A.3. ■

5 The Role of the Corporate Form for Selling Insurance, Raising Capital, and Sharing Risk

5.1 Managerial Incentives to Expand

Proposition 4 provides insights into managerial incentives to expand the number of customers under the two different organizational forms. Suppose that a manager has to exert privately costly effort to increase the number of policyholders, n. The manager derives a private benefit of control as long as the insurer is solvent and therefore has an incentive to avoid financial distress. The sequence of events is as follows in case of a stock insurer: at the beginning of the period, shareholders provide capital C, the manager then chooses his effort level, which determines n, and policyholders pay a fair premium $P_{fair}(n, C)$ given n and C. At the end of the period, policyholders incur their losses, the insurer is insolvent if $\sum_{i=1}^{n} X_i > nP_{fair}(n, C) + C$ and solvent otherwise, and total funds are distributed accordingly (see Section 2). In case of a mutual insurer, a small group of households forms a mutual at the beginning of the period and employs a manager to sell insurance policies at a premium P. At the end of the period, policyholders incur their losses, the insurer is insolvent if $\sum_{i=1}^{n} X_i > nP$ and solvent otherwise, and total funds are distributed accordingly.

For a stock insurer, where shareholders provide capital C, Proposition 4 shows that the manager has no incentives to exert effort. Indeed, selling only very few insurance policies may assure continued operation. In contrast, the manager of a mutual has high incentives to increase the number of insurance policies n with a premium $P > E[X]$, as increasing n reduces the probability of financial distress.

The differences in incentives stem from the differences in how capital is raised. For a stock insurer, a fixed amount of capital is raised first, then policies are sold. Increasing the number of policies then reduces the average capital available for each policy. This effect dominates the benefit of a reduced variance of the average claim. For a mutual, raising capital is tied to selling insurance policies. Since the total of fair premium and capital for ownership rights is constant, the benefit of reducing the variance of the average claim dominates.

One might wonder whether reversing the sequence of activities in a stock corporation would not help. For a given number of policies sold, the probability of financial distress decreases in the amount of capital raised. However, incentives to sell policies are still slim because a low number of policies nevertheless reduces the probability of financial distress for any level of capital to be raised later.

5.2 Policyholders' Incentives to Provide Capital and Risk Sharing

The tying of ownership rights and insurance policies is an important characteristic of a mutual insurer. Thereby, capital for the ownership rights and the premium for the insurance policy are raised simultaneously from the same group of individuals. Thus, risk sharing is achieved within the pool of policyholders. For a stock insurer, risk is transferred to the investors in the stock corporation. If the level of capital is high enough such that the insurer is never insolvent and if full insurance contracts are sold, risk is completely transferred to investors.

The mutuality principle states that if shareholders and insureds are risk averse, both should participate in the risk. This can be achieved either through participating insurance contracts or policyholders also being shareholders in the stock insurer. In a CAPM world, where all individuals hold the market portfolio, policyholders are also shareholders.[9] Through their shares, they participate in the insurer's risk and also in their own losses. If all households hold the market portfolio and buy full insurance from a sufficiently well capitalized stock insurer who is never insolvent, optimal risk sharing is achieved. Thus, in an efficient capital market, where raising capital from dispersed shareholders is frictionless, a stock insurer dominates a mutual insurer.

If it is costly to raise capital from a dispersed group of investors, capital may be so low that policyholders have to bear a substantial amount of risk. In this case, it may be optimal for policyholders to raise capital in order to achieve more efficient risk sharing (see Proposition 3). Capital in a stock insurance company can be raised by policyholders through two channels: increasing the premium, i.e. paying a loading, or buying shares in the stock corporation. Both ways of raising capital are costly. Loading is costly because part of the additional premium may accrue to shareholders. By assumption, raising additional capital through shares is costly as otherwise optimal risk sharing is achieved by raising unlimited capital.

As we have shown in Proposition 3, if capital is very low, policyholders prefer a premium that exhibits loading to a fair premium. This loading can equally be interpreted as additional capital raised by policyholders through shares at unfair terms, i.e., capital with a negative expected return. Providing this capital collectively is optimal for policyholders despite its negative expected return because it provides additional risk sharing which benefits policyholders. More generally, customers are willing to accept a lower return on their capital than pure investors, because customers derive a utility from the additional benefits to risk sharing. That is, there are two important aspects for policyholders when providing capital: the return on the capital and the additional risk sharing.

[9] Thus, the separation of owners and customers is generally not complete in a stock corporation, however, policyholders are generally only a small subset of owners.

In addition to this trade-off between the benefit and cost of providing additional capital, there is a crucial difference between the two ways of raising capital, i.e., loading and shares, which arises from a free rider problem. For the benefit of additional risk sharing, it is crucial that all policyholders provide the additional capital collectively since, if only one policyholder provides additional capital, then risk sharing is not improved for that policyholder. Policyholders would collectively agree to provide additional capital despite higher cost because of the additional risk sharing benefit. However, if policyholders cannot commit to providing additional capital it is individually rational for each policyholder not to invest, but instead free ride on the others' investment. While it is difficult to restrict selling policies to those who hold a minimum amount of shares in the insurer, loading and forming a mutual assures that policyholders also invest a minimum amount of additional capital.

Loading and mutual insurance can thus be interpreted as ways to tie raising capital and insurance policies to overcome this free-rider problem. In addition, loading has another positive effect. It raises the willingness of shareholders to provide additional capital as it raises the return on their investment in the insurer. In equilibrium, the cost of additional capital equals the increase in the value of shares from loading.

6 Conclusion

In this paper, we emphasize the distinction between mutual and stock insurers in organizing risk sharing. In a stock corporation, the efficiency of risk sharing is inherently linked to the level of capital provided by shareholders and the degree to which shareholders are diversified. In a mutual corporation, risks are shared between policyholders only and the efficiency of risk sharing therefore depends on the size of the pool of policyholders. In an efficient capital market, where the cost of holding capital is low and where shareholding is dispersed, risk sharing can optimally be organized through a stock insurer. This is true in particular, if policyholders' risks are highly correlated or the number of policyholders is small. If raising capital is costly, then a stock insurer or a mutual insurer may have a competitive advantage in entering the market depending on whether it is relatively easier to raise capital from a diversified group of shareholders or to sell a large amount of policies. If incentives to sell a high number of policies are important, then a mutual insurer has an advantage over a stock insurer. For a stock insurer with low level of capital, it is beneficial for policyholders to collectively raise additional, costly capital as it improves risk sharing. We argue that raising costly capital through a premium loading and thereby tying additional capital to policies overcomes the free rider problem that is inherent in raising capital through shares. Alternatively, capital provision and insurance can be tied through tying insurance and ownership as in a mutual.

Understanding this difference in degree and structure of organizing risk sharing is important for analyzing the competitive structure of the insurance industry, regulation, understanding the limits to risk sharing (uninsurable risks), and for determining insurance premiums.

A Appendix: Proofs

A.1 Proof of Proposition 1

The actuarially fair premium $P_{fair}(C)$ for a policyholder as a function of capital provided by shareholders is implicitly defined by

$$P_{fair}(C) = E\left[X_i \cdot 1_{\{\Sigma_i X_i \leq nP_{fair}(C)+C\}} + I_i(X_1,...,X_n) \cdot 1_{\{\Sigma_i X_i > nP_{fair}(C)+C\}}\right].$$

Summing over all policies yields

$$\begin{aligned} nP_{fair}(C) &= E\left[\sum_{i=1}^n X_i \cdot 1_{\{\Sigma_i X_i \leq nP_{fair}(C)+C\}} + (nP_{fair}(C)+C) \cdot 1_{\{\Sigma_i X_i > nP_{fair}(C)+C\}}\right] \\ &= \int_0^{nP_{fair}(C)+C} x dF^n(x) + (nP_{fair}(C)+C)(1-F^n(nP_{fair}(C)+C)), \end{aligned} \tag{1}$$

where F^n is the n-fold convolution of F^1 and thus the distribution function of the aggregate loss $\sum_{i=1}^n X_i$. For $C=0$ we have

$$nP_{fair}(0) = \int_0^{nP_{fair}(0)} x dF^n(x) + nP_{fair}(0)(1-F^n(nP_{fair}(0)))$$

which is satisfied for $P_{fair}(0)=0$. For any $P_{fair}(0)>0$ we deduce

$$\int_0^{nP_{fair}(0)} x dF^n(x) + nP_{fair}(0)(1-F^n(nP_{fair}(0))) < nP_{fair}(0).$$

$P_{fair}(0) = 0$ is therefore the unique solution to (1).
For $C = +\infty$, the company is never insolvent and the actuarially fair premium is given by $P_{fair}(\infty) = E[X_1]$.
For any $0 < C < \infty$, define the expected loss for all policyholders from each paying a premium P as

$$f(P) = nP - \int_0^{nP+C} x dF^n(x) - (nP + C)(1 - F^n(nP + C)).$$

The actuarially fair premium $P_{fair}(C)$ is thus characterized by

$$f(P_{fair}(C)) = 0.$$

We have

$$f(0) = -\int_0^C x dF^n(x) - C(1 - F^n(C)) < 0$$

and

$$f(\infty) = \infty$$

for all $0 < C < \infty$. Furthermore

$$f'(P) = n - n(1 - F^n(nP + C)) = nF^n(nP + C) > 0.$$

As $f(\cdot)$ is a continuous function in P the intermediate value theorem implies that there exist a unique solution $P_{fair}(C) > 0$ to $f(P_{fair}(C)) = 0$.
Implicitly differentiating (1) with respect to C yields

$$nP'_{fair}(C) = \left(nP'_{fair}(C) + 1\right)(1 - F^n(nP_{fair}(C) + C))$$

which implies

$$P'_{fair}(C) = \frac{1}{n}\frac{1 - F^n(nP_{fair}(C) + C)}{F^n(nP_{fair}(C) + C)} > 0$$

for all $C > 0$. The actuarially fair premium is thus strictly increasing in the amount of risk capital.

A.2 Proof of Proposition 3

Suppose that the insolvency rule specifies a pro-rata rule, i.e.

$$I_i(X_1, ..., X_n) = \frac{X_i}{\sum_{i=1}^n X_i}\left(nP + \bar{C}\right),$$

and let Δ denote the loading in excess of the fair premium.[10] The final level of wealth of a policyholder, policyholder 1 let's say, is then given by

$$W_1(\Delta) = \begin{cases} w_S(\Delta) = w_0 - P - \Delta & \text{if } \sum_{i=1}^n X_i \leq n(P + \Delta) + \bar{C} \\ w_{IS}(\Delta) = w_0 - P - \Delta - X_1 + \frac{X_1}{\sum_{i=1}^n X_i}\left(n(P + \Delta) + \bar{C}\right) & \text{if } \sum_{i=1}^n X_i > n(P + \Delta) + \bar{C} \end{cases}.$$

[10] For expositional purposes, we focus on the pro-rata rule as bankruptcy rule. The results, however, are robust to any "reasonable" bankruptcy rule that allow to create the form of coinsurance decsribed above. More precisely, the bankruptcy rule must be such that the marginal benefit of an extra dollar under bankruptcy is increasing in the realized size of the loss.

where $w_S(\Delta)$ and $w_{IS}(\Delta)$ are the levels of final wealth in solvency- and insolvency-states respectively. The fair premium $P = P_{fair}(\bar{C})$ is implicitly defined by (1). The policyholder's expected utility of final wealth is given by

$$\begin{aligned}
& E\left[u\left(W_1(\Delta)\right)\right] \\
= \;& u\left(w_S(\Delta)\right) F^n\left(n(P+\Delta)+\bar{C}\right) + \int_0^\infty u\left(w_{IS}(\Delta)\right) dF^n\left(\textstyle\sum_{i=1}^n x_i\right) \\
= \;& u\left(w_S(\Delta)\right) F^n\left(n(P+\Delta)+\bar{C}\right) + \int_0^\infty \int_{n(P+\Delta)+\bar{C}-x_1}^\infty u\left(w_{IS}(\Delta)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right),
\end{aligned}$$

where $x_{-1} = \sum_{i=2}^n x_i$. Differentiating expected utility with respect to Δ yields

$$\begin{aligned}
\frac{\partial E\left[u\left(W_1(\Delta)\right)\right]}{\partial \Delta} = \;& -u'\left(w_S(\Delta)\right) F^n\left(n(P+\Delta)+\bar{C}\right) \\
& + \int_0^\infty \int_{n(P+\Delta)+\bar{C}-x_1}^\infty \left(-1+\frac{n x_1}{x_1+x_{-1}}\right) u'\left(w_{IS}(\Delta)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right).
\end{aligned}$$

The second derivative is given by

$$\begin{aligned}
\frac{\partial^2 E\left[u\left(W_1(\Delta)\right)\right]}{\partial \Delta^2} = \;& u''\left(w_S(\Delta)\right) F^n\left(n(P+\Delta)+\bar{C}\right) - n u'\left(w_S(\Delta)\right) f^n\left(n(P+\Delta)+\bar{C}\right) \\
& + \int_0^\infty \int_{n(P+\Delta)+\bar{C}-x_1}^\infty \left(-1+\frac{n x_1}{x_1+x_{-1}}\right)^2 u''\left(w_{IS}(\Delta)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right) \\
& - n u'\left(w_S(\Delta)\right) \int_0^\infty \left(-1+\frac{n x_1}{n(P+\Delta)+\bar{C}}\right) f^{n-1}\left(n(P+\Delta)+\bar{C}-x_1\right) dF^1\left(x_1\right) \\
= \;& u''\left(w_S(\Delta)\right) F^n\left(n(P+\Delta)+\bar{C}\right) \\
& - \frac{n^2}{n(P+\Delta)+\bar{C}} u'\left(w_S(\Delta)\right) \int_0^\infty x_1 f^{n-1}\left(n(P+\Delta)+\bar{C}-x_1\right) dF^1\left(x_1\right) \\
& + \int_0^\infty \int_{n(P+\Delta)+\bar{C}-x_1}^\infty \left(-1+\frac{n x_1}{x_1+x_{-1}}\right)^2 u''\left(w_{IS}(\Delta)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right) \\
< \;& 0.
\end{aligned}$$

Expected utility is thus globally concave in Δ and any inner solution $\Delta^*\left(\bar{C}\right)$ to the FOC

$$\frac{\partial E\left[u\left(W_1(\Delta)\right)\right]}{\partial \Delta}\Big|_{\Delta=\Delta^*(\bar{C})} = 0 \tag{2}$$

is therefore the unique global maximum. For $\bar{C} = \infty$, we have $P_{fair}(\infty) = E[X_1]$ and the first derivative is given by

$$\frac{\partial E\left[u\left(W_1(\Delta)\right)\right]}{\partial \Delta}\Big|_{\bar{C}=\infty} = -u'\left(w_0 - E[X_1] - \Delta\right) < 0.$$

As expected utility is decreasing in Δ, we get the corner solution $\Delta^*(\infty) = 0$.[11] As $\Delta^*\left(\bar{C}\right)$ is continuous in $\bar{C}$, $\Delta^*\left(\bar{C}\right) = 0$ for large values of $\bar{C}$. For $\bar{C} = 0$, we have $P_{fair}(0) = 0$ (Proposition 1) and the first

[11] The participation constraint for risk-neutral shareholders providing capital imposes $\Delta \geq 0$.

derivative is given by

$$\begin{aligned}
& \frac{\partial E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial\Delta}|_{\bar{C}=0} \\
= \; & -u'\left(w_0-\Delta\right)F^n\left(n\Delta\right) \\
& +\int_0^\infty\int_{n\Delta-x_1}^\infty\left(-1+\frac{nx_1}{x_1+x_{-1}}\right)u'\left(w_0-\Delta-x_1\left(1-\frac{1}{x_1+x_{-1}}\left(n\Delta\right)\right)\right)dF^{n-1}\left(x_{-1}\right)dF^1\left(x_1\right).
\end{aligned}$$

Evaluating this derivative at $\Delta=0$ yields

$$\begin{aligned}
\frac{\partial E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial\Delta}|_{\bar{C}=0,\Delta=0} &= \int_0^\infty\int_0^\infty\left(-1+\frac{nx_1}{x_1+x_{-1}}\right)u'\left(w_0-x_1\right)dF^{n-1}\left(x_{-1}\right)dF^1\left(x_1\right) \\
&= E\left[u'\left(w_0-X_1\right)\left(-1+\frac{nX_1}{X_1+X_{-1}}\right)\right] \\
&= Cov\left(u'\left(w_0-X_1\right),\frac{nX_1}{X_1+X_{-1}}\right)+E\left[u'\left(w_0-X_1\right)\right]E\left[-1+\frac{nX_1}{X_1+X_{-1}}\right].
\end{aligned}$$

We have

$$\begin{aligned}
E\left[-1+\frac{nX_1}{X_1+X_{-1}}\right] &= -1+nE\left[\frac{X_1}{\Sigma_{i=1}^n X_i}\right] \\
&= -1+\sum_{i=1}^n E\left[\frac{X_i}{\Sigma_{i=1}^n X_i}\right] \\
&= 0
\end{aligned}$$

and therefore

$$\frac{\partial E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial\Delta}|_{\bar{C}=0,\Delta=0}=Cov\left(u'\left(w_0-X_1\right),\frac{nX_1}{X_1+X_{-1}}\right)>0.$$

This implies $\Delta^*(0)>0$. Again, as $\Delta^*\left(\bar{C}\right)$ is continuous in $\bar{C}$, $\Delta^*\left(\bar{C}\right)>0$ for small values of $\bar{C}$. Total differentiation of the FOC (1) with respect to $\bar{C}$ and Δ implies

$$\frac{d\Delta^*\left(\bar{C}\right)}{d\bar{C}}=-\frac{\frac{\partial^2 E[u(W_1(\Delta))]}{\partial\Delta\partial\bar{C}}|_{\Delta=\Delta^*\left(\bar{C}\right)}}{\frac{\partial^2 E[u(W_1(\Delta))]}{\partial^2\Delta}|_{\Delta=\Delta^*\left(\bar{C}\right)}}.$$

As expected utility is globally concave in Δ we derive

$$sign\left(\frac{d\Delta^*\left(\bar{C}\right)}{d\bar{C}}\right)=sign\left(\frac{\partial^2 E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial\Delta\partial\bar{C}}|_{\Delta=\Delta^*\left(\bar{C}\right)}\right). \tag{3}$$

Recall that $P = P_{fair}\left(\bar{C}\right)$, i.e.

$$\begin{aligned}
& \frac{\partial E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial \Delta} \\
= & -u'\left(w_0 - P_{fair}\left(\bar{C}\right) - \Delta\right) F^n\left(n\left(P_{fair}\left(\bar{C}\right) + \Delta\right) + \bar{C}\right) \\
& + \int_0^\infty \int_{n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)+\bar{C}-x_1}^\infty \left(-1 + \frac{n x_1}{x_1 + x_{-1}}\right) \\
& \quad \cdot u'\left(w_0 - P_{fair}\left(\bar{C}\right) - \Delta - x_1\left(1 - \frac{1}{x_1 + x_{-1}}\left(n\left(P_{fair}\left(\bar{C}\right) + \Delta\right) + \bar{C}\right)\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right).
\end{aligned}$$

The cross-derivative is then given by

$$\begin{aligned}
& \frac{\partial^2 E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial \Delta \partial \bar{C}} \\
= & P'_{fair}\left(\bar{C}\right) u''\left(w_S\left(\Delta\right)\right) F^n\left(n\left(P_{fair}\left(\bar{C}\right) + \Delta\right) + \bar{C}\right) \\
& - \left(n P'_{fair}\left(\bar{C}\right) + 1\right) u'\left(w_S\left(\Delta\right)\right) f^n\left(n\left(P_{fair}\left(\bar{C}\right) + \Delta\right) + \bar{C}\right) \\
& + \int_0^\infty \int_{n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)\ +\bar{C}-x_1}^\infty \left(-1 + \frac{n x_1}{x_1 + x_{-1}}\right) \\
& \quad \cdot \left(-P'_{fair}\left(\bar{C}\right) + \frac{x_1}{x_1 + x_{-1}}\left(n P'_{fair}\left(\bar{C}\right) + 1\right)\right) u''\left(w_{IS}\left(\Delta\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right) \\
& - \left(n P'_{fair}\left(\bar{C}\right) + 1\right) u'\left(w_S\left(\Delta\right)\right) \\
& \quad \cdot \int_0^\infty \left(-1 + \frac{n x_1}{n\left(P_{fair}\left(\bar{C}\right) + \Delta\right) + \bar{C}}\right) f^{n-1}\left(n\left(P_{fair}\left(\bar{C}\right) + \Delta\right) + \bar{C} - x_1\right) dF^1\left(x_1\right) \\
= & P'_{fair}\left(\bar{C}\right) u''\left(w_S\left(\Delta\right)\right) F^n\left(n\left(P_{fair}\left(\bar{C}\right) + \Delta\right) + \bar{C}\right) \\
& + \int_0^\infty \int_{n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)\ +\bar{C}-x_1}^\infty \left(-1 + \frac{n x_1}{x_1 + x_{-1}}\right) \\
& \quad \cdot \left(-P'_{fair}\left(\bar{C}\right) + \frac{x_1}{x_1 + x_{-1}}\left(n P'_{fair}\left(\bar{C}\right) + 1\right)\right) u''\left(w_{IS}\left(\Delta\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right) \\
& - \frac{n\left(n P'_{fair}\left(\bar{C}\right) + 1\right)}{n\left(P_{fair}\left(\bar{C}\right) + \Delta\right) + \bar{C}} u'\left(w_S\left(\Delta\right)\right) \int_0^\infty x_1 f^{n-1}\left(n\left(P_{fair}\left(\bar{C}\right) + \Delta\right) + \bar{C} - x_1\right) dF^1\left(x_1\right).
\end{aligned}$$

In Proposition 1, we have shown that $P'_{fair}\left(\bar{C}\right) > 0$ which implies

$$\begin{aligned}
\frac{\partial^2 E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial \Delta \partial \bar{C}} < & \int_0^\infty \int_{n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)\ +\bar{C}-x_1}^\infty \left(-1 + \frac{n x_1}{x_1 + x_{-1}}\right) \\
& \cdot \left(-P'_{fair}\left(\bar{C}\right) + \frac{x_1}{x_1 + x_{-1}}\left(n P'_{fair}\left(\bar{C}\right) + 1\right)\right) u''\left(w_{IS}\left(\Delta\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right).
\end{aligned}$$

Introducing the constant coefficient of absolute risk aversion $R_a = -\frac{u''(w)}{u'(w)}$ yields

$$\begin{aligned}\frac{\partial^2 E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial\Delta\partial\bar{C}} &< -R_a \int_0^\infty \int_{n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)+\bar{C}-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) \\ &\quad \cdot \left(-P'_{fair}\left(\bar{C}\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(\bar{C}\right)+1\right)\right) u'\left(w_{IS}\left(\Delta\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right).\end{aligned}$$

For $-1+\frac{nx_1}{x_1+x_{-1}} > 0$ we have

$$-P'_{fair}\left(\bar{C}\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(\bar{C}\right)+1\right) > \frac{1}{n}$$

and thus

$$-\left(-1+\frac{nx_1}{x_1+x_{-1}}\right)\left(-P'_{fair}\left(\bar{C}\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(\bar{C}\right)+1\right)\right) < -\frac{1}{n}\left(-1+\frac{nx_1}{x_1+x_{-1}}\right)$$

For $-1+\frac{nx_1}{x_1+x_{-1}} < 0$ we have

$$-P'_{fair}\left(\bar{C}\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(\bar{C}\right)+1\right) < \frac{1}{n}$$

and thus

$$-\left(-1+\frac{nx_1}{x_1+x_{-1}}\right)\left(-P'_{fair}\left(\bar{C}\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(\bar{C}\right)+1\right)\right) < -\frac{1}{n}\left(-1+\frac{nx_1}{x_1+x_{-1}}\right).$$

This implies

$$\frac{\partial^2 E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial\Delta\partial\bar{C}} < -R_a\frac{1}{n}\int_0^\infty \int_{n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)+\bar{C}-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) u'\left(w_{IS}\left(\Delta\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right).$$

The FOC (2) for $\Delta^*\left(\bar{C}\right)$ implies

$$\begin{aligned}&\int_0^\infty \int_{n\left(P+\Delta^*\left(\bar{C}\right)\right)+\bar{C}-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) u'\left(w_{IS}\left(\Delta^*\left(\bar{C}\right)\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right) \\ =\;& u'\left(w_S\left(\Delta^*\left(\bar{C}\right)\right)\right) F^n\left(n\left(P+\Delta^*\left(\bar{C}\right)\right)+\bar{C}\right)\end{aligned}$$

and therefore

$$\begin{aligned}\frac{\partial^2 E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial\Delta\partial\bar{C}}\Big|_{\Delta=\Delta^*\left(\bar{C}\right)} &< -R_a\frac{1}{n}u'\left(w_S\left(\Delta^*\left(\bar{C}\right)\right)\right) F^n\left(n\left(P_{fair}\left(\bar{C}\right)+\Delta^*\left(\bar{C}\right)\right)+\bar{C}\right) \\ &< 0.\end{aligned}$$

Finally, (3) implies

$$\frac{d\Delta^*\left(\bar{C}\right)}{d\bar{C}} < 0.$$

A.3 Proof of Proposition 4

For a stock insurer with limited capital $\bar{C}$ and implied fair premium $P_{fair}\left(n,\bar{C}\right)$, the probability of financial distress is given by

$$prob\left(\sum_{i=1}^{n} X_i > nP_{fair}\left(n,\bar{C}\right)+\bar{C}\right).$$

Applying the Central Limit Theorem (CLT) yields

$$prob\left(\sum_{i=1}^{n} X_i > nP_{fair}\left(n,\bar{C}\right)+\bar{C}\right) = prob\left(Z > z\left(n\right)\right)$$

where Z is standard normally distributed and

$$z\left(n\right) = \frac{\sqrt{n}}{\sigma\left(X_1\right)}\left(P_{fair}\left(n,\bar{C}\right) - E\left[X_1\right]\right) + \frac{1}{\sqrt{n}\sigma\left(X_1\right)}\bar{C}.$$

This implies that

$$sign\left(\frac{\partial}{\partial n} prob\left(\sum_{i=1}^{n} X_i > nP_{fair}\left(n,\bar{C}\right)+\bar{C}\right)\right) = -sign\left(z'\left(n\right)\right). \tag{4}$$

Recall equation (1) which defines $P_{fair}\left(n,\bar{C}\right)$ through

$$nP_{fair}\left(n,\bar{C}\right) = E\left[\sum_{i=1}^{n} X_i \cdot 1_{\left\{\Sigma_i X_i \le nP_{fair}\left(n,\bar{C}\right)+\bar{C}\right\}} + \left(nP_{fair}\left(n,\bar{C}\right)+\bar{C}\right)\cdot 1_{\left\{\Sigma_i X_i > nP_{fair}\left(n,\bar{C}\right)+\bar{C}\right\}}\right].$$

Applying the CLT to this equation yields

$$z\left(n\right) - \frac{1}{\sqrt{n}\sigma\left(X_1\right)}\bar{C} = \int_{-\infty}^{z(n)} z dN\left(z\right) + z\left(n\right)\left(1 - N\left(z\left(n\right)\right)\right),$$

where $N\left(\cdot\right)$ is the cumulative distribution function of the standard normal distribution. By implicitly differentiating with respect to n we deduce

$$z'\left(n\right) + \frac{1}{2n\sqrt{n}\sigma\left(X_1\right)}\bar{C} = z'\left(n\right)\left(1 - F\left(z\left(n\right)\right)\right)$$

which implies

$$z'\left(n\right) = -\frac{1}{2n\sqrt{n}\sigma\left(X_1\right)F\left(z\left(n\right)\right)}\bar{C} < 0.$$

Equation (4) then proves

$$\frac{\partial}{\partial n} prob\left(\sum_{i=1}^{n} X_i > nP_{fair}\left(n,\bar{C}\right)+\bar{C}\right) > 0.$$

For a mutual insurer with fixed premium $P > E\left[X_1\right]$ and $C = 0$, the probability of financial distress is given by

$$prob\left(\sum_{i=1}^{n} X_i > nP\right).$$

Applying the CLT yields

$$prob\left(\sum_{i=1}^{n} X_i > nP\right) = prob\left(Z > \frac{\sqrt{n}}{\sigma\left(X_1\right)}\left(P - E\left[X_1\right]\right)\right),$$

which implies

$$\frac{\partial}{\partial n} prob\left(\sum_{i=1}^{n} X_i > nP\right) < 0.$$

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