Life Annuity Insurance versus Self-Annuitization: An Analysis from the Perspective of the Family

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ABSTRACT

When comparing investment in an immediate life annuity with a payout-equivalent investment fund decumulation plan (self-annuitization), previous research focused on shortfall probabilities of self-annuitization. Chances of self-annuitization (i.e., bequests) typically have not been addressed. We argue that heirs might be willing to bear the shortfall risk of the retiree's self-annuitization since they might benefit from a bequest. Our article proposes a "family strategy" in which heirs receive the remaining investment fund on the retiree's death, but are obliged to finance the retiree if the fund becomes exhausted. We estimate the chance and risk profile of this "family strategy" from the heirs' perspective using German capital and annuity market data. We show that in many cases, our "family strategy" offers enormous chance potential with low shortfall risk. Finally, we discuss some limitations of the proposed "family strategy" when putting the concept into practice.

Keywords:

Annuity Market

Self-Annuitization

Individual Risk Management and Retirement

Longevity Risk

Intertemporal Consumption Model

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INTRODUCTION

Government-organized pay-as-you-go pension schemes face serious challenges in most Western countries. The need for additional private savings and retirement vehicles has produced much fruitful research in the past few years (see, in particular, Blake et al., 2003; Mitchell et al., 1999). In the following we focus on the decumulation phase after retirement and consider two alternatives: the purchase of an immediate life annuity or self-annuitization. The life annuity gives the retiree a constant income stream as long as he or she lives. When self-annuitizing, the retiree invests his or her money in an investment fund and periodically withdraws money to finance needs. This gives the retiree more flexibility in structuring his or her consumption stream, but exposes the retiree to individual longevity risk. Thus, on the one hand, the retiree may outlive his or her financial resources by living longer than expected and/or by poor mutual investment fund performance (shortfall risk). On the other hand, the retiree may be able to bequeath substantial wealth.

Annuities offered by insurance companies are usually priced to cover operating costs and costs of adverse selection[1] (Mitchell et al., 1999). This led several authors to explore "beat the annuity" strategies (Milevsky et al., 1997; Milevsky, 1998; Milevsky and Robinson, 2000; Albrecht and Maurer, 2002; Milevsky and Young, 2003). These authors demonstrated that, dependent on asset allocation or annuity purchase age, it is possible to minimize but never to fully eliminate shortfall risk. In other words—and this is not surprising—there is no easy arbitrage possibility in the annuity market.[2] From a retiree's viewpoint, knowing the shortfall risk magnitude is not very helpful in making in-

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1 People who buy life annuities tend to live longer than average people. This forces insurance providers to raise prices, making life-annuity insurance expensive for people with average or below-average life expectancy.

2 If financial instruments that duplicate individual mortality were available, other arbitrage possibilities could emerge (see Charupat and Milevsky, 2001; Richter and Russ, 2002).

vestment decisions. Analogizing to a lottery, this is like giving someone the probabilities of winning and losing, but providing no information about possible gains or losses, and then asking the person whether he or she would like to gamble on that lottery. However, it is not easy to clearly see the entire chance and risk profile of the self-annuitization strategy, because information about the utility of the bequest may be necessary to evaluate the chances. Unfortunately, the literature does not provide much information about the strength of bequest motives or how such bequest motives could be modeled (see, in particular, Bernheim et al., 1985; Brown, 2001).

Therefore, in what follows we will avoid assumptions about the utility of bequest for the retiree. Our argument is that if heirs receive all benefits in the case of self-annuitization (the bequest), they might be willing to bear all risks as well. Hence, our strategy—the *family strategy*—is based on an agreement (or contract) between the retiree and his or her heirs: Heirs receive the remaining investment fund on the retiree's death, but they are obligated to finance the retiree if the fund becomes exhausted. We model the family strategy in such a way that the retiree is never put in a position worse than he or she would be with a life annuity.

Our intra-family annuity provision does not simply shift the life annuity/self-annuitization decision from the retiree to the heirs. In the case of multiple heirs, the possible shortfall will be less severe since they will have pooled their individual longevity risks (Kotlikoff and Spivak, 1981). Of course, anybody could underwrite the retiree's shortfall risk, but outsiders, for example, insurers, typically confront the problem of adverse selection and therefore demand a risk premium. Inside the family, information about the retiree's health will be fairly exact, which enables the family to save the insurer's cost of adverse selection (Kotlikoff and Spivak, 1981) and other transaction costs.

The paper is organised as follows. In the next section we describe the model and the data from the German capital and annuity market. Then, simulation results based on different strategies are presented, and some limitations of our approach are discussed. In conclusion, we summarize our findings and make some suggestions for future research.

SIMULATION MODEL AND INPUT DATA

Our example involves either a 65-, 75-, or 80-year-old retiree who has wealth (W_0) of € 100,000. He or she is subject to a marginal tax rate of either 0% or 36%. The benchmark investment is an immediate life annuity that pays a nominal constant amount (A) at the beginning of each year.[3] For our calculations we use offers sold in Germany by the Standard Life Insurance Company. For € 100,000, Standard Life offers for a 65-year-old male an annual payout A of € 6,421 (0% marginal tax rate) or € 5,797 (36% marginal tax rate).[4/5]

Let the initial wealth W_0 be invested in an investment fund and the amount A withdrawn every year as long as the retiree lives. The investment fund consists of two index funds: one is based on a German stock index (DAX) and the other is based on a German bond index (REXP). Table 1 gives the estimated returns, standard deviations, and correlations of the two indices.[6]

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3 The life annuity contains no further guarantees, e.g., there will be no payments to the heirs when the bequeather dies.

4 We would have liked to use broader market data for annuity payouts, but in Germany almost all products are participating (with-profit) annuities. To make our results comparable to previous research (see, in particular, Milevsky et al., 1997), we decided to use the Standard Life offer, which we discovered when we were looking for constant annuities. The annuity payout after tax is calculated according to German income tax law. The taxation depends on the annuitant's age in the year of the first payout of the annuity. If the annuitant is 65 years old, then 27% of every future payout is treated as taxable income. Hence, in the case of a 65-year-old male retiree with a marginal tax rate of 36%, one gets € 6,421 · (1 – 0.27 · 0.36) ≈ € 5,797. If he or she is 75 (80) years old, 16% (11%) is treated as taxable income.

5 The Standard & Poor's rating for the Standard Life Insurance Company is A+ (as of 2004). Thus we decided to omit the possibility of the insurer default in our simulation model.

6 We are greatly indebted to Professor R. Stehle, Ph.D., Chair of Banking and Stock Exchanges, Humboldt-Universität zu Berlin (Germany), for providing us with the time series of the German stock index (DAX) and the German bond index (REXP). Our estimation in Table 1 is based on annual data from 1953 to 2003. The performance of the DAX and REXP after tax is calculated according to German income tax law. The estimates reported here stand for a representative investor with either 0% or 36% marginal tax rate. The method used for the calculation is described in Stehle and Grewe, 2001. It incorporates the different tax treatment of capital gains, dividends, and interest as well as other specific German taxation issues. In general, according to the German income tax law, interest and dividends are subject to taxation whereas capital gains are not taxed.

TABLE 1
Expected continuous annual return $E(r_i)$ of stocks (DAX) and bonds (REXP), annual return standard deviation $\sigma(r_i)$, and correlation $\rho(r_i, r_j)$ depending on marginal tax rate

Marginal tax rate 0%						
DAX:	$E(r_1)$	=	10.46%	$\sigma(r_1)$	=	25.66%
REXP:	$E(r_2)$	=	6.55%	$\sigma(r_2)$	=	4.31%
Correlation:	$\rho(r_1, r_2)$	=	0.214			
Marginal tax rate 36%						
DAX:	$E(r_1)$	=	9.17%	$\sigma(r_1)$	=	25.64%
REXP:	$E(r_2)$	=	4.21%	$\sigma(r_2)$	=	4.12%
Correlation:	$\rho(r_1, r_2)$	=	0.196			

Over time (measured in years), the investment fund value W_t evolves according to the following formula:

$$W_t = [(1-f) \cdot W_{t-1} - I_{t-1} \cdot A] \cdot R . \qquad (1)$$

The variable f denotes annual management fees of 0.6% p.a. (paid upfront).[7] I_{t-1} stands for an indicator variable that takes the value 1 (0) if the retiree is at time t-1 alive (not alive). This variable is modeled using a cohort life-table that reflects expected average population mortality in Germany (see Schmithals and Schütz, 1995). We assume that there are no dependencies between mortality and the returns of the investment funds (see Milevsky et al., 1997). The investment fund's return R depends on the chosen composition α (with $0 \le \alpha \le 100\%$) of stocks and bonds. As in Milevsky et al., 1997, the stocks and bonds follow a geometric Brownian motion (see, e.g., Hull, 2003). Hence, given a constant asset allocation that is rebalanced annually, one gets for the investment fund's return:

$$R = \alpha \cdot e^{r_1} + (1-\alpha) \cdot e^{r_2} , \qquad (2)$$

with normal distributed returns r_1 (stocks) and r_2 (bonds) from Table 1.

[7] After consulting various online brokers, we found that this annual management fee is the competitive price (as of 2004).

In the case of an exhausted investment fund (e.g., $W_t < A$), the heirs are required to provide amount A out of their own pockets. In our model, we assume that if $W_t < A$, the heirs will buy a life annuity that gives the retiree a payout of A. This works as a sort of loss limit for the heirs. Since annuities become cheaper with age, the maximum loss for the heirs will always be less than € 100,000.[8] Future annuity prices are calculated on an actuarial basis with data from the German annuity market.[9]

Measuring the bequest is not easy. Bequests occur at different points of time. So as to make them comparable, we decided to compound them to an identical point in time. We take the maximum age of the retiree (111 years, according to the German life-table) as the point of comparison. The price of the annuity that the heirs must buy in case of shortfall (think of it as a negative bequest) is also compounded to this point. Both (real) bequests and losses (negative bequests) are compounded using the investment fund's return R. The compounded losses can be interpreted as opportunity costs. If the heirs had not been obliged to buy the annuity, they could have placed that money into an investment fund with return R.

SIMULATION RESULTS AND DISCUSSION

The following section is divided into six subsections. First, we look at our base family strategy scenario, which involves a 65-year-old retiree. Second,

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8 To make sure that heirs are able to pay for the annuity if $W_t < A$, they must have some collateral. We take a charge of € 337,27 at the beginning of the family strategy. This amount reflects the transaction costs of registering a cautionary mortgage in Germany (as of 2004). Hence, the fact that heirs sometimes die before their parents can be excluded from our analysis.

9 On the basis of the German annuitant life-table (Schmithals and Schütz, 1995), Standard Life Insurance Company's offer leads to a certain internal rate of return (IRR). For example, an immediate life annuity offered by Standard Life for a 65-year-old male with a premium of € 100,000 and annual payments A of € 6,421 leads to an IRR of 2.76%. We used the IRR and the German annuitant life-table to calculate all future annuity premiums. This life-table (as well as the life-table used for simulating the survival process of the retiree) is a system of cohort life-tables. Survival probabilities thereby depend on both the age and the year of birth (cohort) of the annuitant. We also took into account the fact that, according to German income tax law, the proportion of taxable income of annuity payouts decreases with age (as explained in footnote 4).

we show how the results change when the family strategy involves an older retiree, either 75 or 80 years old. Following, we look at the results for a 65-year-old retiree with an "alternative strategy"—a self-annuitization strategy in which an additional change to a life annuity is made whenever the market value of the investment fund exceeds the premium for the annuity. In the fourth subsection we show simulation results for a 65-year-old retiree having a higher than average life expectancy. Next, we present simulation outcomes for a 65-year-old retiree with average life expectancy in a case where future annuity prices are not by assumption deterministic. In all sections we distinguish male from female retirees. Except for the last two subsections, we will also differentiate cases where the marginal tax rate is either 0% or 36% for both the retiree and heirs. In the final subsection we discuss the main characteristics and limitations of the family strategy.

65-year-old retiree

Let us first look at a 65-year-old retiree with a marginal tax rate of 0%. Using the offer of the Standard Life Insurance Company, the benchmark investment—an immediate life annuity with a premium of € 100,000—leads to an annual payment A of € 6,421 (male) and € 5,607 (female). As mentioned above, we take the maximum age (111 years) of the retiree given by the life-table as the point of comparison. Hence, the chance and risk profile of the family strategy is given by the wealth distribution after 46 years of investment (111 years minus the current age of the retiree). Table 2 illustrates the wealth distribution for the heirs based on a Latin Hypercube simulation (McKay et al., 1979).[10]

[10] All simulations in this article are based on 100,000 iterations. To ensure that the simulation results can be accurately compared with each other, we used the same sequence of random numbers for all simulations.

TABLE 2

Statistical figures of the wealth distribution for the family strategy for a 65-year-old retiree with a marginal tax rate of 0%

	Proportion α of investment in stocks					
	0%	20%	40%	60%	80%	100%
Male, 65						
Mean	532.1	1,251.2	2,752.9	5,810.1	11,905.0	23,853.2
Std	446.3	1,053.2	3,441.9	11,677.0	39,534.1	133,241.0
LPM_0	5.51%	2.85%	4.50%	7.21%	10.38%	14.03%
LPM_1	7.3	6.6	23.0	77.8	235.2	657.4
Female, 65						
Mean	531.0	1,276.2	2,843.0	6,040.8	12,424.9	24,955.2
Std	373.1	983.5	3,464.3	12,036.2	41,078.3	138,766.7
LPM_0	1.52%	1.08%	2.89%	5.67%	9.40%	13.51%
LPM_1	1.3	1.7	11.0	48.8	168.8	505.1

Mean in thousand € (T€); Std = standard deviation in T€; LPM_0 = lower partial moment 0; LPM_1 = lower partial moment 1 in T€[11]

Table 2 shows that both mean and standard deviation of the wealth distribution increase steadily with higher proportions of stocks in the portfolio. The highest expected value of the wealth distribution (approximately € 25 million) is found for a female retiree when the capital is completely invested in stocks. LPM_0 denotes the probability that the family strategy is inferior to investing in an immediate life annuity. This probability is around 14% with a 100% stock investment. The smallest risk of the family strategy measured by LPM_0 is found with a 80% bond investment. In that case, LPM_0 is less than 2.9% (male) and 1.1% (female). The smallest risk measured by LPM_1 occurs with a 80% investment in bonds (male) and a 100% investment in bonds (female). The family strategy is more profitable (except at 100% bond investment) and less risky for female retirees, even so the annuities for male and female retirees are almost equally priced (when using the internal rate of return (IRR) concept).[12] This is because the expected payout stream differs: Female retirees receive lower annual payouts than male retirees, but for longer time. Since the volume of the investment funds for females is on average higher, they benefit

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11 Given a random variable X, LPM_0 (or shortfall risk) is defined as $\mathrm{Prob}(X<0)$. LPM_1 is given by $E\left[\max(0-X,0)\right]$.

12 Based on the German annuitant life-table, Standard Life Insurance Company's offer for a 65-year-old male retiree leads to an IRR of 2.76% (see also footnote 9). Given the offer for a 65-year-old female retiree, one gets an IRR of 2.79%.

on the difference between the (on average higher) return of the investment fund and the return of the annuity more intensively than do male retirees. Table 2 clearly shows the attractiveness of the family strategy and we can see that leaving out these chances when hedging longevity risk via a life annuity is in general expensive for people with average life expectancy.

In our example, measurement of the wealth distribution takes place 46 years from now. Therefore, to get a better idea about the expected value of this wealth distribution it makes sense to adjust the values given in Table 2 for inflation. If we assume, for instance, a constant inflation rate of 2% p.a., the expected value of the wealth distribution in real terms is given by multiplying the expected values given in Table 2 by 0.422 ($\approx 1.02^{-46}$). For example, in the case where the entire investment is in bonds (stocks), the expected value of the family strategy in real terms is roughly € 0.2 million (€ 10 million).

The results for the case of a marginal tax rate of 36% are shown in Table 3 (again in nominal terms).

TABLE 3

Statistical figures of the wealth distribution for the family strategy for a 65-year-old retiree with a marginal tax rate of 36%

	Proportion α of investment in stocks					
	0%	20%	40%	60%	80%	100%
Male, 65						
Mean	146.4	431.3	1,107.7	2,654.2	6,083.5	13,511.2
Std	177.2	417.6	1,453.4	5,443.9	20,334.3	75,124.7
LPM_0	18.95%	8.03%	7.48%	8.95%	11.32%	14.14%
LPM_1	14.7	9.7	19.4	51.1	139.8	377.6
Female, 65						
Mean	135.5	425.3	1,122.8	2,725.8	6,291.3	14,025.7
Std	147.8	380.3	1,445.1	5,576.2	21,032.2	77,921.1
LPM_0	14.19%	4.99%	5.70%	7.98%	10.83%	14.17%
LPM_1	7.6	4.4	11.4	36.1	107.8	306.0

Mean in thousand € (T€); Std = standard deviation in T€; LPM_0 = lower partial moment 0; LPM_1 = lower partial moment 1 in T€

The probability that the family strategy will be inferior to investing in an immediate life annuity (LPM_0) is now significantly higher, especially for bond-oriented investments. However, the risk of the family strategy measured by

LPM_1 is only higher for the bond-leaning investments. Furthermore, in the 36% tax scenario, mean and standard deviation of the wealth distribution are considerably smaller. The relative underperformance of the family strategy for investors with a 36% marginal tax rate is a consequence of German income tax law. Only a small fraction of an annuity payout is treated as taxable income and annuities have a comparative advantage (cf. footnote 4). This advantage is more pronounced relative to bond investments since the major source of income (interest) is taxed, whereas capital gains of stocks are in general not taxed; however, dividends are subject to taxation (cf. footnote 6). Again it can be seen that the family strategy is more attractive for female retirees.

75- and 80-year-old retiree

In the case of a 75-year-old person, the benchmark investment—an immediate life annuity with a premium of € 100.000 as offered by Standard Life—leads to an annual payment A of € 9,521 (male) and € 8,092 (female). For an 80-year-old person, we get annual payments A of € 11,745 (male) and € 10,016 (female).

Table 4 (5) shows the results for a 75-(80)-year-old retiree with a marginal tax rate of 0%. Once again we simulated the chance and risk profile of the family strategy at the maximum age of 111 of the retiree. Hence the wealth distribution is given after 36 years (111 – 75) or 31 years (111 – 80) of investment.

TABLE 4

Statistical figures of the wealth distribution for the family strategy for a 75-year-old retiree with a marginal tax rate of 0%

	Proportion α of investment in stocks					
	0%	20%	40%	60%	80%	100%
Male, 75						
Mean	236.3	485.2	925.0	1,692.7	3,013.5	5,251.2
Std	321.6	557.8	1,259.0	3,193.4	8,376.6	22,313.8
LPM_0	21.77%	14.80%	13.05%	13.84%	15.54%	17.85%
LPM_1	42.9	39.2	56.3	101.8	196.0	381.2
Female, 75						
Mean	238.1	496.4	962.0	1,777.5	3,174.2	5,483.7
Std	268.0	485.3	1,196.6	3,245.0	8,430.1	21,465.7
LPM_0	16.09%	9.14%	9.31%	11.52%	13.90%	17.27%
LPM_1	19.8	15.7	27.6	61.8	136.3	290.8

Mean in thousand € (T€); Std = standard deviation in T€; LPM_0 = lower partial moment 0; LPM_1 = lower partial moment 1 in T€

TABLE 5

Statistical figures of the wealth distribution for the family strategy for a 80-year-old retiree with a marginal tax rate of 0%

	Proportion α of investment in stocks					
	0%	20%	40%	60%	80%	100%
Male, 80						
Mean	175.5	326.6	570.7	961.3	1,580.8	2,553.8
Std	272.2	424.6	808.6	1,751.0	3,999.9	9,274.0
LPM_0	23.37%	18.43%	15.93%	15.63%	16.46%	17.97%
LPM_1	49.7	50.9	63.4	94.6	153.9	259.0
Female, 80						
Mean	175.0	330.2	581.8	987.9	1,633.0	2,650.8
Std	234.7	372.4	750.3	1,698.2	3,975.0	9,325.7
LPM_0	21.34%	14.78%	13.14%	13.79%	15.49%	17.67%
LPM_1	31.5	27.7	37.2	61.8	112.0	201.2

Mean in thousand € (T€); Std = standard deviation in T€; LPM_0 = lower partial moment 0; LPM_1 = lower partial moment 1 in T€

Although annuities for 65-, 75-, and 80-year-old retirees are almost equally priced when using the IRR concept, the risk of the family strategy measured by LPM_0 is—compared to 65-year-old retirees—drastically higher for 75- and, especially, 80-year-old retirees. Again, this is because the expected payout streams differ: 65-year-old retirees receive lower annual payouts for a longer time than 75- and 80-year-old retirees. On average, the size of the investment fund is larger and the time span until a shortfall occurs is, on average, significantly longer for 65-year-old retirees. Therefore, these persons benefit from the difference between the (average) return of the investment fund and the return of the annuity more intensively than do 75- and 80-year-old retirees. The chances of the family strategy expressed by the mean of the wealth distribution and the risk expressed by the standard deviation and LPM_1 cannot be compared directly between the different cases shown in Tables 2, 4, and 5 because of the different maturities of the different family strategies: for example, a 65-year-old retiree has 46 years until the maximum age of 111, compared to 31 years for an 80-year-old person.

The results for 75-year-old retirees and 80-year-old retirees with a marginal tax rate of 36% are shown in Tables 6 and 7.

TABLE 6
Statistical figures of the wealth distribution for the family strategy for a 75-year-old retiree with a marginal tax rate of 36%

	Proportion α of investment in stocks					
	0%	20%	40%	60%	80%	100%
Male, 75						
Mean	80.1	199.5	437.1	893.4	1,749.9	3,324.5
Std	158.3	284.0	656.9	1,765.4	4,971.3	14,209.3
LPM_0	29.23%	21.03%	16.86%	16.08%	16.78%	18.35%
LPM_1	34.4	34.7	44.7	73.5	135.2	260.4
Female, 75						
Mean	73.5	196.2	441.9	918.1	1,815.4	3,470.4
Std	136.2	247.8	616.1	1,745,0	5,042.9	14,595.8
LPM_0	29.62%	17.92%	14.42%	14.55%	16.09%	18.26%
LPM_1	25.3	20.6	27.7	50.0	100.8	206.5

Mean in thousand € (T€); Std = standard deviation in T€; LPM_0 = lower partial moment 0; LPM_1 = lower partial moment 1 in T€

TABLE 7

Statistical figures of the wealth distribution for the family strategy for a 80-year-old retiree with a marginal tax rate of 36%

	Proportion α of investment in stocks					
	0%	20%	40%	60%	80%	100%
Male, 75						
Mean	69.5	149.2	293.7	545.4	976.7	1,703.2
Std	147.7	241.0	469.6	1,058.7	2,553.4	6,275.3
LPM_0	27.83%	22.87%	19.21%	17.77%	17.74%	18.63%
LPM_1	36.6	41.4	50.8	72.7	114.9	192.4
Female, 75						
Mean	63.7	145.4	293.3	552.9	998.7	1,760.7
Std	131.3	213.5	433.1	1,019.1	2,525.1	6,206.1
LPM_0	29.41%	21.97%	17.84%	16.79%	17.36%	19.18%
LPM_1	29.8	29.1	34.9	52.1	88.1	158.7

Mean in thousand € (T€); Std = standard deviation in T€; LPM_0 = lower partial moment 0; LPM_1 = lower partial moment 1 in T€

Keeping in mind the effects of taxation as discussed in the previous section (see Table 3), the results of Tables 6 and 7 are not very surprising. The family strategy becomes less attractive in the case of a marginal tax rate of 36%. Again, this holds true especially for portfolios containing a large fraction of bonds.

Alternative strategy

Up to this point we have assumed that the heirs must buy a life annuity if $W_t < A$, which will be the case when the retiree's consumption needs can no longer be covered by the investment fund. We keep this assumption, but now also require that the heirs buy the annuity if $W_t > \pi_t$, where π_t stands for the annuity premium at time t, which again gives the amount A. This strategy immediately locks in possible profits.[13] Table 8 (marginal tax rate 0%) and Table 9 (marginal tax rate 36%) give the simulation results for this alternative strategy for 65-year-old retirees.

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13 In general, we have an American option with underlying W_t and strike price π_t. The time to maturity depends on the (stochastic) lifetime of the retiree. As mentioned before, a preference-free valuation requires financial instruments that duplicate individual mortality. Since such instruments are usually not available, Milevsky and Young (2003) suggest a preference-based valuation approach.

TABLE 8

Statistical figures of the wealth distribution for the family strategy for a 65-year-old retiree at marginal tax rate 0%; alternative strategy

	Proportion α of investment in stocks					
	0%	20%	40%	60%	80%	100%
Male, 65						
Mean	107.9	275.7	720.4	1,703.0	3,788.1	8,093.9
Std	266.5	564.8	1,505.9	4,685.6	15,840.8	53,471.8
LPM_0	1.03%	0.85%	1.79%	3.13%	4.62%	6.24%
LPM_1	1.9	2.7	12.4	45.8	142.8	408.7
Female, 65						
Mean	82.4	232.6	647.3	1,589.1	3,624.9	7,896.0
Std	178.8	408.7	1,233.7	4,240.3	15,156.2	52,954.3
LPM_0	0.28%	0.32%	1.15%	2.43%	4.02%	5.70%
LPM_1	0.4	0.7	5.9	29.1	103.1	310.0

Mean in thousand € (T€); Std = standard deviation in T€; LPM_0 = lower partial moment 0; LPM_1 = lower partial moment 1 in T€

TABLE 9

Statistical figures of the wealth distribution for the family strategy for a 65-year-old retiree at marginal tax rate 36%; alternative strategy

	Proportion α of investment in stocks					
	0%	20%	40%	60%	80%	100%
Male, 65						
Mean	47.5	114.3	309.0	793.4	1,935.7	4,528.8
Std	113.8	234.6	645.9	2,192.4	8,052.7	29,615.6
LPM_0	6.03%	2.56%	2.95%	3.87%	4.97%	6.23%
LPM_1	5.5	4.0	10.0	29.2	82.8	229.1
Female, 65						
Mean	32.6	90.9	270.4	729.1	1,836.2	4,393.3
Std	77.2	167.7	521.3	1,953.5	7,700.3	29,259.1
LPM_0	3.65%	1.52%	2.24%	3.31%	4.56%	5.89%
LPM_1	2.5	1.8	6.0	20.8	63.9	182.9

Mean in thousand € (T€); Std = standard deviation in T€; LPM_0 = lower partial moment 0; LPM_1 = lower partial moment 1 in T€

Compared with the original strategy (see Tables 2 and 3), mean and standard deviation of the wealth distribution are significantly lower under both tax scenarios. The risk of the family strategy—expressed by LPM_0 and LPM_1, respectively—is drastically reduced. For example, in the case of a 0% marginal tax rate, there is a 98.97% (male) or a 99.72% (female) chance that the family strategy with a 100% investment in bonds is superior to the immediate life annuity. In the case of a male retiree and a marginal tax rate of 0%, we find that with an 80% bond investment there is a significant risk reduction (e.g., the probability of being better off without the family strategy is reduced from 2.85% to 0.85%). However, the chances measured by the mean of the wealth distribution are heavily reduced as well (€ 1.25 million to € 0.27 million).

Retiree with higher than average life expectancy

There is empirical evidence that wealthier people have higher than average life expectancies (see Brown, 2003). Therefore, it can be argued that a potential annuity buyer with wealth of € 100,000 has no average life expectancy but does have typical annuitant mortality, in which case this mortality should be used to calculate the chance and risk profile of the family strategy. The annuitant life-table we have used so far does account for higher annuitant life expectancy (caused by adverse selection). Furthermore, it has significant additional safety loadings (see Schmithals and Schütz, 1995). For this reason, the simulation results shown in Table 10 should be interpreted as a lower bound for the attractiveness of the family strategy.

TABLE 10

Statistical figures of the wealth distribution for the family strategy for a 65-year-old male retiree with annuitant mortality at marginal tax rate 0%

	Proportion α of investment in stocks					
	0%	20%	40%	60%	80%	100%
Male, 65						
Mean	412.0	1,064.8	2,463.5	5,359.4	11,199.5	22,747.9
Std	415.6	978.2	3,266.2	11,293.4	38,732.6	131,580.3
LPM_0	11.54%	5.42%	7.01%	9.98%	13.57%	17.52%
LPM_1	13.4	10.6	31.5	97.2	278.5	748.7
Female, 65						
Mean	451.4	1,158.2	2,666.5	5,773.9	12,018.2	24,312.6
Std	343.9	928.8	3,359.1	11,834.3	40,716.5	138,283.7
LPM_0	3.33%	1.86%	4.01%	7.26%	11.30%	15.77%
LPM_1	2.3	2.5	13.5	56.4	187.5	567.2

Mean in thousand € (T€); Std = standard deviation in T€; LPM_0 = lower partial moment 0; LPM_1 = lower partial moment 1 in T€

It is clear from Table 10 that for a 65-year-old retiree—compared to the situation in Table 2—the chances of the family strategy are reduced and the risk increases. However, the family strategy should still be seriously considered as an alternative to annuitization. Once again, this is especially true for female retirees.

Stochastic annuity prices

Up to now the model has assumed a deterministic path of future annuity prices. In the real world, of course, this will not be the case as insurance companies typically invest a large fraction of their assets in the bond market. Hence, when interest rates fall, one would expect annuity prices to rise. However, we do not believe that this effect dramatically influences our results because the wealth distributions seen in the tables above will be changed only by simulation paths with shortfall events (i.e., in cases where $W_t < A$ and the heirs therefore must pay for an annuity)—an event that happens rarely. Furthermore, when this does happen, typically the retiree is much older than he or she was when the family strategy was put in place and thus the annuity price is low. Clearly, shortfall scenarios will now tend to accompany higher than average annuity prices, but we do not expect this effect to radically change the results set out so far.

How does the connection between annuity prices and interest rates really look? Milevsky, 1998, for example, specifies a stochastic process for the IRR in order to determine future annuity prices based on historical data from the Canadian bond market. However, there are no comparable German data, thus making Milevsky's approach infeasible. The reason for this lack of data is that in the past only participating (with-profit) products were offered in the German market and the interest rate used to calculate the guaranteed part of the payout was directly set by the regulatory authority.

Even though we do not know how Standard Life Insurance Company really prices the annuities it offers in the German market, it is possible to make some plausible assumptions about how annuity prices will be influenced by the development of the bond market and thus evaluate its effects on the family strategy. As previously stated, the Standard Life Insurance Company offer for a 65-year-old retiree leads to an IRR of nearly 2.8% when using the German annuitant life-table. Let us now assume that this will serve as the mean of the IRR distribution for future annuity prices. Furthermore, let the IRR be normally distributed with a standard derivation of 1% and a lower bound of 0%.

Since one would expect a strong link between the bond market and annuity prices, we presume the correlation coefficient between the rate of return of the bond market and the IRR of the annuity to be 0.75.

Future annuity prices will now be stochastic and strongly linked to the rates of return of the bond market. Let us look again at the family strategy situation involving a 65-year-old retiree. Should it be necessary to switch to the annuity after one year, the price for the annuity will now have a mean of € 97,966 for a male retiree (female retiree: € 98,444) and a standard deviation of € 9,668 (female retiree: € 10,917). Table 11 shows the results for this scenario.

TABLE 11

Statistical figures of the wealth distribution for the family strategy for a 65-year-old retiree at marginal tax rate 0% and stochastic future annuity prices

	Proportion α of investment in stocks					
	0%	20%	40%	60%	80%	100%
Male, 65						
Mean	531.6	1,252.2	2,762.2	5,847.0	12,025.0	24,220.3
Std	447.6	1,060.7	3,463.3	11,624.4	38,659.0	127,897.6
LPM_0	5.51%	2.85%	4.50%	7.21%	10.38%	14.03%
LPM_1	7.4	6.5	23.1	80.7	243.0	656.3
Female, 65						
Mean	530.3	1,275.7	2,848.5	6,069.6	12,522.9	25,253.9
Std	372.3	985.6	3,470.3	11,926.7	40,003.0	132,759.7
LPM_0	1.52%	1.08%	2.89%	5.67%	9.40%	13.51%
LPM_1	1.3	1.6	10.5	49.2	175.0	525.8

Mean in thousand € (T€); Std = standard deviation in T€; LPM_0 = lower partial moment 0; LPM_1 = lower partial moment 1 in T€

The probability that the family strategy will be inferior to investing in an immediate life annuity (LPM_0) remains unchanged compared to the base situation shown in Table 2. Additionally, other moments of the wealth distribution (mean, standard deviation, and LPM_1) have changed only slightly because in the rare event of an exhausted investment fund, the retiree will be on average almost 91 years old (male) or 95 years old (female).[14] If changing to an annuity is necessary at that age, the annuity price will have a mean of € 38,838 for a male retiree (female retiree: € 30,572) and a standard deviation of € 1,626

[14] These figures are calculated for the family strategy with 100% bond investment.

(female retiree: € 1,110). In the case set out in Table 11, even if shortfall scenarios tend to accompany higher than average annuity prices, there is no dramatic change in the performance of the family strategy.

However, the situation is different if we combine the "alternative strategy" described above with stochastic annuity prices. In the alternative strategy, a change to an annuity will be made if $W_t < A$ or if $W_t > \pi_t$, where π_t stands for the annuity premium at time t. This alternative strategy, which immediately locks in possible profits, leads to the results shown in Table 12.

TABLE 12

Statistical figures of the wealth distribution for the family strategy for a 65-year-old retiree at marginal tax rate 0% and stochastic future annuity prices; alternative strategy

	Proportion α of investment in stocks					
	0%	20%	40%	60%	80%	100%
Male, 65						
Mean	188.9	400.7	883.8	1,941.8	4,185.8	8,802.2
Std	279.5	608.2	1,636.9	4,991.8	15,718.0	50,250.5
LPM_0	0.54%	0.53%	1.38%	2.65%	4.26%	5.91%
LPM_1	1.1	1.7	9.7	42.3	139.1	384.1
Female, 65						
Mean	180.2	385.0	851.0	1,876.0	4,082.7	8,617.5
Std	213.4	487.8	1,418.8	4,505.0	15,341.9	48,641.3
LPM_0	0.14%	0.19%	0.82%	1.99%	3.55%	5.29%
LPM_1	0.2	0.4	4.3	25.0	99.4	305.2

Mean in thousand € (T€); Std = standard deviation in T€; LPM_0 = lower partial moment 0; LPM_1 = lower partial moment 1 in T€

Compared to the situation shown in Table 8, the results in Table 12 changed in favour of the family strategy. This is simply explained by the fact that the randomness of future annuity prices leads to more switching situations.

Discussion

Our examples have demonstrated that the family strategy offers heirs enormous chance potential while leaving the retiree in a position financially equal to that under the life annuity. Heirs can expect to inherit a positive bequest; their risk is not zero, of course, but it is a considerably low risk. We showed the influence of possible controls, asset allocation and switching strategy, which can provide specific structuring of the chance and risk profile. For investors with a 36% marginal tax rate, the family strategy is not quite as good as it is for the 0% marginal tax rate investors. Also, the older the retiree is at the beginning of the family strategy, the more the shortfall risk increases. Furthermore, the family strategy is generally less attractive for male retirees.

A limitation of the family strategy is that heirs, to be good guarantors of the annuity, need to give collateral, which will not always be possible. Also, by giving collateral, heirs will lose some financial flexibility. The extent of this disadvantage will depend, most likely, on the heirs' individual financial situation. Unfortunately, the wealthier and the more flexible heirs are, the more likely it is that the retiree also belongs to the wealthier and longer-living part of the population and, as we showed, this reduces the attractiveness of the family strategy.

There is also some model uncertainty in the family strategy, for example, assumptions about future life expectancy or the distributions of asset returns. This uncertainty cannot be avoided if heirs want to take a chance on receiving a bequest. The only alternative would be to buy the life annuity immediately, in which case uncertainty would be reduced to the rare instance of the insurer going into default. Another thing to be noted is that the family strategy could give rise to a moral hazard. For example, if the fund is nearly exhausted, the heirs might demand that the retiree renegotiate the deal.

In general, the attractiveness of the family strategy will depend on how many family members participate. The more who participate, the more their individual longevity risk will be reduced through pooling (Kotlikoff and Spivak, 1981). On the other hand, an increase in the number of participants may increase transaction costs (e.g., cost of negotiation) if all possible heirs want to participate in the family strategy.

Finally, the family strategy will gain attractiveness for male retirees in the context of compulsory unisex annuity pricing, as recently proposed by the European Commission.

SUMMARY AND CONCLUSIONS

In this article we developed a model of self-annuitization based on the fact that in the family context there exists the possibility of pooling longevity risk. This risk pooling can be integrated into a retirement plan, what we have called the family strategy. Our family strategy is based on an agreement between the retiree and his or her heirs in which the heirs receive the remaining investment fund on the retiree's death but in return are obligated to finance the retiree if the fund becomes exhausted. We modeled our family strategy in such a way that the retiree is never put in a worse position than he or she would be had the retiree instead purchased a life annuity. Using a simulation model and data from the German capital and annuity market, we showed for a variety of scenarios that the family strategy offers substantial chance potential with low shortfall risk.

We believe that exploration of the chance and risk profile of a self-annuitization strategy is an important step in deriving a sound financial-planning solution. In future research, intra-family risk pooling should be integrated into a preference-based approach. The challenge will be to model the utility of the retiree, possible bequest motives, the utility of heirs, and the individual tax situations of all those involved in the retirement decision.

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