Liquidity, Volume, and Price Behavior: The Impact of Order vs. Quote Based Trading

Katya Malinova* University of Toronto

Andreas Park† University of Toronto

November 21, 2008

Abstract

Intra-day financial market trading is typically organized using two major mechanisms: quote-driven and order-driven trading. Under the former, all orders are arranged through dealers; under the latter, orders are usually arranged via a public limit order book. These two mechanisms generate different outcomes with respect to trading activity, liquidity, and price behavior, as has been well-documented empirically. We provide a theoretical model to compare the two systems and show that the mechanism-intrinsic sequencing of trades and quotes generates most of the observed differences in trading outcomes, even in a frictionless environment. Our analysis shows in particular that large orders in the quote-driven segment of a hybrid market have lower information content, that price impacts are larger for trades in the order-driven segment, and that the quote-driven segment absorbs most large trades, whereas the order-driven segment absorbs small trades.

*Email: katya.malinova@utoronto.ca; web: http://individual.utoronto.ca/kmalinova/.

†Email: andreas.park@utoronto.ca; web: http://www.chass.utoronto.ca/~apark/.

1 Introduction

The literature on financial market microstructure distinguishes two trading mechanisms for the arrangement of intra-day trading: order- and quote-driven trading. Order-driven markets are usually exchanges with a public limit order book, the cleanest examples being Electronic Communication Network (ECN) platforms such as INET or Archipelago and the consolidated limit order books of the Toronto Stock Exchange (TSX) or Paris Bourse. The textbook definition for a quote-driven market is a trading system in which only dealers are allowed to post quotes so that all trades are arranged through these institutions (see, for example, Harris (2003)). Examples of quote-driven markets are bond- or foreign exchange markets (until very recently) or Nasdaq before 1997.

There are several potential conceptual differences between quote and order driven systems, including the level of market transparency, the degree of competition, the importance of repeat interactions among market participants, and the physical speed of order execution. Many of these differences result from market imperfections and the literature has focusses primarily on the impact of these frictions. Yet as new technologies eliminate imperfections, it becomes all the more important to identify the core conceptual differences between the mechanisms that underly the specific trading platforms and to understand how these differences affect major economic variables such as prices, trading costs, liquidity (and measures of liquidity), price efficiency, price volatility, and trading volume.

Absent frictions, the key conceptual difference between order and quote driven trading is the sequencing of actions. To see this, observe that a trade requires two parties: one who wishes to trade and thus demands liquidity, and one who then supplies liquidity.[1] In the order-driven market, liquidity is supplied without knowledge of the liquidity demand. In a quote-driven market, liquidity is supplied after the demand is revealed.[2] Therefore, in order-driven markets, quotes precede orders, in quote-driven markets it is the reverse.

The contribution of our paper is to provide a frictionless model that highlights this core conceptual difference. We study the mechanisms and their implications both when they are operated in isolation and when they compete in a hybrid market, and we obtain sharp predictions about the sequencing's impact on the aforementioned major observable economic variables. Results from our analysis will allow the profession to understand what measurable differences between quote and order driven trading remain

[1]An exception is the case where two liquidity demanders meet accidentally and their orders 'cross'.
[2]In Appendix A we discuss some institutional features in detail.

when controlling for all frictions.

Before we explain the details of our approach, we will change the expositional language. The terms "order-driven" and "quote-driven" do not admit an unambiguous intuition for their definition — one can argue that whether a market is driven by quotes or orders is in the eye of the beholder. As the terms are also semantically close, we shall avoid confusion by identifying the mechanisms with the environments in which they commonly occur. We will thus refer to order-driven markets as *limit order markets* and to quote-driven markets as *dealer markets*.

Our model has the following structure. Liquidity in both the limit order and the dealer markets is supplied by uniformed and risk-neutral institutions which compete for the order flow. This is the case in the standard market microstructure models in the tradition of Kyle (1985) and Glosten and Milgrom (1985). Liquidity demanders trade either for reasons outside the model (e.g., to rebalance their portfolio or inventory, to hedge, or to diversify), or they have private information about the fundamental value of the security.

In the dealer-market, the liquidity providers observe the order flow and then compete for the order in a Bertrand fashion. The equilibrium price then aggregates the information contained in the order flow, and the liquidity provider absorbs the order by taking the other side of the transaction. In the limit order market, liquidity providers post a schedule of buy- and sell limit orders, each for the purchase or sale of a specific number of units. These prices incorporate the information revealed when the respective limit order is "hit" by a market order.[3] One additional contribution of our paper is thus in formulating a model that tractably integrates both a limit order and a dealer market in a Glosten-Milgrom sequential trading setup.

Informed investors receive private binary signals of *heterogenous* precisions, or qualities, and the quality of investor i's signal is i's *private* information. The underlying continuous structure allows a simple and concise characterization of the equilibrium by *marginal trading types*.[4] In equilibrium, the price is set so that traders best-respond by buying (selling) if their private signal is sufficiently encouraging (discouraging). Moreover, the higher is the quality of their private information, the larger is the size of their order.

[3] Other authors (e.g. Back and Baruch (2007)) refer to such prices as "upper-tail"– and "lower-tail"– expectations as each price accounts for the fact that the true order size may be larger.

[4] On a theoretical level, this is a 'purification' approach. One can construct a model in which people receive signals of different, discrete qualities. When using such a setup, however, one may have to describe behavior with mixed strategies and all their inherent (interpretational) complications. Our approach delivers cleaner insights.

When the mechanisms operate in isolation, we find that —from the investor's perspective— small size orders are cheaper in a dealer market, whereas large size orders are cheaper in a limit order market. Volume is higher in limit order markets, and, if there are sufficiently many traders, prices in the limit order market are more volatile and less informationally efficient.

The intuition underlying these results is that liquidity providers in dealer markets are intrinsically better at pin-pointing the information content of a transaction because they know the order size before setting the price. While this lowers traders' information rents, those with lower quality information are better off being identified. For in the limit order market, very well informed traders hide among the less well informed ones and thus earn a rent at their expense.[5] Since well informed traders submit orders more aggressively, volume in limit order markets is larger, while the dealer's superior information extraction power causes price volatility to be smaller in dealer markets.

The above result on the cost difference has implications for standard (empirical) measures for liquidity. We predict that the effective spread, which is the spread implied by the transaction prices (and not by the quotes, which may be merely indicative) is smaller in the dealer market. The price impact, which is commonly understood as the change in the price triggered by the order flow, is larger in the limit order market.

Our analysis further shows that the behavior of traders in a dealer market is not stationary. For instance, as prices drop, favourably informed traders submit buy orders more aggressively, thus acting as *contrarians*. Unfavourably informed investors submit sell-orders less aggressively. Consequently, one must be careful not to make implicit stationarity assumptions when estimating the probability of informed trading (PIN), a standard measure for the information content of trades (see, Easley, Kiefer, O'Hara, and Paperman (1996)).

While the classification of trading mechanisms into quote- and order driven is useful as a benchmark, most real-world markets share features of both trading systems and are thus hybrids. NYSE, for instance, is a hybrid market because traders can either send orders directly to the limit order book or they can arrange trades via floor brokers. Similarly, on the TSX, trades can be send to the consolidated limit order book or they can be arranged via an 'upstairs' dealer. On Nasdaq, trades can be made on INET (an ECN) or with a dealer. Moreover, even systems that can be classified as a pure limit order or pure dealer market may compete with a venue of a different kind, so that the two markets together are a hybrid. One example is Paris Bourse (a limit order market)

[5]The result that small trades are cheaper on dealer markets has been previously noted by, for instance, Glosten (1994) or Seppi (1997).

and London Stock Exchange (a dealer market until 1997) which compete for cross-listed stocks (see de Jong, Nijman, and Roell (1995)). In other words, the two trading systems usually co-exist and compete.

It is therefore important to understand the impact of each mechanism when *both* are in operation in a hybrid market. In our frictionless setting, equilibrium trading costs in different segments of the hybrid market must coincide. Then two kinds of equilibria emerge: in the first, there is market specialization in the sense that all large orders are traded in one system and all small orders in the other. In the second, which is economically more appealing, orders of all sizes are traded in each market segment, but the information content of trades differs endogenously. Specifically, large trades are more informative in the limit order market segment and small trades are more informative in the dealer market segment. As a corollary, we show that there will be more large orders in the dealer segment of the market and more small orders in the limit order segment.

Standard empirical findings from hybrid market are as follows: De Jong, Nijman and Roell (1995) find that markets are deeper and that there are more large transactions on LSE relative to Paris Bourse. According to Viswanathan and Wang (2002), large orders on NYSE are filled by dealers, whereas small orders are filled by involving the limit order book. Smith, Turnbull, and White (2001), Booth, Lin, Martikainen, and Tse (2002), and Bessembinder and Venkataraman (2004), respectively, find for the Toronto Stock Exchange, the Helsinki Stock Exchange, and Paris Bourse that upstairs trades (in the dealer segment) have lower information content and smaller price impacts than downstairs trades (in the limit order segment). The main insight of our theoretical analysis is that the sequencing of orders and quotes alone explains these empirically observed differences in trading outcomes.

The remainder of this paper is organized as follows. In Section 2, we discuss the related theoretical literature. In Section 3, we introduce the general model. In Section 4, we derive the limit order market equilibrium, in Section 5, the dealer market equilibrium. In Section 6, we compare the two mechanisms when they operate in isolation. In Section 7, we discuss hybrid markets. Section 8 concludes. The appendix contains the details of the institutional background, the signal distributions and the proofs.

2 Related Theoretical Literature

Our dealer-market setup is similar to Easley and O'Hara (1987) who employ a setting in the tradition of Glosten and Milgrom (1985) and allow trades of different sizes. In their setup the trader, equipped with a signal of a fixed quality, chooses whether to submit a large or a small order size and usually chooses a mixed strategy. The trader behavior in our dealer market is, effectively, a purification of the mixed-strategy behavior in Easley-O'Hara.

The mechanics of the limit order market pricing schedules in our paper coincide with those in Glosten (1994), which is one of the first papers to formally model limit order markets.[6] While Glosten (1994) contains a system comparison, the broker there strategically acts against the electronic book, potentially trying to undercut certain quotes. Consequently, this dealer still operates under a limit order mechanism.

The two classic comparative theoretical studies of trading mechanisms are Grossman (1992) and Seppi (1990). Grossman studies the relation of upstairs (or dealer) markets and downstairs (or limit order) markets. In his model some traders choose not to publicly reveal their willingness to trade. Leaving a non-binding indication to trade allows the knowing upstairs traders to tap into this "unexpressed liquidity" if occasion arises. Grossman finds that if sufficiently many traders submit orders upstairs, then an informational advantage of an upstairs dealer about the displayed order flow is so large that the upstairs market will dominate.

In Seppi (1990) the non-anonymous nature of upstairs trading and repeat interactions ensure that informed and uninformed trading is segmented into downstairs and upstairs markets, respectively. Routing an informed order to an upstairs liability trader may trigger a penalty on future transactions. In equilibrium, informed trades are sent to the limit order book, while uninformed trades clear upstairs.

The contribution of our paper is on a different level. We assume that all liquidity is observed and we abstract from repeat interactions. Our focus is on the inherent outcomes that are generated by the two trading mechanisms in isolation and in combination. This affords a clear view of the impact of the systems themselves, the bottom line being that even if, say, a revolutionary new regulation could force the display of all liquidity and remove the consequences of repeat interactions, there would still be differences.

Two more recent comparative studies are Viswanathan and Wang (2002) and Back and Baruch (2007). Viswanathan and Wang focus on non-information based trading and the effects of risk aversion. Back and Baruch study a model in which a single, perfectly informed trader chooses whether to work an order (i.e. submit a series of small trades) or whether to submit a large block trade. They find an equivalence between the dealer and limit order market. Specifically, the prices charged on the dealer market resemble the upper-tail-expectation prices in the limit order market as dealers anticipate

[6] Biais, Martimort, and Rochet (2000) is a more recent contribution that studies a version of Glosten with imperfect competition.

that the informed trader works his order. Our study complements theirs in several aspects. First, Back and Baruch have a single informed trader, while we have a series of imperfectly informed traders who, however, cannot work orders.[7] Second, Back and Baruch allow traders in the dealer market to remain anonymous even when repeatedly submitting orders. We rule this out deliberately so that in our model an order in a dealer market could not be worked. Our major focus is to understand and explain the observed differences with respect to price impacts and the information content of trades, and we show how these differences prevail even in a frictionless hybrid market.

Comparisons between different systems have also been made regarding the level of market transparency: Pagano and Roell (1996) compare transparency of a uniform price auction with a dealer market system. Brown and Zhang (1997) formulate a model-hybrid of a Kyle (1985)-style and a Rational Expectations-style setup to study dealers' decisions to participate in a market, and to describe how dealers' decisions to supply liquidity affect the informational efficiency of prices.

Another strand of the theoretical market microstructure literature studies the strategic provision of liquidity (whereas liquidity suppliers in our model are non-strategic). Most papers on the strategic provision of liquidity are based on either Parlour (1998) or Foucault (1999). Examples of studies that compare different market structures in this framework are Seppi (1997), Parlour and Seppi (2003), and Buti (2007); for an extensive up-to-date survey of the literature on limit order markets see Parlour and Seppi (2008). Our framework complements these studies by considering the effects of strategic market order submission by heterogeneously informed investors.

The important insight that our analysis provides is that the sequencing of events, namely, whether liquidity is offered continuously or supplied only on demand, naturally generates observed patterns in order submission behavior and price characteristics. While other market characteristics most certainly play a role, we believe that it is important to fully understand the benchmark case that we provide here.

3 The Basic Setup

General Market Organization: We consider a stylized version of security trading in which informed and uninformed investors trade a single asset by submitting market orders. Liquidity is supplied by uninformed, risk-neutral liquidity providers who compete

[7] Ideally, one would like a a combination of both our and their setups to one with multiple informed traders who can trade repeatedly. Yet this is highly non-trivial and, to the best of our knowledge, such a model with endogenous trade-timing with multiple informed traders has not yet been successfully attempted (see also the experimental paper by Bloomfield, O'Hara, and Saar (2005) for a discussion).

for order order flow. At each discrete point in time there is exactly one trader who arrives at the market according to some random process. This individual trades upon their arrival and only then. Short positions are filled at the true value. In the limit order market, the liquidity providers post a series of limit buy orders at which they are willing to buy the security and a series of sell-orders at which they are willing to sell the security. The former constitutes a series of ask-prices, the latter a series of bid-prices. Each price is for a single unit (i.e. a round lot). The investor posts his market order after observing these prices.[8] In the dealer market, the investor posts his market order first and the liquidity providing dealers then compete in a Bertrand fashion in the price for this order. In both systems, liquidity providers are subject to perfect competition and thus set prices so that they make zero expected profits.

Security: There is a single risky asset with a liquidation value V from a set of two potential values $\mathbb{V} = \{\underline{V}, \overline{V}\} \equiv \{0, 1\}$, with $\mathsf{Pr}(\overline{V}) = 1/2$. The prior distribution over $\mathbb{V}$ is common knowledge.

Investors: There is an infinitely large pool of investors out of which one is drawn at each point in time at random. Each investor is equipped with private information with probability $\mu > 0$; if not informed, an investor becomes a noise trader (probability $1 - \mu$). The informed investors are risk neutral and rational.

Noise traders have no information and trade randomly. These investors are not necessarily irrational, but they trade for reasons outside of this model, for example to obtain cash by liquidating a position.[9] To simplify the exposition, we assume that noise traders make trades of either direction and size with equal probability.

Trade Size: All trades are market orders for round lots and thus the number of shares traded is discrete. The order at time t is denoted by o_t where $o_t < 0$ indicates a sell-order and $o_t > 0$ is a buy-order. Traders can also abstain from trading, so that $o_t = 0$. We focus on order sizes $o_t \in \{-2, \ldots, 2\}$.

Public Information. The structure of the model is common knowledge among all market participants. The identity of an investor and his signal are private information, but everyone can observe the history of trades and transaction prices. The public information H_t at date $t > 1$ is the sequence of orders o_t and realized transaction prices at all dates prior to t: $H_t = ((o_1, \mathsf{p}_1), \ldots, (o_{t-1}, \mathsf{p}_{t-1}))$. H_1 refers to the initial history before trades occurred.

Whether a trader is informed or noise at time t is independent of the past history H_t

[8]When referring to a liquidity provider in singular, we will use the female form and for liquidity demanders we will use the male form.

[9]Assuming the presence of noise traders is common practice in the literature on micro-structure with asymmetric information to prevent "no-trade" outcomes à la Milgrom-Stokey (1982).

and the underlying fundamental V, as is the quality of each informed investor's information. Payoff-relevant public information at time t can thus be summarized by a time-t *public belief* that the security's liquidation value is high ($V = 1$). Since the liquidity providers must break even in expectation, this public belief coincides with the time-($t-1$) transaction price, denoted by p_{t-1}.

Liquidity Providers' Information. In the dealer market, the liquidity providers know the public history H_t and the order o_t. In the limit order market, liquidity providers do not know which order will be posted at time t, and their information is only H_t.

Informed Investors' Information. We follow most of the GM sequential trading literature and assume that investors receive a binary signal about the true liquidation value V. These signals are private, and they are independently distributed, conditional on the true value V. Specifically, informed investor i is told "with chance q_i, the liquidation value is High/Low (h/l)" where

Pr(signal|true value)	$V = 0$	$V = 1$
signal $= l$	q_i	$1 - q_i$
signal $= h$	$1 - q_i$	q_i

In contrast to most of the GM literature, we assume that these signals come in a *continuum* of qualities. This q_i is the *signal quality* and it is trader i's private information. The distribution of qualities is independent of the asset's true value and can be understood as reflecting, for instance, the distribution of traders' talents to analyze securities. Figure 1 illustrates the distribution of noise and informed traders and the information structure.

In the subsequent analysis it will be convenient to combine the signal and its quality in a single variable, which is trader i's *private belief* $\pi_i \in (0, 1)$ that the asset's liquidation value is high ($V = 1$). This belief is the trader's posterior on $V = 1$ after he learns his signal quality and sees his private signal but *before* he observes the public history.

This private belief is obtained by Bayes Rule and coincides with the signal quality if the signal is h, $\pi_i = \mathsf{Pr}(V = 1|h) = q_i/(q_i + (1 - q_i)) = q_i$; or if the signal is l, it is $\pi_i = 1 - q_i$. In what follows we will use the distribution of these private beliefs. Let $f_1(\pi)$ be the density of beliefs if the true state is $V = 1$, and likewise let $f_0(\pi)$ be the density of beliefs if the true state is $V = 0$. Appendix B fleshes out how these densities are obtained from the underlying distribution of qualities.

Example of private beliefs. Figure 1 depicts an example where the signal quality q is uniformly distributed. Conditional densities are $f_1(\pi) = 2\pi$ and $f_0(\pi) = 2(1 - \pi)$, yielding distributions $F_1(\pi) = \pi^2$ and $F_0(\pi) = 2\pi - \pi^2$. The figure also

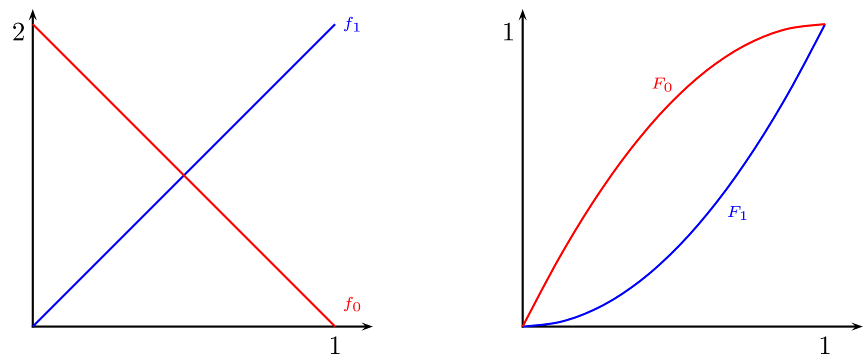


Figure 1: **Plots of belief densities and distributions.** *Left Panel:* The densities of beliefs for an example with uniformly distributed qualities. The densities for beliefs conditional on the true state being 1 and 0 respectively are $f_1(\pi) = 2\pi$ and $f_0(\pi) = 2(1-\pi)$; *Right Panel:* The corresponding conditional distribution functions : $F_1(\pi) = \pi^2$ and $F_0(\pi) = 2\pi - \pi^2$.

illustrates the important principle that signals are informative: recipients in favor of state $V = 0$ are more likely to occur in state $V = 0$ than in state $V = 1$.

4 Trading in Limit Order Markets

Prices in the Limit Order Market. The liquidity providers post a series of limit orders that constitute the bid- and ask-prices: the first ask-price, ask_t^1 is for the first unit that is being sold by liquidity providers, and ask_t^2 is for the second unit; similarly for bid-prices. At each price, the zero-profit condition must hold so that the price coincides with the expectation of the true value, conditional on the trading history and on the current transaction. In particular, the ask-price ask_t^2 accounts for the fact that the trade size is 2 (not 1). With zero expected profits, it must hold that

$$\begin{aligned}
\mathsf{ask}_t^1 &= \mathsf{E}[V|\text{ investor buys } o_t \geq 1 \text{ units at } \{\mathsf{ask}_t^1, \mathsf{ask}_t^2\},\ \text{public info at } t], \\
\mathsf{ask}_t^2 &= \mathsf{E}[V|\text{ investor buys } o_t = 2 \text{ units at } \{\mathsf{ask}_t^1, \mathsf{ask}_t^2\},\ \text{public info at } t].
\end{aligned}$$

This equilibrium pricing rule is common knowledge.

A market order in a order driven market 'walks the book', that is, it picks up the best-priced limit orders on its side of the market. Once this is accomplished, all posted limit orders adjust to reflect the information contained in the recent market order.

The total execution cost of a buy-order $o_t \geq 1$ and the total cash flow from a sell-order $o_t \leq -1$ is then computed as the sum of all the prices at which portions of the order are cleared:

$$C_t = \sum_{n=1}^{o_t} \mathsf{ask}_t^n \quad \text{or} \quad C_t = \sum_{n=1}^{|o_t|} \mathsf{bid}_t^n.$$

Examples for limit order markets are the NYSE's public limit order book (maintained by the specialist) or the TSX consolidated limit order book (note that prices listed on Nasdaq are usually not considered standing limit orders but 'trade-indications').

Trader's Decision in a Limit Order Market. An informed investor receives his private signal, observes all past trades, and can only trade in period t. Upon observing the posted prices he chooses the order size to maximize his expected profits. He abstains from trading if he expects to make negative trading profits. To simplify the exposition, we will from now on focus on the buy-side of the market. Analogous theorems apply to the sale-side of the market.

We focus on monotone decision rules. Namely, we assume that an insider uses a 'threshold' rule: he buys two units if his private belief π_i is at or above the time-t buy threshold π_t^2, $\pi_i \geq \pi_t^2$, he buys one unit if his belief is above π_t^1 but below π_t^2, $\pi_i \in [\pi_t^1, \pi_t^2)$, and he does not buy otherwise; similarly for selling.

Consider the decisions of the marginal buyer π_t^2, who is indifferent between buying 2 and 1 units, and type π_t^1, who is indifferent between trading and not trading. We will compress notation by writing $\mathsf{E}[V|H_t, \pi_t^j] = \mathsf{E}_t\pi_t^j$. For these marginal types it must hold that

$$2 \cdot \mathsf{E}_t\pi_t^2 - (\mathsf{ask}_t^1 + \mathsf{ask}_t^2) = 1 \cdot \mathsf{E}_t\pi_t^2 - \mathsf{ask}_t^1 \quad \Leftrightarrow \quad \mathsf{ask}_t^2 = \mathsf{E}_t\pi_t^2, \quad \text{and} \quad \mathsf{E}_t\pi_t^1 - \mathsf{ask}_t^1 = 0.$$

The liquidity providers anticipate the traders' behavior and post prices that take the information content of the trades into account, given the marginal trading types π_t^1, π_t^2.

Let $\lambda := (1-\mu)/4$ denote the probability of a noise buy or sell of any size order and let β_v^j denote the probability that there is a buy of $j \in \{1, 2\}$ units when the value of the security is $v \in \{0, 1\}$. Then

$$\begin{aligned}
\beta_v^1 &= \lambda + \mu(1 - F_v(\pi_t^2)) + \lambda + \mu(F_v(\pi_t^2) - F_v(\pi_t^1)) = 2\lambda + \mu(1 - F_v(\pi_t^1)), \\
\beta_v^2 &= \lambda + \mu(1 - F_v(\pi_t^2)).
\end{aligned}$$

The probability that a given trader is informed is independent of other traders' identities and the security's liquidation value. Since private beliefs are independent conditionally on the security's value, so are investors' actions. The ask prices for unit $j \in \{1, 2\}$ and the expectation of type π_t^j can then be written explicitly as

$$\mathsf{ask}_t^j = \frac{\beta_1^j p_t}{\beta_1^j p_t + \beta_0^j (1-p_t)}, \quad \mathsf{E}_t \pi_t^j = \frac{\pi_t^j p_t}{\pi_t^j p_t + (1-\pi_t^j)(1-p_t)}, \tag{1}$$

and $\mathsf{E}_t \pi_t^j = \mathsf{ask}_t^j$ simplifies to

$$\pi_t^j = \frac{\beta_1^j}{\beta_1^j + \beta_0^j}. \tag{2}$$

Hence, in any equilibrium in Period t, the threshold decision rules are independent of the public belief about the security's liquidation value, p_t. In other words, investors' *actions* in Periods $t = 1, \dots, t-1$ do not affect actions in Period t.

Since necessarily the level of noise trading for the first unit is larger than for the second unit, it intuitively holds that $\pi_t^1 < \pi_t^2$ and thus $\mathsf{ask}_t^1 < \mathsf{ask}_t^2$. Equilibrium thresholds are determined by two independent equations.

Theorem 1 (Trading in Limit Order Markets)
There exist a unique symmetric equilibrium with marginal buying types $\frac{1}{2} < \pi_t^1 < \pi_t^2 < 1$ and monotone decision rules so that investors with private belief $\pi < \pi_t^1$ do not buy, investors with private belief $\pi \in [\pi_t^1, \pi_t^2)$ buy one unit and investors with belief $\pi \in [\pi_t^2, 1]$ buy two units. Thresholds π_t^1, π_t^2 are time invariant: for $t' \neq t$, $\pi_t^i = \pi_{t'}^i$, $i = 1, 2$.

In limit order markets, both buying-thresholds maximize the ask-price as a function of the marginal trading type.[10]

Lemma 1 (Thresholds maximize the Bid-Ask-Spread)
The ask-price ask_t^j, $j \in \{1, 2\}$, as a function of the marginal buying type, is maximized at the equilibrium buying threshold π_t^j.

The above result is intuitive. In equilibrium, the ask price is set so that it averages signal qualities over a range and coincides with the marginal trader's expectation. Loosely speaking, the average quality (plus noise) must thus coincide with the marginal quality. As in many economic problems this occurs when the average (i.e. the ask price as a function of the marginal trader's belief) is maximal.

[10]This result is similar to one in Herrera and Smith (2006) who derive it in a different context; they do not use it to compare trading systems but to analyse trade-timing in a unit-size system. They kindly allowed us to study their private notes; we attempted a proof for our setting after we observed their result and we do not claim novelty; the proof techniques differ. The intuition for their result and ours coincides, and we borrow the intuition that they provide.

5 Trading in Dealer Markets

Prices in the Dealer Market. Since liquidity providers are competitive, they make zero expected profits. Consequently, prices at date t are set to coincide with the expectation of the fundamental, conditional on all available information

$$\mathsf{p}_t = \mathsf{E}[V|o_t, H_t].$$

Traders thus pay a uniform price p_t for each unit that they buy and they receive a uniform price p_t for each unit that they sell. The total execution costs of order o_t is thus

$$C_t = |o_t| \cdot \mathsf{p}_t.$$

This pricing rule is identical to the one used in Easley and O'Hara (1987) and it is also similar to the one in Kyle (1985), except that in our model the liquidity providers know precisely how much each single trader orders.

It is without loss of generality that traders do not know the price of their transaction before posting an order. In what follows we will show that the trader can perfectly anticipate the ultimate transaction price. We comment more on this issue and on the institutional background in Appendix A.

In light of this, we will henceforth refer to prices that are set for buy orders, $o_t \in \{1, 2\}$ as ask-prices, and prices that are set for sell-orders as bid-prices. As before, we will focus on the buy-side of the market. Then,

$$\begin{aligned}
\mathsf{ask}_t^1 &= \mathsf{E}[V|\text{ investor buys } o_t = 1 \text{ unit at } \{\mathsf{ask}_t^1\}, \text{ public info at } t], \\
\mathsf{ask}_t^2 &= \mathsf{E}[V|\text{ investor buys } o_t = 2 \text{ units at } \{\mathsf{ask}_t^2\}, \text{ public info at } t].
\end{aligned}$$

The Trader's Decision in the Dealer Market. An informed investor enters the market in period t, receives his private signal and observes history H_t. He then chooses the order size o_t to maximize his expected profits. The trader publicly posts the desired quantity (a market order) and the liquidity suppliers respond with a zero-profit price that they offer for the order quantity o_t.

To describe the equilibrium we seek two marginal belief-types. The first is trader type, π_t^2, is indifferent between purchasing quantity $o_t = 2$ and quantity $o_t = 1$. The next trader type π_t^1 is indifferent between purchasing 1 unit and not trading at all. Since we assume that noise traders trade any quantity with equal probability, denoted by λ as before, the probabilities of a buy order of size $j \in \{1, 2\}$ when the true value of the security is $v \in \{0,1\}$ are as follows

$$\beta_v^2 = \lambda + \mu(1 - F_v(\pi_t^2)), \quad \beta_v^1 = \lambda + \mu(F_v(\pi_t^2) - F_v(\pi_t^1)), \quad \text{with } \lambda = (1-\mu)/4.$$

In equilibrium, the trader perfectly anticipates the prices. Consequently, type π_t^2 must be indifferent between buying two shares at price ask_t^2 and one share at price ask_t^1:

$$2 \cdot \left(\mathsf{E}_t\pi_t^2 - \mathsf{ask}_t^2\right) = 1 \cdot \left(\mathsf{E}_t\pi_t^2 - \mathsf{ask}_t^1\right) \;\Leftrightarrow\; \mathsf{ask}_t^2 = \frac{1}{2}\,(\mathsf{E}_t\pi_t^2 + \mathsf{ask}_t^1).$$

Further, for $o_t = 1$ the marginal trader is indifferent between buying one unit and abstaining. Thus we have

$$\mathsf{E}_t\pi_t^1 = \mathsf{ask}_t^1.$$

These two conditions yield a system of two equations with two unknowns. We show that there exists a unique symmetric solution to the system, which is consequently the equilibrium.

Theorem 2 (Equilibrium in the Dealer Market)
For any prior $p_t \in (0,1)$ there exist a unique symmetric equilibrium with marginal buying types $1/2 < \pi_t^1 < \pi_t^2 < 1$ and monotone decision rules such that investors with private belief $\pi < \pi_t^1$ do not buy, investors with belief $\pi \in [\pi_t^1, \pi_t^2)$ buy one unit and investors with belief $\pi \in [\pi_t^2, 1]$ buy two units.

Note also that π_t^1 maximizes the ask-price for the single quantity in the same way as depicted in Lemma 1 for the limit-order ask-prices; π_t^2, however, has no such property as we argue in what follows.

6 Comparison of the Two Trading Mechanisms

In what follows, we will omit time subscripts and use L for outcomes in the limit order market and D for those in the dealer market. In this section, we assume that the two markets operate in isolation and do not compete against each other. Further, $C_i(n)$ denotes the execution cost of an order of size n under trading regime $i \in \{\mathsf{L}, \mathsf{D}\}$.

At first blush it may seem that the outcomes and the behavior in limit order and dealer markets must be the same. Traders pay $\mathsf{ask}_\mathsf{L}^1$ and $\mathsf{ask}_\mathsf{D}^1$ for the first unit or the small size order in the limit and dealer markets respectively, and they pay $C_\mathsf{L}(2) = \mathsf{ask}_\mathsf{L}^1 + \mathsf{ask}_\mathsf{L}^2$ and $C_\mathsf{D}(2) = 2 \cdot \mathsf{ask}_\mathsf{D}^2$ for two units in the limit order and dealer markets. One might think that prices satisfy $C_\mathsf{L}(1) = C_\mathsf{D}(1)$ and $C_\mathsf{L}(2) = C_\mathsf{D}(2)$ so that $\mathsf{ask}_\mathsf{D}^1 = \mathsf{ask}_\mathsf{L}^1$ and

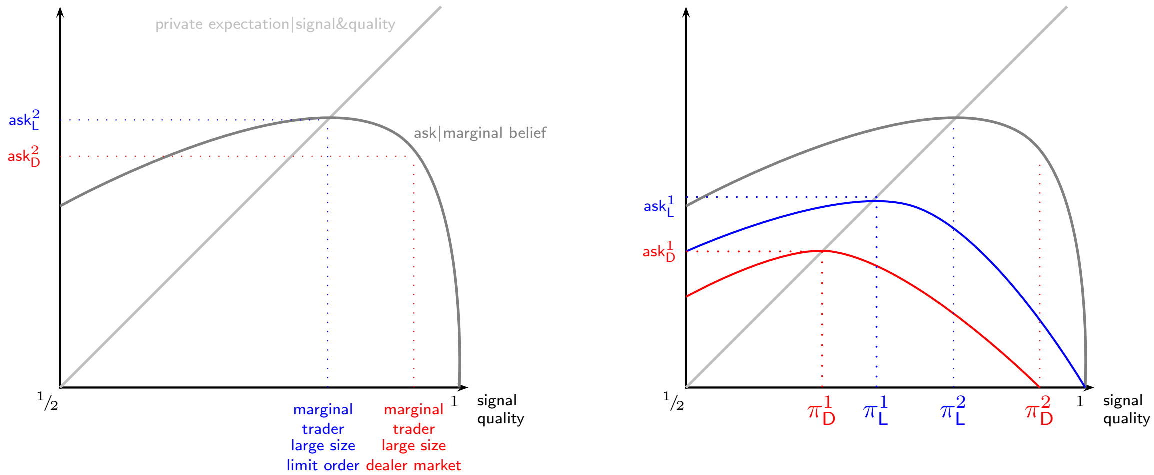


Figure 2: **Marginal Limit Order and Dealer Markets.** Both panels plot the private expectation of a trader as a function of his belief π; this is the monotonically increasing line. The curves depict the ask price as a function of the marginal buyer's signal quality. The left panel plots the price-expectation relation for the two-unit price (dealer market) or the price of the second unit (limit order market) (the two have the same functional form). The right panel plots the respective ask price curves for the first unit in the limit order market (blue) and the dealer market (red) and illustrates that the single or first ask-price is lower in the dealer market.

$\mathsf{ask}^2_\mathsf{D} = (\mathsf{ask}^1_\mathsf{L} + \mathsf{ask}^2_\mathsf{L})/2$. Alas, looking at systems in isolation, this naive intuition is incorrect — price $\mathsf{ask}^1_\mathsf{D}$ assumes that the single unit is sold exclusively to investors with lower quality signals whereas in the limit order market, $\mathsf{ask}^1_\mathsf{L}$ assumes that the first unit is also traded by traders with high quality information. Figure 2 illustrates the difference in prices, Proposition 1 offers a formal comparison of the execution costs.

We will first compare the trading mechanisms with respect to their impact on the bid-ask spread and price impact, which are the standard liquidity measures used in the literature. The bid-ask-spread is the difference between the lowest quoted ask price and the highest quoted bid price. The price impact of a trade is the change in the price that a transaction triggers. Finally, we also look at the execution cost of an order, which is the total dollar cost of trading quantity o_t.

Comparing the outcomes of the two different trading arrangements we find

Proposition 1 (Liquidity Measures and Execution Costs)

(a) *Bid-ask-spreads are larger in the limit order market,* $\mathsf{ask}^1_\mathsf{L} - \mathsf{bid}^1_\mathsf{L} > \mathsf{ask}^1_\mathsf{D} - \mathsf{bid}^1_\mathsf{D}$.

(b) *The price impact of a trade of any size is smaller in the dealer market,* $\mathsf{ask}^2_\mathsf{L} > \mathsf{ask}^2_\mathsf{D}$ *and* $\mathsf{ask}^1_\mathsf{L} > \mathsf{ask}^1_\mathsf{D}$.

(c) *The total execution cost for small trades is higher in the limit order market,* $C_\mathsf{L}(1) > C_\mathsf{D}(1)$, *and for large trades in the dealer market,* $C_\mathsf{L}(2) < C_\mathsf{D}(2)$.

Marginal traders of two units make a profit in both markets. In the dealer market, this marginal trader makes a profit on each of the units, and thus his expectation exceeds the ask-price for the two units. In the limit order market, the marginal trader breaks even on the second unit but benefits on the first. As the price for the second unit is maximal when it coincides with the expectation of the marginal two-unit buyer by Lemma 1, we have that in equilibrium $\mathsf{ask}_\mathsf{L}^2 > \mathsf{ask}_\mathsf{D}^2$. Figure 2's left panel illustrates this argument.

The result on trading costs may seem counterintuitive because the price impact of each unit in the limit order market is larger than in the dealer market. To see why the result holds, consider the perspective of a liquidity provider *after* the order has been executed in the limit order market. Suppose that the order was for a single unit. Then after the fact, the liquidity provider knows that she sold to someone who had information quality in $[\pi_1, \pi_2)$. Had she known this, she would have charged a lower price (due to perfect competition) — but now she actually made a profit on this trade. Likewise, if the quantity traded was the large one, then the liquidity provider knows that she sold the first unit too cheaply and thus made a loss on it.

The second unit, however, is priced "fairly" in the sense that the liquidity provider would charge the same price before and after observing the trade. Since the profits of the liquidity providers are losses for the traders and vice versa, on average traders make a profit on large quantities and a loss on small quantities. In contrast, in the dealer market, there is no difference between the ex-post and the ex-ante payoff of the liquidity provider — it is always zero.

Dynamic Behavior. In the limit order market, the history of trades does not affect a trader's decision to buy or sell. In the dealer market the behavior is history dependent.

Proposition 2 (Behavioral Dynamics in Dealer and Limit Order Markets) *In the dealer market, as p_t traverses from 0 to 1, all trading thresholds increase. In the limit order market, the prior does not affect traders' behavior.*

One could imagine that as the prior favours a trader's opinion, he becomes more convinced and needs a lower signal quality to trade (a "herding" effect). Alternatively, one can also argue that the value of a trader's information decreases so that he needs better quality information (a "contrarian" effect). In the dealer market, the latter effect dominates. To a degree, this is in line with empirical observations in that (allegedly) informed traders (such as institutional investors) tend to act as contrarians.

An immediate implication of this proposition is that one must be careful when aggregating or collecting trades of different sizes during the trading day. Depending on

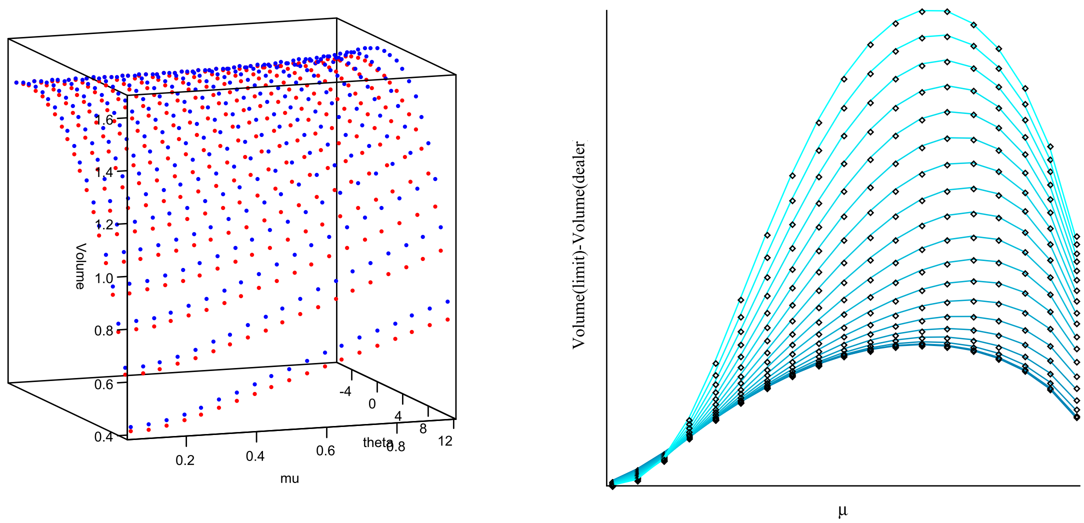


Figure 3: **Volume in Order and Quote-Driven Markets.** Both panels are based on the class of quadratic quality distribution functions that are centered at and symmetric around 1/2. The left panel plots the volume for both limit order (blue dots) and dealer markets (red dots) as a function of the amount of noise μ and the distribution parameter θ. The right panel plots the difference between the volume proxy for the limit order market and the one for the dealer market. As can be seen from both graphs, volume is larger in the limit order market.

trading histories, these trades would have been initiated by different signal types and thus have different informational contents. Any analysis of the "Probability of Informed Trading" (PIN) as introduced by Easley, Kiefer, O'Hara, and Paperman (1996) must thus account for this dynamic behavior.

Volume. A simple measure for the extent of trading activity is volume. While our model describes single trader arrivals each period, we can still proxy volume by the probabilities of the different units being traded. We can then determine the expected quantity traded in each period. For instance, a small sale will be initiated by noise traders or by informed traders with signal in $(1-\pi^2, 1-\pi^1]$ so that the probability of a small sale in state v is $\lambda + \mu(F_v(1-\pi^1) - F_v(1-\pi^2))$. Combining all cases we have

$$\begin{aligned} \text{volume} &= \sum_{v=0}^{1} (2\lambda + \mu(F_v(\pi^2) - F_v(\pi^1) + F_v(1-\pi^1) - F_v(1-\pi^2)))\mathsf{Pr}(V=v) \\ &\quad +2 \cdot \sum_{v=0}^{1} (2\lambda + \mu(1 - F_v(\pi^2) + F_v(1-\pi^2))\mathsf{Pr}(V=v). \end{aligned}$$

While we have not found an analytical result, we computed volume numerically, using the quadratic quality distributed that is outlined in Appendix B. Figure 3 illustrates the finding.

Numerical Observation (Volume)
The expected volume in each period is larger in limit order than in dealer markets.

Information Efficiency. Prices in our framework follow a martingale process, irrespective of the trading mechanism. Consequently, prices converge to the truth when there are sufficiently many trades. Yet the question remains if one trading system is faster or more accurate at revealing the true value.

We approached this question by simulations and asked two questions. First, does one trading mechanism yield average prices that are closer to the true value and second, does one mechanism display persistently larger price dispersion?

The first question can be answered by analyzing whether the average end-price of a sequence of trades is closer to the true value for one mechanism than for the other. To answer the second question, we first compared the variances of end-prices and found that the price variance in the limit order market is larger. We then expanded the analysis and investigated whether the higher dispersion in limit order markets is systematic in the sense of second order stochastic dominance. Formally, let the empirical distribution of prices under trading mechanism x be $F_x(\mathsf{p})$. Then the empirical distribution F_y of prices second order stochastically dominates the empirical distribution F_x if and only if

$$\int_0^z [F_x(\mathsf{p}) - F_y(\mathsf{p})]\, d\mathsf{p} \geq 0 \quad \forall z \leq 1. \tag{3}$$

Denote by p_L and p_D the end-prices for limit order and dealer markets respectively, and use F_L and F_D for the respective distributions of end-prices and $av(\mathsf{p}), stdev(\mathsf{p})$ for the averages and standard errors.

We employed the following data generation procedure for the simulations: we fixed the true value to $V = 1$, the level of informed trading to $\mu = .5$. So prices are closer to the true value if they are larger. We then obtained 50,000 observations for each of the Poisson arrival rates $\rho \in \{7, 10, 13, 17, 20, 25, 30\}$.[11] The underlying signal quality distribution was uniform.

For each series, we first drew the number of traders for the session and performed the random allocation of traders into noise and informed.[12] The informed traders were then equipped with a signal quality and a draw of the high or low signal for that quality, conditional on $V = 1$. Noise traders were assigned a random trading role. We then determined a random entry order (the order is irrelevant for limit order markets because

[11] A Poisson arrival rate of, for instance, $\rho = 20$ implies that, on average, there are 20 traders.
[12] Thus overall there were, for instance, about 1,000,000 trades for $\rho = 20$ and 500,000 for $\rho = 10$.

ρ	average price		standard error	
entry rate	limit order	dealer	limit order	dealer
7	.7625	.7570	.2395	.2238
10	.8211	.8173	.2308	.2160
13	.8617	.8603	.2180	.2028
17	.9025	.9025	.1921	.1780
20	.9246	.9254	.1741	.1601
25	.9521	.9534	.1407	.1278
30	.9668	.9682	.1212	.1096

Table 1: **Means and Variance of the Price Distributions.** This table is based upon the simulations described in the main text; the underlying true value is $V = 1$. As can be seen, for low entry rates (i.e. a small expected numbers of traders), the prices from the limit order trading on average are closest to the truth, those for dealer markets are furthest. Yet the average price under limit order trading deviates further from the truth as the entry rate increases. The variance of the limit order prices is always larger.

marginal types are history invariant, but it matters in the dealer market). Finally, we computed the end-price that would result from the two trading sessions under both trading regimes and compared these final prices. Table 1 displays the details of the simulation results.

Numerical Observation (Price Informativeness and Dispersion)

(a) *Prices are more efficient in the limit order market for small entry rates,* $av(\mathsf{p_L}) > av(\mathsf{p_D})$*, and in the dealer market for large entry rates,* $av(\mathsf{p_L}) < av(\mathsf{p_D})$*.*

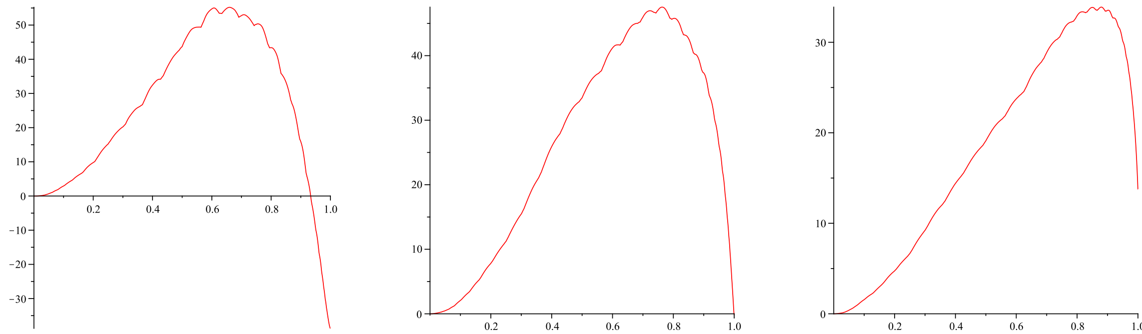
(b) *Prices in the limit order market are more volatile,* $stdev(\mathsf{p_L}) > stdev(\mathsf{p_D})$*.*

(c) *For a high enough entry rate,* $\rho \geq 17$*,* F_{D} *second order stochastically dominates* F_{L}*.*

To understand the finding, observe that theoretically a trade in the dealer market reveals a trader's information more precisely. Consequently, if there are sufficiently many trades, as occurs for large ρ, then dealer market prices should be closer to the mark. Limit order trading has a higher variance because, by Proposition 1, prices react stronger to each trade and thus move faster towards the extremes.

For part (c) observe first that price impact of a large trade is stronger in the limit order market. This generates two opposing effects: 'correct' trades push prices stronger towards the correct value, but 'wrong' trades push prices away stronger. Since the trading threshold for large sales is relatively less extreme in the limit order market, $1 - \pi_{\mathsf{D}}^2 < 1 - \pi_{\mathsf{L}}^2$, a large trade there is also more likely to be in the 'wrong' direction.

Figure 4 displays the plots for the left hand side of equation (3) applied to the empirical price distributions, with x being the limit order market and y being the dealer market. We observe that full-fleshed second order stochastic dominance first $\rho \geq 17$. For



$\rho = 10$ $\rho = 17$ $\rho = 25$

Figure 4: **Second Order Stochastic Dominance of Prices.** The three panels plot equation (3) for three entry rates. In the left panel, $\rho = 10$, cumulation distribution weight for large prices is relatively large for quote driven prices and thus the value for equation (3) dips below zero for large prices. For $\rho = 17$ (middle panel) this no longer occurs and for $\rho = 25$ (right panel) the relation is very clean.

smaller ρ the left hand side of equation (3) applied to F_{L} and F_{D} dips below zero for high prices. This outcome is intuitive: if there are only a few people, then each investor's trade has a relatively large price impact. Since price movements with limit order trading are more extreme, prices will move faster towards both 'wrong' and 'correct' values, leading to a higher concentration of probability weight at the extremes.

In summary, prices in the limit order market are more volatile and on average less efficient than those in the dealer market.

7 Equilibrium in Hybrid Markets

So far we have assumed that the two trading systems operate in isolation. Most real world markets, however, are hybrid markets where investors have the choice of either arranging their trade with a dealer or posting it to the limit order book.[13] For instance, on the TSX or Paris Bourse traders can approach an upstairs dealer or they can send their order directly to the consolidated limit order book. It is therefore natural to ask, what happens when investors can choose the trading regime for their order.

Two types of equilibria arise. First, there could be corner solutions, in which market systems specialize in the quantity that they trade. The second, and arguably more realistic scenario is one where both small and large quantities are traded in both market segments.

[13]Some systems are more convoluted: for instance, on NYSE, a market order that arrives at the specialist's desk could be filled with the current book, with the specialist, or with floor brokers who opt to participate, or the specialist may auction the order to floorbrokers. In Nasdaq, small orders are routed to dealers according to much debated systems of rules. We abstract from these institutional subtleties, and focus on the main distinction between the two general systems.

7.1 Corner Solutions: Market Specialization.

Observe first that the pricing problem in a combined market with specialization resembles that of the dealer market so that the ask price for one unit is always $\mathsf{ask}_{\mathsf{D}}^1$ and that for the two units is $\mathsf{ask}_{\mathsf{D}}^2$, irrespective of the trading mechanism.

In the first corner solution, all single unit trades occur in the dealer segment at price $\mathsf{ask}_{\mathsf{D}}^1$. Two unit trades occur in the limit order segment at price $\mathsf{ask}_{\mathsf{D}}^2$, which is charged for both the first and the second unit. The price for the two unit trade in the dealer segment is anywhere strictly above $\mathsf{ask}_{\mathsf{D}}^2$.

The second corner solution is the reverse of the first. All small trades occur in the limit order segment, all large orders in the dealer segment. In the limit order segment, the equilibrium ask-price for first unit is $\mathsf{ask}_{\mathsf{D}}^1$, and any price strictly above $2\mathsf{ask}_{\mathsf{D}}^2 - \mathsf{ask}_{\mathsf{D}}^1$ is an equilibrium price for the second unit. The ask-price for a single unit in the dealer segment is any price that is strictly larger than $\mathsf{ask}_{\mathsf{D}}^1$, and the price for a two unit trade is $\mathsf{ask}_{\mathsf{D}}^2$. The following proposition summarizes.[14]

Proposition 3 (Hybrid Market Equilibrium with Market Specialization) *There are two kinds of equilibria with specialization in hybrid markets: in the first, all large trades occur in the limit order market, all small trades in the dealer market. In the second, all large trades occur in the dealer market and all small trades in the limit order market.*

7.2 Non-Specialization.

The economically more appealing outcome is one in which trades of all sizes are negotiated in all market segments. In such an interior solution, the costs of order execution must be identical across the two market segments, otherwise traders would switch to the cheaper market. Trade cost equalization then implies that the marginal informed investors are the same in both markets. To see this, observe that traders have the choice between markets and between large and small orders. If a trade of a particular size is cheaper in one market, then traders will choose that market for their trade.

Without loss of generality, we assume that informed investors trade in each system with equal probability and compute the proportions of noise traders in each market segment that ensure equal costs. Let λ_i^j denote the probability that an order of size $j \in \{1, 2\}$ in market $i \in \{\mathsf{L}, \mathsf{D}\}$ stems from a noise trader.

[14] Technically, the equilibrium involved is the Perfect Bayesian Equilibrium and thus requires out-of-equilibrium beliefs and best-responses. For the simplicity of the exposition we follow the tradition of omitting the full and notationally involved description.

Theorem 3 (Hybrid Market Equilibrium without Specialization)
There exists a unique equilibrium in which both quantities are traded on both exchanges. In equilibrium, trading costs coincide for both mechanisms, prices for the small size orders coincide for both trading mechanisms, $\mathsf{ask}_\mathsf{L}^1 = \mathsf{ask}_\mathsf{D}^1$, *and prices for the large size orders satisfy* $2 \cdot \mathsf{ask}_\mathsf{D}^2 = \mathsf{ask}_\mathsf{L}^1 + \mathsf{ask}_\mathsf{L}^2$. *There are more noise traders for small size orders in the limit order segment,* $\lambda_\mathsf{L}^1 > \lambda_\mathsf{D}^1$, *and more noise traders for the large size orders on the dealer segment,* $\lambda_\mathsf{D}^2 > \lambda_\mathsf{L}^2$.

The proof of this result requires a careful construction because changes in trading behavior for one trade size affect the prices for both sizes. Yet the intuition is simple. Consider the market segments in isolation. The ask price for two units in the dealer segment is lower than the ask price for the second unit in the limit order segment. At the same time, the cost of the two unit trade is higher in the dealer segment. To equalize costs, the cost in the limit order segment must rise and it must fall in the dealer segment. Intuitively, to lower cost for trade in the dealer segment the noise level there must rise. This leads to $\lambda_\mathsf{L}^2 < \lambda_\mathsf{D}^2$. A similar argument applies to the single unit case.

Notably, as $2 \cdot \mathsf{ask}_\mathsf{D}^2 = \mathsf{ask}_\mathsf{L}^1 + \mathsf{ask}_\mathsf{L}^2$ and $\mathsf{ask}_\mathsf{L}^1 = \mathsf{ask}_\mathsf{D}^1$, we have $\mathsf{ask}_\mathsf{L}^2 > \mathsf{ask}_\mathsf{D}^2$. In other words, the price for the second unit in the limit order segment is larger than the price for the two-unit trade in the dealer segment. So a two unit trade in the limit order market continues to cause a higher price impact.

Theorem 3 provides a theoretical basis for the empirical finding that upstairs markets —which loosely correspond to our dealer markets— are better at identifying uninformed trades. Our result shows that the co-existence of different structures of trading *necessarily* implies that more uninformed traders seek to trade large quantities on a dealer market.

The information content of trades implied by Theorem 3 is similar to that in Seppi (1990), but the driving force in our model is different. In Seppi (1990), dealer markets are 'better at screening out the informed traders' due to repeated interactions. In our model, traders *self-select* into different markets in such a manner that the information content of large trades is smaller in dealer than in limit order markets.

Volume. While the fraction of the informed traders in either market segment is the same, there are more small noise trades in the limit order segment and more large noise trades in the dealer segment of the hybrid market. This immediately yields the following corollary of Proposition 3.

Corollary (Volume in Hybrid Markets)
In a hybrid market, there are more large transactions in the dealer segment and more small transactions in the limit order segment.

Dynamic Behavior. We have already established that the behavior of traders in the dealer market is not stationary. The hybrid market exhibits a similar non-stationarity.

Corollary (Behavioral Dynamics in Hybrid Markets)
As p_t traverses from 0 to 1, the proportion of large order noise buyers increases in the dealer segment, and the proportion of small order noise buyers increases in the limit order segment; the reverse is true for noise sellers.

The intuition is analogous to that for Proposition 2, though more subtle. Suppose p_t increases to $p_t+\Delta$. If the dealer market were operated in isolation, then by Proposition 2 the trading thresholds in the dealer market would increase. Since in the hybrid market, thresholds are identical for both dealer and limit order segments, the threshold in the limit order segment must also increase. This requires that noise trading there decreases.

8 Conclusion

To the best of our knowledge, we are the first to provide a comprehensive theoretical comparison of order and quote driven markets using a unifying frictionless model. Stripped of all possible market imperfections, the major conceptual difference between the two mechanisms is the sequencing of actions. In quote-driven markets, dealers know the size of the order before setting a price, whereas liquidity providers in an order-driven markets learn the full size of an order that hit their quote only after the fact. Since large orders are posted usually by better informed traders, dealers in quote-driven markets can better assess the information content of the order flow. Our analysis then shows that many empirically identified differences in the trading outcomes of the two mechanisms, regarding price impacts, volume or information contents of orders, can be traced back to the intrinsic difference of the sequencing of orders and quotes.

A Appendix: The Institutional Background

Our treatment of price formation in the dealer market is stylized: effectively, people submit their market orders without knowing the price and there are no standing quotes from dealers. Of course, in real markets dealers do post quotes, but they usually quote only a single bid and a single ask price. Moreover, for most markets, dealers are commonly required to trade a guaranteed minimum number of units at this price (for instance, on Nasdaq a quote must be good for 1000 shares for most stocks). Alternatively, on exchanges such as the TSX or Paris Bourse, the upstairs dealers are required to trade at the best bid or offer (BBO) that is currently on the book, unless the size of the trade is very large. Finally, trading systems or exchanges that include small-order routing (i.e. small orders are given to different dealers according to a pre-determined set of routing rules) require dealers to do price improvement, that is they require dealers to give small order customers the best price that is currently quoted.

None of these institutional details contradict our setup. First, the defining feature of dealer markets is that the dealer will know the size of the trade when quoting the uniform price for the order. Thus the dealer quotes cannot be 'hit' in the same way as a standing limit order in a consolidated limit order book. Next, in the theoretical analysis of our paper we describe that the dealer charges different prices for different quantities. The bid- and ask-prices that she quotes would be for the minimum quantity that he must trade — and this quantity may well be "large"; in other words, the quoted ask price may be $\mathsf{ask}_{\mathsf{D}}^2$, but when facing a small order, the dealer may offer $\mathsf{ask}_{\mathsf{D}}^1$. Third, in our model traders accurately anticipate the price that they will be quoted. Consequently, quotes will be self-fulfilling. Finally, the BBO requirement in upstairs-downstairs markets is trivially satisfied in the hybrid market that we discuss in Section 7.

The important distinction between the two market mechanisms is that when posting prices in a limit order market, the liquidity provider only knows the distribution of order sizes whereas on the dealer market the liquidity provider observes the realized order size prior to posting the price. The main insight of our paper is that this difference alone yields many of the empirical observations in the literature, for instance concerning the information content or the price impact of trades under different trading systems.

B Appendix: Quality and Belief Distributions

The standard parametric approach to signal quality has the quality in $[1/2, 1]$. Yet to derive the distributions of beliefs it is mathematically convenient to normalize the quality so that it is distributed on $[0, 1]$ according to distribution $G(q)$ with continuous density $dG(q) = g(q)$. We further assume that g is symmetric around $1/2$. With this specification, signal qualities q and $1-q$ are equally useful for the individual: if someone receives signal h and has quality $1/4$, then this signal has 'the opposite meaning', i.e. it has the same meaning as receiving signal l with quality $3/4$. Assuming symmetry around

1/2 is thus without loss of generality. Signal qualities are assumed to be independent across agents, and independent of the security's liquidation value V.

Beliefs are derived by Bayes Rule, given signals and signal-qualities. Specifically, if a trader is told that his signal quality is q and receives a high signal then his belief is $q/[q + (1 - q)] = q$ (respectively $1 - q$ if he receives a low signal), because the prior is 1/2. The belief π is thus held by people who receive signal h and quality $q = \pi$ and by those who receive signal l and quality $q = 1 - \pi$. Consequently, the density of individuals with belief π is given by $f_1(\pi) = \pi[dG(\pi) + dG(1 - \pi)]$ in state $V = 1$ and analogously by $f_0(\pi) = (1 - \pi)[dG(\pi) + dG(1 - \pi)]$ in state $V = 0$.

Smith and Sorensen (2008) prove the following property of private beliefs:

Lemma 2 (Symmetric beliefs, Smith and Sorensen (2008))
With the above the signal quality structure, private belief distributions satisfy $F_1(\pi) = 1 - F_0(1 - \pi)$ *for all* $\pi \in (0, 1)$.

Proof: Since $f_1(\pi) = \pi[dG(\pi) + dG(1 - \pi)]$ and $f_0(\pi) = (1 - \pi)[dG(\pi) + dG(1 - \pi)]$, we have $f_1(\pi) = f_0(1 - \pi)$. Integrating, $F_1(\pi) = \int_0^\pi f_1(x)dx = \int_0^\pi f_0(1 - x)dx = \int_{1-\pi}^1 f_0(x)dx = 1 - F_0(1 - \pi)$. □

The belief densities also satisfy the monotone likelihood ratio property because

$$\frac{f_1(\pi)}{f_0(\pi)} = \frac{\pi[dG(\pi) + dG(1 - \pi)]}{(1 - \pi)[dG(\pi) + dG(1 - \pi)]} = \frac{\pi}{1 - \pi}$$

is increasing in π.

While not used in the paper, the reader may be interested to know how one can obtain the distribution of qualities on $[1/2, 1]$ from the above specification. Denote this distribution by $\tilde{G}$ and note that since g is symmetric, $G(1/2) = 1/2$. Then

$$\tilde{G}(q) = \int_{\frac{1}{2}}^{q} g(s)ds + \int_{1-q}^{\frac{1}{2}} g(s)ds = 2\int_{\frac{1}{2}}^{q} g(s)ds = 2G(q) - 2G(1/2) = 2G(q) - 1.$$

In text we already discussed that a uniformly distributed quality can be employed. A more general example that we used for some simulations is the following quadratic quality distribution with density

$$g(q) = \theta\left(q - \frac{1}{2}\right)^2 - \frac{\theta}{12} + 1, \quad q \in [0, 1] \text{ and } \theta \in [-6, 12]. \tag{4}$$

Note that this class includes the uniform density (for $\theta = 0$).

C Appendix: Omitted Proofs

C.1 Existence in the Limit Order Market: Proof of Theorem 1

We focus on the buy-side of the market; thresholds for the sell-side are determined analogously and are symmetric (this follows from the symmetry of private beliefs). Further, we also focus only on the equilibrium for the first unit; the proof for the second unit is analogous. First,

$$F_1(\pi) = \int_0^\pi f_1(s)\ ds = \int_0^\pi s \cdot (g(s) + g(1-s))\ ds = 2\int_0^\pi s \cdot g(s)\ ds,$$

where the last step is due to the symmetry of g around $1/2$. Integrating by parts,

$$F_1(\pi) = 2\int_0^\pi s \cdot g(s)\ ds = 2sG(s)|_0^\pi - 2\int_0^\pi G(s)\ ds = 2\pi G(\pi) - 2\int_0^\pi G(s)\ ds.$$

Then

$$F_1(\pi) + F_0(\pi) = 2\int_0^\pi s \cdot g(s)\ ds + 2\int_0^\pi (1-s)\cdot g(s)\ ds = 2G(\pi).$$

Further

$$\beta_1^1 + \beta_0^1 = (1-\mu)/2 + \mu(1 - F_1(\pi)) + (1-\mu)/2 + \mu(1 - F_0(\pi)) = 1 + \mu - 2\mu G(\pi) \quad (5)$$

Thus

$$\pi^1 = \frac{\beta_1^1}{\beta_1^1 + \beta_0^1} \ \Leftrightarrow\ \pi^1(1+\mu-2\mu G(\pi^1)) = (1-\mu)/2 + \mu\left(1 - 2\pi^1 G(\pi^1) + 2\int_0^{\pi^1} G(s)\ ds\right).$$

Further simplification leads to

$$\frac{2\pi^1 - 1}{4}\ \frac{\mu+1}{\mu} = \int_0^{\pi^1} G(s)\ ds. \quad (6)$$

Both sides of (6) are continuous in π^1. At $\pi^1 = 1/2$, the left-hand side of (6) is 0, while the right-hand side is positive. At $\pi^1 = 1$, the ordering is reversed: the left-hand side is at least $1/2$ ($\mu \leq 1$ is the probability that a given trader is informed), while the right-hand side is exactly $1/2$ (we show this in the appendix). Consequently, there exists a π^1 that warrants equality.

To prove uniqueness it suffices to show that the left hand side of (6) is steeper than the right hand side for all $\pi^1 \in (1/2, 1)$. The slope of the left hand side in π^1 is simply

$(1+\mu)/2\mu \geq 1$. The slope of the right hand side is $G(\pi^1) < 1$ for all $\pi^1 \in (1/2, 1)$, since G is a cdf.

C.2 Equilibrium Threshold Beliefs Maximize the Spread: Proof of Lemma 1

To simplify the exposition we omit the superscripts for trade-size. One can define the ask-price as a function of a hypothetical marginal trader π so that

$$\mathsf{ask}(\pi) = \frac{\beta_1(\pi)p_t}{\beta_1(\pi)p_t + \beta_0(\pi)(1-p_t)}.$$

The Lemma states that the equilibrium π maximizes the ask-price. The first order condition, $\frac{\partial}{\partial\pi}\mathsf{ask}(\pi) = 0$, is equivalent to

$$\frac{\partial}{\partial\pi}\frac{\beta_1(\pi)p_t}{\beta_1(\pi)p_t + \beta_0(\pi)(1-p_t)} = 0 \quad\Leftrightarrow\quad \frac{\beta_0(\pi)}{\beta_1(\pi)} = \frac{\beta_0'(\pi)}{\beta_1'(\pi)}. \tag{7}$$

Now recall that $\beta_v(\pi) = \lambda + \mu(1 - F_v(\pi))$. Thus the right hand side of the second relation above is

$$\frac{\beta_0'(\pi)}{\beta_1'(\pi)} = \frac{f_0(\pi)}{f_1(\pi)}.$$

In Appendix B we argue that $f_1(\pi) = \pi[dG(\pi) + dG(1-\pi)]$, and $f_0(\pi) = (1-\pi)[dG(\pi) + dG(1-\pi)]$. Thus $f_0/f_1 = (1-\pi)/\pi$. With this we have

$$\frac{\beta_0'(\pi)}{\beta_1'(\pi)} = \frac{1-\pi}{\pi}. \tag{8}$$

Next, consider the equilibrium condition $\mathsf{E}\pi = \mathsf{ask}(\pi)$. Rearranging and simplifying, threshold π must satisfy

$$\frac{\beta_0(\pi)}{\beta_1(\pi)} = \frac{1-\pi}{\pi}. \tag{9}$$

Combining equations (7) and (8) delivers the same relation as equation (9) and thus the equilibrium condition yields $\arg\max \mathsf{ask}(\pi)$.

C.3 Existence in the Dealer Market: Proof of Theorem 2

We will drop all time subscripts. The proof proceeds in two steps. In the first step, we show that for all π^2, there exists a unique π^1 such that (a) π^1 solves $\mathsf{E}\pi^1 = \mathsf{ask}^1(\pi^1, \pi^2)$, and (b) π^1 is strictly increasing in π^2.

In the second step we show that there exists a unique π^2 that solves $\mathsf{E}\pi^2 + \mathsf{E}\pi^1(\pi^2) = 2\mathsf{ask}^2(\pi^2)$.

Step 1: Showing the existence of π^1 is akin to showing the existence of an equilibrium in the limit order market: π^1 must satisfy

$$\pi^1 = \frac{\beta_1^1}{\beta_1^1 + \beta_0^1} \quad \Leftrightarrow \quad \pi^1 = \frac{\lambda + \mu(F_1(\pi^2) - F_1(\pi^1))}{\lambda + \mu(F_1(\pi^2) - F_1(\pi^1)) + \lambda + \mu(F_0(\pi^2) - F_0(\pi^1))}. \tag{10}$$

Trade informativeness immediately implies that π^1 must be at least 1/2. What remains to show is existence and uniqueness of $\pi^1 \in [1/2, \pi^2)$.

$$\beta_1^1 + \beta_0^1 \;\; = \;\; 2\lambda + 2\mu(G(\pi^2) - G(\pi^1)). \tag{11}$$

We can then simplify (10) to

$$G(\pi^2)(\pi^2 - \pi^1) - \frac{\lambda}{2\mu}\,(2\pi^1 - 1) = \int_{\pi^1}^{\pi^2} G(s)\; ds. \tag{12}$$

Both sides of (12) are continuous in π^1. At $\pi^1 = \pi^2$, the right-hand side of (12) is 0, while the left-hand side is $-(1-\mu)/4/(2\mu) < 0$. Now consider $\pi^1 = 1/2$. Then

$$G(\pi^2)(\pi^2 - 1/2) > \int_{\frac{1}{2}}^{\pi^2} G(s)ds,$$

where the inequality obtains because G is increasing. At $\pi^1 = 1/2$ the right-hand side is smaller than the left-hand side. Since both left-hand side and right-hand side are continuous in π^1, there exists a π^1 that warrants equality.

To prove *uniqueness* it suffices to apply Lemma 1: by the same arguments as in the proof there, the ask price ask^1 is maximal at π^1. And since $\mathsf{E}\pi$ is monotonic in π, there is exactly one intersection of the ask^1 and $\mathsf{E}\pi$.

We now show that π^1 increases strictly in π^2. The equilibrium condition for the small size buying-threshold, equation (2), can be reformulated to $\beta_0^1(\pi^1, \pi^2)/\beta_1^1(\pi^1, \pi^2) = (1 - \pi^1)/\pi^1$. It thus suffices to compute the derivative of $\beta_0^1(\pi^1, \pi^2)/\beta_1^1(\pi^1, \pi^2)$ with respect to π^2 and show that it is strictly negative:

$$\frac{\partial}{\partial \pi^2} \frac{\beta_0^1(\pi^1, \pi^2)}{\beta_1^1(\pi^1, \pi^2)} < 0 \;\Leftrightarrow\; f_0(\pi^2)\beta_1^1 - f_1(\pi^2)\beta_0^1 < 0 \;\Leftrightarrow\; \frac{f_0(\pi^2)}{f_1(\pi^2)} < \frac{\beta_0^1}{\beta_1^1}.$$

The RHS, however, is just $(1 - \pi^1)/\pi^1$ by the equilibrium condition; moreover, the definition of f_v ensures that $f_0/f_1 = (1 - \pi^2)/\pi^2$. The above can then be simplified to $(1 - \pi^2)/\pi^2 < (1 - \pi^1)/\pi^1$, which certainly holds as $\pi^2 > \pi^1$.

Step 2: We now turn to determine the unique π^2. We already know from Step 1, that for every π^2 there exists a unique π^1 such that $\mathsf{ask}^1(\pi^1, \pi^2) = \mathsf{E}\pi^1$. The equilibrium π^2 must then satisfy

$$\mathsf{E}\pi^2 + \mathsf{E}\pi^1 = 2\mathsf{ask}^2(\pi^2). \tag{13}$$

Both the right hand side and the left hand side of this equation are functions of π^2. Let $\pi^* := \arg\max_\pi \mathsf{ask}^2(\pi)$. Since π^1 increases in π^2 and since in equilibrium,

$$\frac{1-\pi^1}{\pi^1} = \frac{\beta_0^1(\pi^1, \pi^2)}{\beta_1^1(\pi^1, \pi^2)}, \tag{14}$$

we have that for any $\pi^2 < 1$ the corresponding π^1 that satisfies (14) also has $\pi^1 < \pi^*$ and for $\pi^2 = 1$ we have $\pi^1 = \pi^*$. Consequently, for $\pi^2 = \pi^*$, the right hand side of (13) is $2\mathsf{E}\pi^*$ and right hand side > left hand side. As $\pi^2 = 1$, the left hand side of (13) is $1 + \mathsf{E}\pi^*$, the right hand side is $2 \times \mathsf{ask}^2(1) = 2 \cdot p_t < 1 + \mathsf{E}\pi^*$ because $\mathsf{E}\pi^* > p_t$ for $\pi^* > 1/2$. Thus at $\pi^2 = 1$ holds right hand side < left hand side.

Since π^2 maximizes ask^2, the right hand side is decreasing in π^2 for $\pi^2 > \pi^*$. Since both $\mathsf{E}\pi^2$ and $\mathsf{E}\pi^1$ increase in π^2, the left hand side increases in π^2. Together with the boundary relations for $\pi^2 \in \{\pi^*, 1\}$, we know that left and right hand sides coincide exactly once for $\pi^2 \in (\pi^*, 1)$. By the proof of Theorem 1 we know that $\pi^2 \geq \pi^*$ because for $\pi < \pi^*$, $\mathsf{E}\pi < \mathsf{ask}^2(\pi)$. Thus the equilibrium threshold pair $\{\pi^1, \pi^2\}$ exists and is unique.

C.4 Liquidity Measures: Proof of Proposition 1

(a) Since both $\mathsf{ask}^1_\mathsf{L}$ and $\mathsf{ask}^1_\mathsf{D}$ are single-peaked and since the respective peaks mark the equilibrium prices, it suffices to show that $\mathsf{E}\pi^1_\mathsf{D} < \mathsf{ask}^1_\mathsf{L}(\pi^1_\mathsf{D})$. Thus we want to show that

$$\frac{\pi^1_\mathsf{D}}{1-\pi^1_\mathsf{D}} < \frac{\beta^1_{1,\mathsf{L}}(\pi^1_\mathsf{D})}{\beta^1_{0,\mathsf{L}}(\pi^1_\mathsf{D})} \quad \Leftrightarrow \quad \frac{\pi^1_\mathsf{D}}{1-\pi^1_\mathsf{D}} < \frac{2\lambda + \mu(1 - F_1(\pi^1_\mathsf{D}))}{2\lambda + \mu(1 - F_0(\pi^1_\mathsf{D}))}.$$

We know that the equilibrium π^1_D satisfies

$$\frac{\pi^1_\mathsf{D}}{1-\pi^1_\mathsf{D}} = \frac{\lambda + \mu(F_1(\pi^2_\mathsf{D}) - F_1(\pi^1_\mathsf{D})}{\lambda + \mu(F_0(\pi^2_\mathsf{D}) - F_0(\pi^1_\mathsf{D})}. \tag{15}$$

Rearranging the inequality we get

$$2\lambda\pi^1_\mathsf{D} + \mu\pi^1_\mathsf{D} \;\; < \;\; 2\lambda(1-\pi^1_\mathsf{D}) + \mu(1-\pi^1_\mathsf{D}) + \mu\pi^1_\mathsf{D}F_0(\pi^1_\mathsf{D}) - \mu(1-\pi^1_\mathsf{D})F_1(\pi^1_\mathsf{D}). \tag{16}$$

By the same token, we can rearrange (15) to obtain

$$\mu\pi^1_{\mathsf{D}}F_0(\pi^1_{\mathsf{D}}) - \mu(1-\pi^1_{\mathsf{D}})F_1(\pi^1_{\mathsf{D}}) \;=\; \lambda\pi^1_{\mathsf{D}} + \mu\pi^1_{\mathsf{D}}F_0(\pi^2_{\mathsf{D}}) - \lambda(1-\pi^1_{\mathsf{D}}) + \mu(1-\pi^1_{\mathsf{D}})F_1(\pi^2_{\mathsf{D}}).$$

Substituting the above into (16) we obtain

$$2\lambda\pi^1_{\mathsf{D}} + \mu\pi^1_{\mathsf{D}} \;<\; 2\lambda(1-\pi^1_{\mathsf{D}}) + \mu(1-\pi^1_{\mathsf{D}}) + \lambda\pi^1_{\mathsf{D}} + \mu\pi^1_{\mathsf{D}}F_0(\pi^2_{\mathsf{D}}) - \lambda(1-\pi^1_{\mathsf{D}}) + \mu(1-\pi^1_{\mathsf{D}})F_1(\pi^2_{\mathsf{D}})$$

Rearranging and simplifying, we obtain

$$\frac{\pi^1_{\mathsf{D}}}{1-\pi^1_{\mathsf{D}}} \;<\; \frac{\lambda+\mu(1-F_1(\pi^2_{\mathsf{D}})}{\lambda+\mu(1-F_0(\pi^2_{\mathsf{D}})} \;\Leftrightarrow\; \mathsf{E}\pi^1_{\mathsf{D}} < \mathsf{ask}^2_{\mathsf{D}}(\pi^2_{\mathsf{D}}).$$

The last part obviously holds because $\mathsf{E}\pi^1_{\mathsf{D}} < {}^1\!/_2(\mathsf{E}\pi^1_{\mathsf{D}} + \mathsf{E}\pi^2_{\mathsf{D}}) = \mathsf{ask}^2_{\mathsf{D}}$.

(b) We only need to show that $\mathsf{ask}^2_{\mathsf{L}} > \mathsf{ask}^2_{\mathsf{D}}$. Both $\mathsf{ask}^2_{\mathsf{L}}$ and $\mathsf{ask}^2_{\mathsf{D}}$ have the same functional form; further π^2_{L} maximizes $\mathsf{ask}^2_{\mathsf{L}}$. Since $\mathsf{ask}^2_{\mathsf{L}}(\pi) > \mathsf{E}\pi$ for $\pi < \pi^2_{\mathsf{L}}$ (as shown in the proof of Theorem 1), it must hold that $\pi^2_{\mathsf{D}} \geq \pi^2_{\mathsf{L}}$. Suppose that $\pi^2_{\mathsf{D}} = \pi^2_{\mathsf{L}}$. In equilibrium, π^2_{D} solves $2(\mathsf{ask}^2_{\mathsf{D}} - \mathsf{E}\pi^2_{\mathsf{D}}) = \mathsf{E}\pi^2_{\mathsf{D}} - \mathsf{ask}^1_{\mathsf{L}}$. Thus when $\pi^2_{\mathsf{D}} = \pi^2_{\mathsf{L}}$, then the left hand side of the above is 0, whereas the right hand side is positive. Consequently, $\pi^2_{\mathsf{D}} > \pi^2_{\mathsf{L}}$ and thus $\mathsf{ask}^2_{\mathsf{L}} > \mathsf{ask}^2_{\mathsf{D}}$.

(c) In text.

C.5 Behavioral Dynamics: Proof of Proposition 2

We show only the proof for the buy-thresholds; the sell-thresholds are analogous.

For this proof, let $\mathsf{ask}^1(\pi^1(\pi))$ denote the equilibrium price for the a single unit trade that would transpire if $\pi^2 = \pi$ so that $\pi^1(\pi)$ solves (15). To show that the large dealer-market buy threshold π^2 increases in the prior p, it suffices to show that $\delta(p,\pi) := 2\mathsf{ask}^2(\pi) - \mathsf{E}\pi - \mathsf{ask}^1(\pi^1(\pi)))$ decreases for π whenever $\mathsf{E}\pi \geq \mathsf{ask}^2(\pi)$ and increases in p when $\pi = \pi^2$ such that $\delta(p,\pi^2) = 0$.

Fix p and suppose δ is a decreasing function of π. If p increases from p to $p+\epsilon$ then the curve $\delta(p+\epsilon,\pi)$ as a function of π only lies to the North-East of the the curve $\delta(p,\pi)$ increases in p. Consequently, the root of $\delta(p+\epsilon,\pi)$ in π is larger than the root of $\delta(p,\pi)$.

In the proof of Theorem 2 we showed that $\mathsf{E}\pi$ intersects $\mathsf{ask}^2(\pi)$ once from below at π^*, and thus $\mathsf{ask}^2(\pi)$ decreases for $\pi > \pi^*$. Moreover, threshold π^1 increases in π^2 and thus so does $\mathsf{ask}^1(\pi^1(\pi^2))$. Thus δ decreases for the relevant values of π.

To show that δ increases in p we first define $\ell := (1-p)/p$, and thus have to show that

$\partial\delta/\partial\ell < 0$. Also define $\rho_2 := \beta_0^2/\beta_1^2$, $\rho_1 := \beta_0^1/\beta_1^1$ and $\gamma := (1-\pi^2)/\pi^2$. Consequently, $\mathsf{ask}^2 = 1/(1+\rho_2\ell)$ so that $\rho_2 = (1/\ell)\cdot(1-\mathsf{ask}^2)/\mathsf{ask}^2$. Then

$$\begin{aligned}\frac{\partial\delta}{\partial\ell} &= -2\rho_2(\mathsf{ask}^2)^2 + \gamma\mathsf{E}\pi^2 + \rho_1(\mathsf{ask}^1)^2 &(17)\\ &= -2(1-\mathsf{ask}^2)\mathsf{ask}^2\ \frac{1}{\ell} + \mathsf{E}\pi^2(1-\mathsf{E}\pi^2)\ \frac{1}{\ell} + (1-\mathsf{ask}^1)\mathsf{ask}^1\ \frac{1}{\ell}. &(18)\end{aligned}$$

Considering now the value of π so that $\pi = \pi^2$ describes an equilibrium, we can use $\mathsf{ask}^1 = 2\mathsf{ask}^2 - \mathsf{E}\pi^2$ and thus simplify further

$$= -\frac{2}{\ell}\left((\mathsf{ask}^2)^2 + (\mathsf{E}\pi^2)^2 + 2\mathsf{E}\pi^2\mathsf{ask}^2\right) = -\frac{2}{\ell}\left(\mathsf{ask}^2 + \mathsf{E}\pi^2\right)^2 < 0.$$

C.6 Existence of the General Hybrid Market Equilibrium: Proof for Theorem 3

As stated in the main text, without loss of generality we can assume that informed traders act on either venue with equal probability. Next, in equilibrium, the beliefs of the marginal traders in both types of markets must coincide for the same quantity. Using the equilibrium in the isolated markets as a starting point, we construct the equilibrium in the hybrid market by adjusting the distribution of noise trading in such a manner that execution costs are equal on both exchanges; thresholds will then automatically coincide. Note that when computing the prices with the new noise distribution, one must re-normalize the probabilities of buys and sales — yet these normalizing factors trivially drop out.

The proof will employ several variables and functions. Specifically,

(1) Let $\bar{\pi}_j^i$ denote the equilibrium marginal buyer in market $j = \mathsf{L}, \mathsf{D}$ of $i = 1, 2$ units when markets are viewed in isolation; similarly for $\bar{\mathsf{ask}}_j^i$.

(2) Let λ^i define the mass of noise traders that trade i units in the *limit order* segment of the market. In isolated markets, a noise buy of either size occurs with chance λ.

(2) We will use $\mathsf{ask}_j^i(\pi^1, \pi^2, \lambda^1, \lambda^2)$, $i = 1, 2$, $j = \mathsf{L}, \mathsf{D}$, to denote a function of the noise levels and thresholds (as also used in Figure 2). We will drop the arguments from ask_j^i whenever their usage is clear from context.

(3) For any $x \in [0, 1]$ define $\pi^2(x) = \bar{\pi}_\mathsf{L}^2 + x \cdot (\bar{\pi}_\mathsf{D}^2 - \bar{\pi}_\mathsf{L}^2)$. Note that $\pi^2(0) = \bar{\pi}_\mathsf{L}^2$ and $\pi^2(1) = \bar{\pi}_\mathsf{D}^2$.

To construct the equilibrium, we proceed in the following steps

Step 1: $\forall x$ we define $\lambda^2(x)$ as the value for λ^2 so that $\mathsf{E}\pi^2(x) = \mathsf{ask}_\mathsf{L}^2(\cdot, \pi^2(x), \cdot, \lambda^2(x))$.

We show that a unique such $\lambda^2(x)$ exists, that $\lambda^2(x) \in (0, \lambda]$, and that $\frac{d}{dx}\lambda^2(x) < 0$.

Step 2: $\forall x$ and given $\lambda^2(x)$ we construct $\lambda^1(x)$ as the value for λ^1 and $\pi^1(x)$ as the value for π^1 so that

$$\mathsf{E}\pi^1(x) = \mathsf{ask}^1_{\mathsf{L}}(\pi^1(x), \pi^2(x), \lambda^1(x), \lambda^2(x)) = \mathsf{ask}^1_{\mathsf{D}}(\pi^1(x), \pi^2(x), \lambda^1(x), \lambda^2(x))$$

We show that unique such $\lambda^1(x), \pi^1(x)$ exist, that $\lambda^1(x) \in (\lambda, 2\lambda)$, and that $\frac{d}{dx}\pi^1(x) > 0$ in x.

For fixed x we then have $\lambda^1(x), \lambda^2(x)$, $\pi^1(x)$, and $\pi^2(x)$. By construction, $\mathsf{ask}^1_{\mathsf{L}}$, $\mathsf{ask}^1_{\mathsf{D}}$, $\mathsf{ask}^2_{\mathsf{L}}$ are equilibrium prices in the respective market segments when these are considered in isolation.

The key component left to show is that there exists a unique $x^* \in (0,1)$ so that

$$\mathsf{E}\pi^1(x^*) + \mathsf{E}\pi^2(x^*) = 2\mathsf{ask}^2_{\mathsf{D}}(\pi^1(x^*), \pi^2(x^*), \lambda^1(x^*), \lambda^2(x^*)),$$

which is the equilibrium condition for the dealer market. This is accomplished in the remaining two steps.

Step 3: We show $\frac{d}{dx}\mathsf{ask}^2_{\mathsf{D}}(\pi^1(x), \pi^2(x), \lambda^1(x), \lambda^2(x)) < 0$ and $\frac{d}{dx}(\mathsf{E}\pi^1(x) + \mathsf{E}\pi^2(x)) > 0$.

Step 4: We show

$$\begin{aligned}
2\mathsf{ask}^2_{\mathsf{D}}(\pi^1(0), \pi^2(0), \lambda^1(0), \lambda^2(0)) - [\mathsf{E}\pi^1(0) + \mathsf{E}\pi^2(0)] &> 0, \text{ and} \\
2\mathsf{ask}^2_{\mathsf{D}}(\pi^1(1), \pi^2(1), \lambda^1(1), \lambda^2(1)) - [\mathsf{E}\pi^1(1) + \mathsf{E}\pi^2(1)] &< 0.
\end{aligned}$$

Step 5: We show that π^2 cannot be smaller than $\bar{\pi}^2_{\mathsf{L}}$ or larger than $\bar{\pi}^2_{\mathsf{D}}$.

Steps 3 and 4 together yield existence of the desired equilibrium x^*. Further we show that $\pi^2 \in (\bar{\pi}^2_{\mathsf{L}}, \bar{\pi}^2_{\mathsf{D}})$ and $\lambda^1(x^*) > \lambda$ and $\lambda^2(x^*) < \lambda$ so that there is more noise in the limit order book than in the dealer segment of the market for the small units, and the converse is true for large orders.

The details of the steps are as follows.

Step 1: Define $\lambda^2(x)$ as the level λ^2 that solves

$$\frac{\pi^2}{1-\pi^2} = \frac{\lambda^2 + \mu(1 - F_1(\pi^2))}{\lambda^2 + \mu(1 - F_0(\pi^2))}, \tag{19}$$

for $\pi^2 = \pi^2(x)$. To show that $\lambda^2(x)$ exists, we will show that for every $\lambda^2 \in [0, \lambda]$, there exists a unique $\pi^2 \in [\bar{\pi}^2_{\mathsf{L}}, 1]$ that solves (19), that π^2 is strictly decreasing in λ^2, that $\pi^2 = 1$ for $\lambda^2 = 0$, and that $\pi^2 = \bar{\pi}^2_{\mathsf{L}}$ for $\lambda^2 = \lambda$. With such a 1:1 mapping we then know that for every $\pi^2 \in [\bar{\pi}^2_{\mathsf{L}}, 1]$ there exists a corresponding $\lambda^2 \in [0, \lambda]$ that solves (19) and strictly decreases in π^2.

To see this, rewrite (19) to obtain the following equation which is analogous to (6)

$$\frac{\lambda^2 + \mu}{2\mu}\ (2\pi^2 - 1) = \int_0^{\pi^2} G(s)\ ds. \tag{20}$$

We have shown before that (20) has unique solution in π^2 for $\lambda^2 = \lambda$, namely the equilibrium threshold $\bar{\pi}^2_{\mathsf{L}}$. The right hand side is an increasing function of π^2 with slope $G(\pi^2) < 1$ for $\pi^2 < 1$. For $\lambda^2 \in [0, \lambda]$, the left hand side is a linear function in π^2 with slope $1 + \lambda^2/\mu \geq 1$. Next, at $\pi^2 = 1/2$ the left hand side is zero, the right hand side is positive and at $\pi^2 = 1$, the right hand side is $1/2$ (as shown before), the left hand side is $1/2 + \lambda^2/(2\mu) \geq 1/2$ (with equality at $\lambda^2 = 0$). Consequently, as the slope of the left hand side exceeds the slope of the right hand side, the solution to (20) in π^2 is unique for every λ^2. As the slope of the left hand side strictly increases in λ^2, the solution of (20) in π^2 strictly decreases in λ^2.

Step 2: (*a*) *For* $\pi^2 = \pi^2(x)$ *and* $\lambda^2 = \lambda^2(x)$, $\exists \lambda^1(x) \in (\lambda, 2\lambda)$ *so that* $\mathsf{ask}^1_{\mathsf{L}} = \mathsf{ask}^1_{\mathsf{D}}$.

By the same arguments as employed in the existence theorems, there exist unique π^1_{L} and π^1_{D} that solve respectively for any $\lambda^1 \in [\lambda, 2\lambda]$

$$\frac{\pi^1_{\mathsf{L}}}{1 - \pi^1_{\mathsf{L}}} = \frac{\lambda^1 + \lambda^2 + \mu(1 - F_1(\pi^1_{\mathsf{L}}))}{\lambda^1 + \lambda^2 + \mu(1 - F_0(\pi^1_{\mathsf{L}}))}, \quad \frac{\pi^1_{\mathsf{D}}}{1 - \pi^1_{\mathsf{D}}} = \frac{2\lambda - \lambda^1 + \mu(F_1(\pi^2) - F_1(\pi^1_{\mathsf{D}}))}{2\lambda - \lambda^1 + \mu(F_0(\pi^2) - F_0(\pi^1_{\mathsf{D}}))}. \tag{21}$$

Denote these solutions by $\pi^1_{\mathsf{L}}(\lambda^1)$ and $\pi^1_{\mathsf{D}}(\lambda^1)$. We now determine that there exists a unique value λ^1 such that for given $\lambda^2(x)$ and $\pi^2(x)$ we have $\pi^1_{\mathsf{L}}(\lambda^1) = \pi^1_{\mathsf{D}}(\lambda^1)$. This value λ^1 is the desired $\lambda^1(x)$ and then $\pi^1(x) = \pi^1_{\mathsf{L}}(\lambda^1) = \pi^1_{\mathsf{D}}(\lambda^1)$.

(*i*) For $\lambda^1 \in [\lambda, 2\lambda]$, we have $\frac{\partial}{\partial \lambda_1}\pi^1_{\mathsf{L}} < 0$ and $\frac{\partial}{\partial \lambda_1}\pi^1_{\mathsf{D}} > 0$ because thresholds decrease in the level of noise.

(*ii*) At $\lambda^1 = \lambda$, we have $\pi^1_{\mathsf{D}}(\lambda^1) \leq \bar{\pi}^1_{\mathsf{D}}$, because $\pi^2(x) \leq \bar{\pi}^2_{\mathsf{D}}$ and π^1_{D} increases in π^2. At the same time, at $\lambda^1 = \lambda$ we have $\pi^1_{\mathsf{L}}(\lambda^1) \geq \bar{\pi}^1_{\mathsf{L}}$ since $\lambda^2(x) \leq \lambda$ and $\frac{\partial}{\partial \lambda_2}\pi^1_{\mathsf{L}} < 0$. Thus at $\lambda^1 = \lambda$ we have $\pi^1_{\mathsf{D}}(\lambda^1) \leq \bar{\pi}^1_{\mathsf{D}} < \bar{\pi}^1_{\mathsf{L}} \leq \pi^1_{\mathsf{L}}(\lambda^1)$.

(*iii*) At $\lambda^1 = 2\lambda$ we have $\pi^1_{\mathsf{D}}(\lambda^1) = \pi^2$ whereas $\pi^1_{\mathsf{L}}(\lambda^1) < \pi^2$ (for π^2 solves the first equation in (21) for $\lambda^1 = 0$). Consequently at $\lambda^1 = 2\lambda$, $\pi^1_{\mathsf{L}}(\lambda^1) < \pi^1_{\mathsf{D}}(\lambda^1)$.

Combining (*i*), (*ii*), and (*iii*) we have that there exists a unique $\lambda^1 \in (\lambda, 2\lambda)$ so that

$\pi^1_{\mathsf{L}}(\lambda^1) = \pi^1_{\mathsf{D}}(\lambda^1)$. We will henceforth denote this λ^1 as $\lambda^1(x)$ and the corresponding threshold by $\pi^1(x)$.

Step 2: (b) *We show that* $\frac{d}{dx}\pi^1(x) > 0$.

Suppose not. Then there exist $x, \tilde{x}$ with $x < \tilde{x}$ such that $\pi^1(\tilde{x}) \leq \pi^1(x)$. From Step 1 we know that $\lambda^2(\tilde{x}) < \lambda^2(x)$. But $\pi^1_{\mathsf{L}}(\lambda^1(\tilde{x})) \leq \pi^1_{\mathsf{L}}(\lambda^1(x))$ only if $\lambda^1(x) + \lambda^2(x) \leq \lambda^1(\tilde{x}) + \lambda^2(\tilde{x})$. Then it must hold that $\lambda^1(\tilde{x}) > \lambda^1(x)$. Consequently, there is less noise trading in the single-unit dealer market segment for $\tilde{x}$ than for x. Moreover, $\pi^2(\tilde{x}) > \pi^2(x)$. Then it must be that $\pi^1_{\mathsf{D}}(x) < \pi^1_{\mathsf{D}}(\tilde{x})$, a contradiction.

Step 3: (a) *We show that* $\frac{d}{dx}\mathsf{ask}^2_{\mathsf{D}} < 0$. First note that

$$\frac{d}{dx}\mathsf{ask}^2_{\mathsf{D}} < 0 \quad \Leftrightarrow \quad \frac{d}{dx}\frac{2\lambda - \lambda^2(x) + \mu(1 - F_1(\pi^2(x)))}{2\lambda - \lambda^2(x) + \mu(1 - F_0(\pi^2(x)))} < 0.$$

Differentiating with respect to x, we obtain

$$\frac{d}{dx}\frac{2\lambda - \lambda^2(x) + \mu(1 - F_1(\pi^2(x)))}{2\lambda - \lambda^2(x) + \mu(1 - F_0(\pi^2(x)))} < 0$$

which is equivalent to

$$\begin{aligned}-\frac{d\lambda^2}{dx}\mu \cdot \left[F_1(\pi^2(x)) - F_0(\pi^2(x))\right] - \frac{d\pi^2}{dx} \cdot [f_1(\pi^2(x))(2\lambda - \lambda^2(x) + \mu(1 - F_0(\pi^2(x)))) - \\ -f_0(\pi^2(x))(2\lambda - \lambda^2(x) + \mu(1 - F_1(\pi^2(x))))] \quad < \quad 0.\end{aligned}$$

We know that $-\frac{d}{dx}\lambda^2(x) > 0$, $F_1(\pi^2) - F_0(\pi^2) < 0$ and $-\frac{d}{dx}\pi^2(x) < 0$. So if we can show that the last term is positive, then the derivative is indeed negative. In the proof of Lemma 1 we showed that $f_1(\pi^2)/f_0(\pi^2) = \pi^2/(1 - \pi^2)$. Thus

$$\begin{aligned}f_1(\pi^2(x))(2\lambda - \lambda^2(x) + \mu(1 - F_0(\pi^2(x)))) \quad &> \quad f_0(\pi^2(x))(2\lambda - \lambda^2(x) + \mu(1 - F_1(\pi^2(x)))) \\ \Leftrightarrow \quad \frac{\pi^2(x)}{1 - \pi^2(x)} \quad &> \quad \frac{2\lambda - \lambda^2(x) + \mu(1 - F_1(\pi^2(x)))}{2\lambda - \lambda^2(x) + \mu(1 - F_0(\pi^2(x)))}.\end{aligned}$$

The last relation holds because (i) by definition of $\lambda^2(x)$,

$$\frac{\pi^2(x)}{1 - \pi^2(x)} = \frac{\lambda^2(x) + \mu(1 - F_1(\pi^2(x)))}{\lambda^2(x) + \mu(1 - F_0(\pi^2(x)))},$$

(ii) $\lambda^2(x) < 2\lambda - \lambda^2(x)$, which holds because $\lambda^2(x) < \lambda$, and thus (iii)

$$\frac{\lambda^2(x) + \mu(1 - F_1(\pi^2(x)))}{\lambda^2(x) + \mu(1 - F_0(\pi^2(x)))} > \frac{2\lambda - \lambda^2(x) + \mu(1 - F_1(\pi^2(x)))}{2\lambda - \lambda^2(x) + \mu(1 - F_0(\pi^2(x)))}.$$

Step 3: (*b*) *We show that* $\frac{d}{dx}(\mathsf{E}\pi^1(x) + \mathsf{E}\pi^2(x)) > 0$.

We know that $\frac{d}{dx}\pi^2(x) > 0$ by construction, and $\frac{d}{dx}\pi^1(x) > 0$ by Step 2(b). Thus we know that $\frac{d}{dx}(\mathsf{E}\pi^1(x) + \mathsf{E}\pi^2(x)) > 0$.

Step 4: (*a*) *We show that at* $x = 0$, $2\mathsf{ask}_\mathsf{D}^2 - [\mathsf{E}\pi^1(0) + \mathsf{E}\pi^2(0)] > 0$.

At $x = 0$ we have $\pi^2(0) = \bar{\pi}_\mathsf{L}^2$ and $\lambda^2(0) = \lambda$. Then $\mathsf{ask}_\mathsf{D}^2 = \bar{\mathsf{ask}}_\mathsf{L}^2 = \mathsf{E}\bar{\pi}_\mathsf{L}^2 = \mathsf{E}\pi^2(0)$. From Step 2(a) we know that $\pi^1(0) < \pi^2(0)$ so that $\mathsf{E}\pi^1(0) + \mathsf{E}\pi^2(0) < 2\mathsf{ask}_\mathsf{D}^2$.

Step 4:(*b*) *We show that* $2\mathsf{ask}_\mathsf{D}^2 - [\mathsf{E}\pi^1(1) + \mathsf{E}\pi^2(1)] < 0$.

At $x = 1$, we know that $\pi^2(1) = \bar{\pi}_\mathsf{D}^2$, consequently, $\lambda^2(1) < \lambda$. Then $\mathsf{ask}_\mathsf{D}^2 < \bar{\mathsf{ask}}_\mathsf{D}^2$ because the noise in $\mathsf{ask}_\mathsf{D}^2$ is higher than in $\bar{\mathsf{ask}}_\mathsf{D}^2$, $2\lambda - \lambda^2(1) > \lambda$. Moreover by Step 2 (*a*) we know that, $\pi^1(1) = \pi_\mathsf{D}^1(\lambda^1(1)) > \pi_\mathsf{D}^1(\lambda) = \bar{\pi}_\mathsf{D}^1$. Taken together, we have

$$\mathsf{E}\pi^2(1) + \mathsf{E}\pi^1(1) = \mathsf{E}\bar{\pi}_\mathsf{D}^2 + \mathsf{E}\pi^1(1) > \mathsf{E}\bar{\pi}_\mathsf{D}^2 + \mathsf{E}\bar{\pi}_\mathsf{D}^1 = 2\bar{\mathsf{ask}}_\mathsf{D}^2 > 2\mathsf{ask}_\mathsf{D}^2.$$

Step 5: *We show that* π^2 *must be in* $[\bar{\pi}_\mathsf{L}^2, \bar{\pi}_\mathsf{D}^2]$ *by contradiction.*

Suppose $\pi^2 > \bar{\pi}_\mathsf{D}^2$. This implies that $\lambda^2 < \lambda$ and that $\mathsf{ask}_\mathsf{D}^2 < \bar{\mathsf{ask}}_\mathsf{D}^2$. For the indifference condition on the dealer market segment, $2\mathsf{ask}_\mathsf{D}^2 = \mathsf{E}\pi^2 + \mathsf{ask}_\mathsf{D}^1$, to hold, we need that $\mathsf{ask}_\mathsf{D}^1 < \bar{\mathsf{ask}}_\mathsf{D}^1$. But since $\pi^2 > \bar{\pi}_\mathsf{D}^2$ this implies that $\lambda^1 < \lambda$. With $\lambda^2, \lambda^1 < \lambda$, we have that that $\pi_\mathsf{L}^1 > \bar{\pi}_\mathsf{L}^1$. In this case, however, $\mathsf{ask}_\mathsf{D}^1 < \bar{\mathsf{ask}}_\mathsf{D}^1 < \bar{\mathsf{ask}}_\mathsf{L}^1 < \mathsf{ask}_\mathsf{L}^1$, a contradiction.

Next, suppose that $\pi^2 < \bar{\pi}_\mathsf{L}^2$. Then $\lambda^2 > \lambda$. Then for any π, $\mathsf{ask}_\mathsf{D}^2(\cdot, \pi, \cdot, 2\lambda - \lambda^2) > \mathsf{ask}_\mathsf{L}^2(\cdot, \pi, \cdot, \lambda^2)$. Since $\mathsf{ask}_\mathsf{L}^2(\cdot, \pi^2, \cdot, \lambda^2) = \mathsf{E}\pi^2$, we have that $\mathsf{ask}_\mathsf{D}^2(\cdot, \pi^2, \cdot, 2\lambda - \lambda^2) > \mathsf{E}\pi^2$. This violates π^2's participation constraint.

References

Back, K., and S. Baruch (2007): "Working Orders in Limit Order Markets and Floor Exchanges," *Journal of Finance*, 62(4), 1589–1621.

Bessembinder, H., and K. Venkataraman (2004): "Does an electronic stock exchange need an upstairs market?," *Journal of Financial Economics*, 73(1), 3–36.

Biais, B., D. Martimort, and J.-C. Rochet (2000): "Competing Mechanisms in a Common Value Environment," *Econometrica*, 68(4), 799–837.

Bloomfield, R., M. O'Hara, and G. Saar (2005): "The 'make or take' decision in an electronic market: Evidence on the evolution of liquidity," *Journal of Financial Economics*, 75(1), 165–199.

Booth, G. G., J.-C. Lin, T. Martikainen, and Y. Tse (2002): "Trading and Pricing in Upstairs and Downstairs Stock Markets," *The Review of Financial Studies*, 15(4), 1111–1135.

Brown, D. P., and Z. M. Zhang (1997): "Market Orders and Market Efficiency," *The Journal of Finance*, 52(1), 277–308.

Buti, S. (2007): "A Challenger to the Limit Order Book: The NYSE Specialist," Working paper, University of Toronto.

de Jong, F., T. Nijman, and A. Roell (1995): "A comparison of the cost of trading French shares on the Paris Bourse and on SEAQ International," *European Economic Review*, 39(7), 1277–1301.

Easley, D., N. M. Kiefer, M. O'Hara, and J. B. Paperman (1996): "Liquidity, Information, and Infrequently Traded Stocks," *Journal of Finance*, LI(4), 1405–1436.

Easley, D., and M. O'Hara (1987): "Price, Trade Size, and Information in Securities Markets," *Journal of Financial Economics*, 19, 69–90.

Foucault, T. (1999): "Order flow composition and trading costs in a dynamic limit order market," *Journal of Financial Markets*, 2, 99–134.

Glosten, L. R. (1994): "Is the Electronic Open Limit Order Book Inevitable?," *The Journal of Finance*, 49(4), 1127–1161.

Glosten, L. R., and P. R. Milgrom (1985): "Bid, Ask and Transaction Prices in a Specialist Market with Heterogenously Informed Traders," *Journal of Financial Economics*, 14, 71–100.

Grossman, S. J. (1992): "The Informational Role of Upstairs and Downstairs Trading," *The Journal of Business*, 65(4), 509–528.

Harris, L. (2003): *Trading and Exchanges: Market Microstructure for Practitioners.* Oxford University Press, New York, 1 edn.

Herrera, H., and L. Smith (2006): "private notes," , University of Michigan.

Kyle, A. (1985): "Continuous Auctions and Insider Trading," *Econometrica*, 53, 1315–1336.

Pagano, M., and A. Roell (1996): "Transparency and Liquidity: A Comparison of Auction and Dealer Markets with Informed Trading," *The Journal of Finance*, 51(2), 579–611.

Parlour, C. A. (1998): "Price dynamics in limit order markets," *Review of Financial Studies*, 11, 789–816.

Parlour, C. A., and D. J. Seppi (2003): "Liquidity-Based Competition for Order Flow," *The Review of Financial Studies*, 16(2), 301–343.

——— (2008): "Limit Order Markets: A Survey," *Handbook of Financial Intermediation*, forthcoming.

Seppi, D. J. (1990): "Equilibrium Block Trading and Asymmetric Information," *Journal of Finance*, XLV(1), 73–94.

——— (1997): "Liquidity Provision with Limit Orders and Strategic Specialist," *The Review of Financial Studies*, 10(1), 103–150.

Smith, B. F., D. A. S. Turnbull, and R. W. White (2001): "Upstairs Market for Principal and Agency Trades: Analysis of Adverse Information and Price Effects," *The Journal of Finance*, 56(5), 1723–1746.

Smith, L., and P. Sorensen (2008): "Rational Social Learning with Random Sampling," mimeo, University of Michigan.

Viswanathan, S., and J. J. D. Wang (2002): "Market architecture: limit-order books versus dealership markets," *Journal of Financial Markets*, 5(2), 127–167.