The Term Structure of Currency Hedge Ratios

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The Term Structure of Currency Hedge Ratios

Abstract

Firms that export face product price risk in foreign currency, uncertain costs in home currency, and exchange rate risk. If prices and exchange rates in different countries interact, natural hedges of exchange rate risk might result. If the effectiveness of such hedges depends on the hedge horizon, they might affect a firm’s usage of foreign exchange derivatives and lead to a term structure of optimal hedge ratios. We analyze this issue by deriving the variance minimizing hedge position in currency forward contracts of an exporting firm that is exposed to different risks. In an empirical study, we quantify the term structure of hedge ratios for a “typical” German firm that is exporting either to the United States, the United Kingdom, or Japan. Based on cointegrated vector autoregressive models of prices, interest rates, and exchange rates, we show that the hedge ratio decreases substantially with the hedge horizon, reaching values of one half or less for a ten-year horizon. This finding can (partly) explain the severe underhedging of long-term exchange rate exposures that is frequently observed.


1 Introduction

There is evidence that hedging strategies of non-financial firms strongly depend on the hedge horizon. One indication is survey results by Bodnar, Hayt, and Marston (1996, 1998), who show that the percentage of firms using foreign currency derivatives decreases with the time to maturity of the contracts. Of all firms using derivatives, 82% hold at least some contracts with maturities less than 90 days, whereas only 12% hold any contracts with maturities greater than three years.\footnote{See Bodnar, Hayt, and Marston (1998), p. 77 f.} In this sense, we can speak of a decreasing term structure of hedging activity that might well translate into a corresponding term structure of hedge ratios. Such a term structure of hedge ratios is directly observed by Adam, Fernando, and Salas (2007). They show that the proportion of future production that is hedged decreases sharply with the hedge horizon for their sample of gold mining firms.

One can imagine different reasons why financial hedging activity declines with the hedge horizon. One important aspect is that operational hedges can be used instead of financial hedges to manage long-term exposure, as suggested by Brealey and Kaplanis (1995) and Chowdhry and Howe (1999).\footnote{Allayannis, Ihrig, and Weston (2001), Kim, Mathur, and Nam (2006) and Bartram (2008) provide empirical evidence on the interplay between financial and operational hedging.} In addition, long-term exposure might be hedged using dynamic strategies that employ short-term financial contracts. For example, Brennan and Crew (1997), Neuberger (1999), and Bühler, Korn, and Schöbel (2005) analyze different model-based strategies to hedge long-term commodity price exposure with short-term futures contracts.

We must also consider that the uncertainty of a firm’s cash flows is likely to increase with the time horizon. For example, an exporting firm’s revenues in foreign currency are probably better known for the next year than for the next five years. The theoretical literature on corporate risk management has shown that such revenue risk can cause underhedging of exchange rate risk, which could explain a downward sloping term structure of currency hedge ratios. For example, Benninga, Eldor, and Zilcha (1985) and Adam-Müller (1997) analyze hedging strategies with forward contracts. They show that if revenues and exchange rates are uncorrelated and forward markets are unbiased, underhedging occurs for utility functions with positive prudence.

A further explanation for underhedging of exchange rate risk at longer hedge horizons lies in certain imperfections in derivatives contracts, which become more relevant when the hedge horizon increases. One example are different forms of basis

\footnote{See Bodnar, Hayt, and Marston (1998), p. 77 f.}
risk, as theoretically analyzed by Briys, Crouhy, and Schlesinger (1993), Castelino (2000) and Adam-Müller (2006). Furthermore, Castelino (2000) shows empirically that minimum-variance hedge ratios reduce with increasing basis risk. Another example is provided by increasing liquidity needs of long-term hedging strategies with futures contracts, as analyzed by Zhou (1998), Mello and Parsons (2000), and Deep (2002). Finally, Cummins and Mahul (2008) demonstrate that a possible default of OTC derivatives can lead to underhedging. Since default risk usually increases with the time to maturity, the extent of underhedging should increase with the hedge horizon.

In this paper, we look at still another aspect of the interplay between different sources of risk, the potential “natural hedging” of exchange rate risk by offsetting changes in a firm’s revenues and costs. In the extreme case, if revenues move in parallel with the general price level and prices and exchange rates always follow the predictions of Purchasing Power Parity (PPP) theory, there will be a perfect natural hedge and the firm faces no exchange rate risk in real terms. However, this extreme case is surely not realistic, since a large body of literature has shown that PPP does not hold in the short run. Nevertheless, there is evidence for some movement towards PPP in the very long run. These findings suggest that the characteristics of exchange rate risk and hedge ratios depend on the hedge horizon. Even if PPP relations do not play any role, there might still be interactions between revenues, costs and exchange rates which lead to hedge ratios that differ across hedge horizons.

The aim and contribution of this paper is to show what the relations between revenues, costs and exchange rates imply for corporate risk management. In essence, we characterize the term structure of currency hedge ratios; i.e., we ask how much should be hedged at different hedge horizons. In particular, the term structures of currency hedge ratios that we derive and quantify in this paper help us shed light on two important issues. First, they provide evidence on how far the increased underhedging at longer hedge horizons, which we observe for many firms, can be explained by some kind of risk diversification between exchange rates and revenues. Second, they provide some guidance for risk managers to design hedging strategies in certain major currencies.

The starting point of our investigation is a simple model of an exporting firm that we use to derive the variance minimizing hedge position in currency forward contracts.

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3This argument is well known from the risk management literature. See e.g. the critical discussion in Dufey and Srinivasulu (1983).

4The literature on PPP is enormous and we make no attempt to review it. Survey articles on this literature are Breuer (1994), Froot and Rogoff (1995) and Taylor and Taylor (2004).
Based on this analysis, we perform an empirical study to quantify the term structure of currency hedge ratios and the corresponding hedging effectiveness for a German firm that exports either to the United States (US), to the United Kingdom (UK) or to Japan (JP). This study specifies a vector autoregressive (VAR) model with possible cointegration relations between price levels, exchange rate and long-term interest rates. By means of simulated sample paths from this model, generated by a bootstrap algorithm, we quantify hedge ratios and hedging effectiveness for different hedge horizons.

Our main empirical result shows that the term structure of hedge ratios is clearly decreasing for all currencies considered, going down to a half or less for a hedge horizon of ten years. We have found that one explanation is that revenue risk increases more strongly with the hedge horizon than does exchange rate risk. The main reason, however, lies in the correlation structure between different risks that varies with the hedge horizon due to cointegration relations; i.e., we observe natural hedges at long horizons. As a consequence, hedging effectiveness decreases much less with the hedge horizon than hedge ratios.

For long horizons, there can also be substantial differences between currencies. For instance, the ten-years hedge ratio for the British Pound still amounts to 53% in comparison to 34% for the US Dollar. In contrast, the difference for shorter horizons of up to two years is very small.

The remaining part of the paper is organized as follows: Section 2 introduces the model of an exporting firm that hedges with forward contracts. We then derive the variance minimizing hedge ratio and provide some interpretation. Section 3 contains the empirical study. First, the data set is introduced and the study design is briefly explained. Then, we discuss the specification of the VAR model and report the cointegration results. Finally, the results on the term structure of hedge ratios and the hedging effectiveness of the corresponding strategies are presented and discussed. Section 4 completes the paper with a summary, some conclusions and an outlook on further research.

2 Model Analysis

2.1 Model Setup

Our analysis starts with a model of an exporting firm. This firm produces a single good that is sold in a foreign market. Assume that we are currently at time zero.
In each of the following $T$ periods, production takes place and goods are sold at the end of each period. Thus, the firm has a simultaneous exposure to foreign exchange risk at different horizons. For simplicity, assume that the firm has already decided on its per period output quantity, $Q$, which is constant over time.

Both the product prices $\tilde{P}_t$, $t = 1, \ldots, T$, in foreign currency and the corresponding exchange rates $\tilde{X}_t$, $t = 1, \ldots, T$, measured in units of home currency per unit of foreign currency, are exogenous stochastic variables. Since the firm produces in its home country, the exogenous stochastic production costs per period, $\tilde{C}_t$, $t = 1, \ldots, T$, are denominated in the firm’s home currency. Therefore, the firm generates the following profits from operations per period:

$$\tilde{\Pi}_t = \tilde{P}_t Q \tilde{X}_t - \tilde{C}_t, \quad t = 1, \ldots, T.$$  (1)

In a next step, the uncertainty in foreign revenues, costs and exchange rates is specified more explicitly. Denote the current product price by $P_0$ and write the future product prices $\tilde{P}_t$, $t = 1, \ldots, T$, as

$$\tilde{P}_t = P_0 (1 + \tilde{\epsilon}_{f,t}), \quad \text{with} \quad (1 + \tilde{\epsilon}_{f,t}) \equiv \prod_{k=1}^{t} (1 + \tilde{\epsilon}_{f,k-1,k}),$$  (2)

where $\tilde{\epsilon}_{f,k-1,k}$ is the uncertain percentage price change in period $k$. Note that this percentage price change equals the percentage change in revenues in foreign currency under our assumption of a fixed production quantity. In the same way, production costs in different periods are determined by some current cost level $C_0$ and the random percentage changes in costs, $\tilde{\epsilon}_{h,t}$, $t = 1, \ldots, T$.

$$\tilde{C}_t = C_0 (1 + \tilde{\epsilon}_{h,t}), \quad \text{with} \quad (1 + \tilde{\epsilon}_{h,t}) \equiv \prod_{k=1}^{t} (1 + \tilde{\epsilon}_{h,k-1,k}).$$  (3)

The given representation of future prices and costs in terms of per period percentage changes is very useful later on. When we implement our model, we identify the changes in sales prices and revenues as changes in the foreign country’s price level, and the changes in costs as changes in the home country’s price level. Therefore, we implicitly assume that revenues and costs move according to the (production) price level. Although uncertainty in revenues and costs will usually have industry-specific and firm-specific components, our focus on the general price level provides a
reference case that is useful in explaining “average” behavior of firms and provides a starting point for designing hedging strategies in specific situations.

Let the uncertain future exchange rates $\tilde{X}_t$, $t = 1, \ldots, T$, be expressed as follows:

$$
\tilde{X}_t = X_0 \left(1 + \tilde{\epsilon}_{h,t}\right) \left(1 + \tilde{\epsilon}_{f,t}\right) \left(1 + \tilde{u}_t\right),
$$

where $\tilde{u}_t \equiv \prod_{k=1}^{t} \left(1 + \tilde{u}_{k-1,k}\right)$.

Future exchange rates are functions of the current exchange rate $X_0$ and the random variables $\tilde{\epsilon}_{h,t}$, $\tilde{\epsilon}_{f,t}$ and $\tilde{u}_t$, $t = 1, \ldots, T$. Note that the formulation in Equation (4) does not impose any particular restrictions on the distribution of future exchange rates, since no assumptions are made about the distribution of the $\tilde{u}_t$s. Given our interpretation of $\tilde{\epsilon}_{h,t}$ and $\tilde{\epsilon}_{f,t}$ as relative price changes, $\tilde{u}_t$ is the component of relative exchange rate changes that is not driven by relative price changes in the two countries. For example, with $\tilde{u}_t \equiv 0$, the exchange rate would exactly adjust in such a way that relative prices in the two countries are unchanged; i.e., relative PPP would hold. In this sense, the $\tilde{u}_t$s measure the deviations from relative PPP for different period lengths.

Substitution of Equations (2) to (4) into Equations (1) leads to the following per period profits:

$$
\tilde{\Pi}_t = P_0 Q X_0 \left(1 + \tilde{\epsilon}_{h,t}\right) \left(1 + \tilde{u}_t\right) - C_0 \left(1 + \tilde{\epsilon}_{h,t}\right),
$$

for $t = 1, \ldots, T$. (5)

By summing these per period profits, we obtain the firm’s profit from operations for the total period from zero to $T$, which is the assumed planning horizon. We have to consider, however, that some profits occur earlier than others. For reasons of tractability, we assume that the firm invests any early profits in real assets, whose value increases with the price level in the firm’s home country; i.e., the real return is zero and the period $k$ nominal compounding rate equals $\tilde{\epsilon}_{h,k-1,k}$. Under this assumption, the total profit from operations becomes

$$
\tilde{\Pi} = \sum_{t=1}^{T} \left( \tilde{\Pi}_t \prod_{k=t+1}^{T} \left(1 + \tilde{\epsilon}_{h,k-1,k}\right) \right).
$$

So far, hedging of exchange rate risk has not been considered. Assume now that the firm can enter into foreign exchange forwards with different maturity dates $t = 1, \ldots, T$ at time zero. Denote by $H_t$ the number of units of foreign currency sold...
for delivery at time $t$ and by $F_{0,t}$ the corresponding forward price. Then the total profit of the firm, including forward transactions, becomes

$$ \tilde{\Pi} = \sum_{t=1}^{T} \left( [\tilde{\Pi}_t + H_t \left( F_{0,t} - \tilde{X}_t \right)] \prod_{k=t+1}^{T} (1 + \tilde{\epsilon}_{h,k-1,k}) \right). $$ (7)

In the next step, we exploit the covered interest parity relation to determine forward prices. Under standard assumptions, no-arbitrage prices of currency forward contracts are given by

$$ F_{0,t} = X_0 \cdot \frac{(1 + r_{h,t})}{(1 + r_{f,t})}, \quad t = 1, \ldots, T, $$ (8)

where $r_{h,t}$ and $r_{f,t}$ are the current $t$-period risk-free interest rates in the home country and the foreign country, respectively. Substituting the above expressions for the forward prices into Equation (7) and using the representation of future spot exchange rates $\tilde{X}_t$, $t = 1, \ldots, T$, from Equations (4), we finally obtain the following total profit:

$$ \tilde{\Pi} = \sum_{t=1}^{T} \left( [\tilde{\Pi}_t + H_t X_0 \left( \frac{(1 + r_{h,t})}{(1 + r_{f,t})} - \frac{(1 + \tilde{\epsilon}_{h,t})}{(1 + \tilde{\epsilon}_{f,t})} (1 + \tilde{\epsilon}_t) \right)] \prod_{k=t+1}^{T} (1 + \tilde{\epsilon}_{h,k-1,k}) \right). $$ (9)

Investors are ultimately interested in consumption. Therefore, if the firm’s nominal profit is high but the inflation rate is also high, investors might be worse off compared to a lower nominal profit in an environment with low inflation rates.\(^5\) Accordingly, we concentrate on real profits in our analysis; i.e., on profits in the firm’s home currency measured in current prices. Equation (10) provides these real profits, which are obtained by dividing $\tilde{\Pi}$ from Equation (9) by $(1 + \tilde{\epsilon}_{h,T})$.

$$ \tilde{\Pi}_{real} = \sum_{t=1}^{T} \left( P_0 Q X_0 (1 + \tilde{\epsilon}_t) - C_0 ight. $$

$$ + H_t X_0 \left[ \frac{(1 + r_{h,t})}{(1 + r_{f,t})} \frac{1}{(1 + \tilde{\epsilon}_{h,t})} - \frac{1}{(1 + \tilde{\epsilon}_{f,t})} (1 + \tilde{\epsilon}_t) \right]. $$ (10)

The real profit in Equation (10) provides the basis for the firm’s hedging decision. As we can see, the risk of the firm’s real operating profit depends only on the random variables $\tilde{\epsilon}_t$, $t = 1, \ldots, T$, the deviations from PPP. The risk of the forward positions, however, depends on both the deviations from PPP and the development of the price levels in the home country and the foreign country.

\(^5\)Adam-Müller (2000) uses the same argument and analyzes hedging strategies that consider real wealth instead of nominal wealth.
2.2 Hedging Strategy

The firm’s hedging problem is to choose the optimal number of forward positions. Similar problems have been analyzed in the literature. A popular approach maximizes the expected utility of profits according to a concave utility function.\(^6\) In the context of this literature, profits according to Equation (10) resemble a hedging problem with both additive and multiplicative basis risk. Since general results are difficult to obtain in this case,\(^7\) we need additional restrictions on the decision criterion. Due to its tractability and popularity in practice, we use variance minimization as the hedging goal.

To formulate the firm’s decision problem, rewrite real profits as

\[
\tilde{\Pi}_{\text{real}} = \sum_{t=1}^{T} \tilde{A}_t + H_t \tilde{B}_t, \quad \text{with}
\]

\[
\tilde{A}_t \equiv P_0 Q X_0 (1 + \tilde{u}_t) - C_0 \quad \text{and}
\]

\[
\tilde{B}_t \equiv X_0 \left( \frac{1}{1 + \tilde{r}_{h,t}} \frac{1}{1 + \tilde{r}_{f,t}} (1 + \tilde{\epsilon}_{h,t}) - \frac{1}{1 + \tilde{\epsilon}_{f,t}} (1 + \tilde{u}_t) \right).
\]

The firm’s decision problem can then be stated as:

\[
\min_{H_t, t=1,...,T} \text{Var} \left[ \sum_{t=1}^{T} \tilde{A}_t + H_t \tilde{B}_t \right].
\]

(12)

Variance minimization according to our setting is a standard optimization problem that leads to the necessary conditions for optimal forward positions given in the normal equations (13) below. These conditions are also sufficient for a unique minimum, if the variance-covariance matrix of the \(\tilde{B}_t\)s, \(t = 1, \ldots, T\), has full rank; i.e., if none of the forward contracts is a redundant hedging instrument.

\[
\sum_{i=1}^{T} \text{Cov}[\tilde{A}_i, \tilde{B}_t] + H_t \text{Cov}[\tilde{B}_i, \tilde{B}_t] \perp 0, \quad t = 1, \ldots, T.
\]

(13)

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\(^6\)See e.g. Holthausen (1979), Feder, Just, and Schmitz (1980), or Benninga, Eldor, and Zilcha (1984) for classical analyses based on one-period models.

\(^7\)Briys, Crouhy, and Schlesinger (1993) determine some characteristics of the optimal forward position in the case of independent additive basis risk (Case A.2, p. 956 f.). Adam-Müller (2006) derives some results for independent multiplicative basis risk (Case M.2, Section 3). Note, however, that our hedging problem involves multiple forwards contracts with different maturities and both additive and multiplicative basis risk. Moreover, it would be unreasonable to assume that the three random variables in Equation (10) are independent. Adam-Müller (2000) provides some results on underhedging and overhedging for the case of inflation risk in the home country. However, we consider inflation risk in both countries, the home country and the foreign country.
Solutions to the system of linear equations (13) can easily be computed numerically, if the necessary variances and covariances are available. As we see from the optimality conditions, the hedge positions depend on all covariances between the profits from operations in different periods (\(\tilde{A}_t\)) and the payoffs of different forward contracts (\(\tilde{B}_t\)). Moreover, all covariances between the \(\tilde{B}_t\)s enter into the calculation of the forward positions. Note, however, that the optimal forward positions do not depend on the initial cost \(C_0\) and the initial exchange rate \(X_0\). The cost \(C_0\) is an additive non-random term in the profit function, which does not influence the variance of total profits. The initial exchange rate \(X_0\) is just a multiplicative scaling factor that scales all relevant random components of both the \(\tilde{A}_t\)s and the \(\tilde{B}_t\)s.

The optimization problem (12) exploits the complete dependence structure between operating profits in different periods and forward contracts with different maturities. In principle, the approach allows for a general cross hedging between different maturities; i.e., long-term exposure might be hedged to some extent with short-term forwards and short-term exposure to some extent with long-term forwards. However, the information requirements that make such a general cross hedge possible and useful are quite demanding.\(^8\) Moreover, one would expect that the resulting strategies are very sensitive to specification errors with respect to the input parameters\(^9\), because of near multicollinearity between forward contracts written on the same underlying. As a consequence, the resulting hedge positions might be hard to interpret economically and difficult to communicate to the management. Therefore, many firms quantify their foreign exchange exposure separately for different time horizons and use maturity matching contracts to hedge exposure.\(^{10}\) This more realistic approach is also followed here. In our setting, it leads to the restriction that profits occurring at time \(t\) are hedged exclusively with forwards maturing at time \(t\). Under this restriction, hedging problem (12) leads to the following first order conditions:

\[
\text{Cov}[\tilde{A}_t, \tilde{B}_t] + H_t \text{Cov}[\tilde{B}_t, \tilde{B}_t] = 0, \quad t = 1, \ldots, T. \tag{14}
\]

Solving for the \(H_t\)s delivers the following optimal forward positions:

\[
H_t^* = -\frac{\text{Cov}[\tilde{A}_t, \tilde{B}_t]}{\text{Var}[\tilde{B}_t]}, \quad t = 1, \ldots, T. \tag{15}
\]

\(^8\)For example, Loderer’s and Pichler’s (2000) study indicates that one should not be too optimistic about the available information. Their survey results for Swiss firms show that many firms were not even able to quantify their currency risk exposure.

\(^9\)Input parameters are the required variances and covariances. As these moments usually have to be estimated, estimation errors are likely.

\(^{10}\)See Brown (2001), p.411, for an example of such a procedure.
Finally, we can substitute for $\tilde{A}_t$ and $\tilde{B}_t$ in the above equations. As a result, we obtain the following representation of the hedge positions:

$$H_t^* = -\frac{Cov\left[P_0 Q (1 + \tilde{u}_t), \frac{(1+r_{h,t})}{(1+\tilde{e}_{h,t})} \frac{1}{(1+\tilde{e}_{f,t})}(1 + \tilde{u}_t)\right]}{Var\left[\frac{(1+r_{h,t})}{(1+\tilde{e}_{h,t})} \frac{1}{(1+\tilde{e}_{f,t})}(1 + \tilde{u}_t)\right]}, \quad t = 1, \ldots, T. \quad (16)$$

To get some intuition for the optimal forward positions $H_t^*$, it is instructive to look at some extreme cases. Firstly, consider that relative PPP holds exactly, which implies that $Var(\tilde{u}_t) = 0$ for all $t$. Therefore, the $\tilde{A}_t$s would not be stochastic and doing without forward contracts would lead to a total variance of zero. This “no hedge” result is quite intuitive. If PPP holds, the firm faces no risk in real operating profits. Therefore, hedging is not needed. On the contrary, since forwards are written on the nominal exchange rate, hedging would introduce risk in the first place.

A second extreme case would consider non-stochastic product prices and costs; i.e., $Var(\tilde{e}_{h,t}) = Var(\tilde{e}_{f,t}) = 0$ for all $t$. Under this assumption, we can see from Equations (16) that we obtain forward positions $H_t^* = P_0 Q (1 + \epsilon_{f,t})$. Such forward positions represent a “full hedge”. Note that revenues at time $t$ in foreign currency equal $P_0 Q (1+\epsilon_{f,t})$. These revenues are fully hedged with the corresponding currency forward contracts. The following intuition lies behind this result: if movements of the price level in both countries are deterministic, the only remaining source of risk is $\tilde{u}_t$, the deviation from relative PPP. Accordingly, since currency risk is completely independent from the relative price levels, there is no natural hedge component and the firm’s foreign currency position should be fully hedged in the forward market. Such a full hedge would eliminate risk completely.

Irrespective of whether PPP holds or not, the two extreme cases highlight the fact that the term structure of hedge positions will strongly depend on how different sources of risk scale with the hedge horizon. If the “price risks” $\tilde{e}_{h,t}$ and $\tilde{e}_{f,t}$ increase more strongly with the hedge horizon than the “real exchange rate risk” $\tilde{u}_t$, hedging becomes less and less attractive, since forward contracts enhance the first kind of risk and reduce the second one. Thus, the term structure of hedge positions is expected to fall. In addition, the correlation structure of the “price risks” and the “real exchange rate risk” could change with the hedge horizon, which is a second channel by which the term structure of hedge positions could be influenced.

Usually, one analyzes hedging strategies in terms of hedge ratios, normalized values that are often easier to interpret and to compare. Within our model, the hedge ratios for hedge horizons $t = 1, \ldots, T$ are reasonably defined as the ratios of $H_t^*$ and
the expected revenues at time \( t \) in foreign currency, \( P_0 Q E(1 + \tilde{\epsilon}_{f,t}) \). This kind of normalization leads to the following expressions for the optimal hedge ratios:

\[
HR_t^* = - \frac{Cov\left[(1 + \tilde{u}_t), \frac{(1+r_{h,t})}{(1+r_{f,t})} - \frac{1}{(1+\tilde{\epsilon}_{f,t})}(1 + \tilde{u}_t)\right]}{E(1 + \tilde{\epsilon}_{f,t}) Var\left(\frac{(1+r_{h,t})}{(1+r_{f,t})} - \frac{1}{(1+\tilde{\epsilon}_{f,t})}(1 + \tilde{u}_t)\right)}, \quad t = 1, \ldots, T. \tag{17}
\]

The term structure of currency hedge ratios; i.e., the hedge ratios for different hedge horizons \( t \) from Equations (17), will be quantified for different currencies in our empirical study. Note that the hedge ratios do not depend on the quantity \( Q \) and the current product price \( P_0 \). They are solely determined by the current interest rates \( r_{h,t} \) and \( r_{f,t} \) and the joint distributions of the three groups of random variables \( \tilde{\epsilon}_{h,t}, \tilde{\epsilon}_{f,t} \) and \( \tilde{u}_t \).

A comparative static analysis with respect to one of the moments of \( \tilde{\epsilon}_{h,t}, \tilde{\epsilon}_{f,t} \) and \( \tilde{u}_t \) does not in general lead to a distinct conclusion, since all random variables can be arbitrarily correlated. However, we can get some intuition about the effects of a higher price risk in the home country \( (\tilde{\epsilon}_{h,t}) \), a higher price risk in the foreign country \( (\tilde{\epsilon}_{f,t}) \) and a higher risk of a deviation from PPP \( (\tilde{u}_t) \), if we assume independence of the three random variables. Firstly, if the price level in the home country gets more volatile, the numerator in Equation (17) does not change, but the denominator increases. Therefore, the hedge ratio decreases. Secondly, if the volatility of prices in the foreign country increases (without changing first moments), a similar effect results. The numerator of the hedge ratio stays the same, the denominator increases, and the hedge ratio decreases. Finally, an increased volatility of \( \tilde{u}_t \) increases both the covariance in the numerator of the hedge ratio and the variance in the denominator.

However, if the random variable \( 1 + \tilde{\epsilon}_{f,t} \) is greater than one; i.e., if the inflation rate is positive, the effect on the numerator will dominate and the hedge ratio increases. Roughly speaking, we can conclude that a higher inflation risk decreases hedge ratios, while a higher inflation independent currency risk increases hedge ratios. At the limits, we reach the no hedge case and the full hedge case, respectively.

\( ^{11} \)Note that the second extreme case from above leads to a hedge ratio of one, which is intuitive.
3 Empirical Study

3.1 Study Design and Data Set

In our model, the currency specific hedge ratios $HR^*_t$, $t = 1, \ldots, T$, depend crucially on the joint distribution of the three groups of random variables: $\tilde{\epsilon}_{h,t}$, $\tilde{\epsilon}_{f,t}$ and $\tilde{u}_t$. Thus, an econometric model which quantifies the joint distribution at different time horizons is required. In particular, we need an econometric model that realistically captures the dynamics of prices and exchange rates. A typical framework for such an analysis is a cointegrated VAR model. When modelling prices and exchange rates in such a framework, one usually includes interest rates as well, because of the strong economic connection between inflation, exchange rates and interest rates.\footnote{See Juselius and MacDonald (2000, 2004).} We follow the same approach, since the moments that make up the hedge ratios $HR^*_t$ should be interpreted as conditional moments, and interest rates are potentially important conditioning variables.

Based on a specified and estimated VAR model for two countries, the required moments of the random variables $\tilde{\epsilon}_{h,t}$, $\tilde{\epsilon}_{f,t}$ and $\tilde{u}_t$, $t = 1, \ldots, T$, are quantified using a bootstrap algorithm. In this algorithm, we resample residual vectors and construct simulated paths of the corresponding variables for time horizons of up to ten years. From these simulated paths we obtain hedge ratios $HR^*_t$ by calculating the realized moments according to Equations (17).

The data set used for the estimation of the cointegrated VAR model was retrieved from the International Financial Statistics (CD Rom, 3/2006) of the International Monetary Fund (IMF) and the Datastream database. It consists of monthly price levels, interest rates and exchange rates for Germany, the US, the UK and Japan over the period from July 1975 to December 2005. The data period of more than 30 years leaves us with a total number of 366 observations for each data series. Data before 1975 were not taken into account to avoid any influence of the Bretton Woods system of fixed exchange rates. As proxies for product prices and costs we use producer price indices (PPI), which are more appropriate than consumer price indices. Prices in the foreign country and in the home country (Germany) are denoted by $P_f$ and $P_h$, respectively. The corresponding logarithmic prices are $p_f$ and $p_h$. The level of the exchange rate (end-of-month rates) between Germany and the foreign country is denoted by $X$, and the logarithmic exchange rate by $x$. Before 1999, synthetic exchange rates for the Euro are used, which were calculated using
the introductory rate of the Euro to the Deutschmark. Finally, we use long-term government yields as interest rates in the econometric model, which are denoted by $i_f$ and $i_h$.

Figures 4 to 7 in the Appendix give an overview of the time series underlying our analysis. The figures strongly indicate that the series are non-stationary. The degree of integration of different series and possible cointegration relations between different series are very important for the term structure of hedge ratios. Therefore, these properties will be carefully considered in the concrete specification of the econometric model.

### 3.2 Specification of the VAR Model

A p-dimensional cointegrated VAR model with $l$ lags, stated in vector error correction (VEC) form, is defined as follows:

$$
\Delta Y_t = \Gamma_1 \Delta Y_{t-1} + \ldots + \Gamma_{l-1} \Delta Y_{t-l+1} + \Pi Y_{t-1} + \Phi D_t + \xi_t, \quad t = 1, \ldots, \hat{T},
$$

(18)

where $Y_t$ is a p-dimensional random vector of endogenous variables, $\Pi$ and $\Gamma_1, \ldots, \Gamma_{l-1}$ are $p \times p$ coefficient matrices, $D_t$ is a b-dimensional vector of deterministic components like a constant, a linear time trend, seasonal or intervention dummies etc., $\Phi$ is a $p \times b$ coefficient matrix and $\xi_t, \ t = 1, \ldots, \hat{T}$, are p-dimensional vectors of i.i.d. Gaussian error terms. In an I(1) cointegrated VAR model with $r$ linearly independent cointegration equations, the long-run matrix $\Pi$ can be written as

$$
\Pi = \alpha \beta',
$$

(19)

where $\alpha$ and $\beta$ are $p \times r$ coefficient matrices with full column rank and $r \leq p$. As the vector time series $Y_t, \ t = 1, \ldots, \hat{T}$, is assumed to be I(1), its first difference, $\Delta Y_t$, is stationary. In this sense, the matrix $\Pi$ transforms non-stationary series into stationary ones. In particular, the matrix $\beta$ contains the weights of the stationary linear combinations of the I(1) vector time series $Y_t, \ t = 1, \ldots, \hat{T}$, and the matrix $\alpha$ contains the parameters that determine the speed of adjustment to the long-run equilibrium relations.

In the econometric model that we apply to characterize the term structure of currency hedge ratios, the vector $Y_t$ consists of five variables. First, (monthly) inflation rates $\Delta p_h$ and $\Delta p_f$ are used to represent the uncertainty of revenues and costs in the model. The part of exchange rate uncertainty that can not be explained by price changes is captured by the deviation from absolute PPP, which is given by

12
Note that the moments of the three groups of random variables that enter into the hedge ratios according to Equations (17) should be conditional moments. Since interest rates are natural conditioning variables for the interplay between prices and exchange rates, they are additionally included. In summary, the vector \( Y_t \) takes the following form:\(^{14}\)

\[
Y_t = \begin{pmatrix}
    ppp_t \\
    \Delta p_{f,t} \\
    \Delta p_{h,t} \\
    i_{f,t} \\
    i_{h,t}
\end{pmatrix}
\]

(20)

The integration rank of each of the above time series is determined by means of standard unit root tests. The first test that we apply is the Augmented Dickey Fuller (ADF) test\(^{15}\), which has a null hypothesis of non-stationarity. The second one is the test by Kwiatkowski et al. (1992) (KPSS), which has a null hypothesis of stationarity. The test results of Table 1 indicate that all time series are best described as I(1) processes. A graphical inspection of the series confirms these results, which are in line with results of similar analyses in the literature.\(^{16}\) In particular, note that we do not find evidence for stationarity in the three \( ppp \) series. Thus, we have to conclude that PPP in the sense of mean-reversion towards the PPP relation does not hold.

In the next step of model specification, we have to choose the lag lengths of our three VAR models (US, UK, Japan). A choice of two lags is supported by the information criteria of Hannan-Quinn and Schwarz, as given in Table 2. The only exception is a lag length of one for the UK, according to the Schwarz criterion. However, since a lag length of one leaves us with some autocorrelation in the residuals,\(^{17}\) we generally choose two lags.\(^{18}\)

The graphs of the differenced variables in the Appendix show that the normality assumption is not valid for many of the marginal processes. To obtain valid sta-

\(^{13}\)According to Equation (18), changes of this variable are simulated later, which capture deviations from relative PPP; i.e., changes in real exchange rates. The time series behavior of \( ppp \) is shown in Figure 8 in the Appendix.

\(^{14}\)Centered seasonal dummies are used to capture seasonal effects in the data, because the time series are not seasonally adjusted.

\(^{15}\)See Dickey and Fuller (1979, 1981) and Said and Dickey (1984).

\(^{16}\)See Juselius and MacDonald (2000, 2004).

\(^{17}\)Corresponding results of an LM test are not reported here.

\(^{18}\)See also Juselius (2006) p. 72, who stresses that a model with two lags is often the best starting point.
ADF test | KPSS test
---|---
$\Delta p_h$ | -2.676 | 0.600$^*$
$\Delta p_{US}$ | -2.280 | 0.556$^*$
$\Delta p_{UK}$ | -1.953 | 2.698$^{***}$
$\Delta p_{JP}$ | -2.544 | 0.667$^{**}$
$ppp_{US}$ | -1.929 | 0.383$^*$
$ppp_{UK}$ | -2.143 | 1.978$^{***}$
$ppp_{JP}$ | -2.386 | 1.266$^{***}$
$i_h$ | -1.310 | 2.359$^{***}$
$i_{US}$ | -1.047 | 2.764$^{***}$
$i_{UK}$ | -1.250 | 3.827$^{***}$
$i_{JP}$ | -1.422 | 3.653$^{***}$

Note: For the ADF tests, $^*$, $^{**}$ and $^{***}$ mean that the null hypothesis of non-stationarity is rejected at a confidence level of 90%, 95% and 99%. The corresponding critical values are -3.451, -2.870 and -2.571 assuming no linear trend. For the KPSS tests, $^*$, $^{**}$ and $^{***}$ mean that the null hypothesis of stationarity is rejected at a confidence level of 90%, 95% and 99%. The lag truncation parameter is set to 8 according to Kwiatkowski et al. (1992), p. 174, since at this value the test settles down. The critical values are 0.347, 0.463 and 0.739 assuming no linear trend in the data.

Table 1: Unit root tests.

In a next step the cointegration rank $r$ is determined. Results of the trace test, or Johansen test, are reported in Table 3. In every country the largest two eigenvalues are significantly different from zero. The significance of the third eigenvalue is a borderline case. For the US and the UK, the trace test suggests two cointegration relations between the endogenous variables. For countries...
Table 2: Determination of the lag length.

<table>
<thead>
<tr>
<th></th>
<th>p-r</th>
<th>r</th>
<th>Eig.Value</th>
<th>Trace</th>
<th>Trace*</th>
<th>Frac95</th>
<th>P-Value</th>
<th>P-Value*</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>US</strong></td>
<td>5 0</td>
<td>0.348</td>
<td>258.471</td>
<td>252.609</td>
<td>53.956</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 1</td>
<td>0.208</td>
<td>103.230</td>
<td>101.132</td>
<td>35.098</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 2</td>
<td>0.036</td>
<td>18.744</td>
<td>18.357</td>
<td>20.604</td>
<td>0.088</td>
<td>0.098</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 3</td>
<td>0.009</td>
<td>5.437</td>
<td>5.117</td>
<td>9.964</td>
<td>0.252</td>
<td>0.281</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 4</td>
<td>0.006</td>
<td>2.097</td>
<td>1.653</td>
<td>0.000</td>
<td>N.A</td>
<td>N.A</td>
<td></td>
</tr>
<tr>
<td><strong>UK</strong></td>
<td>5 0</td>
<td>0.341</td>
<td>278.597</td>
<td>272.228</td>
<td>65.550</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 1</td>
<td>0.242</td>
<td>127.213</td>
<td>124.521</td>
<td>45.380</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 2</td>
<td>0.034</td>
<td>26.810</td>
<td>26.275</td>
<td>28.317</td>
<td>0.075</td>
<td>0.085</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 3</td>
<td>0.024</td>
<td>14.073</td>
<td>13.072</td>
<td>14.465</td>
<td>0.062</td>
<td>0.086</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 4</td>
<td>0.015</td>
<td>5.305</td>
<td>4.637</td>
<td>3.799</td>
<td>0.021</td>
<td>0.031</td>
<td></td>
</tr>
<tr>
<td><strong>JP</strong></td>
<td>5 0</td>
<td>0.312</td>
<td>278.198</td>
<td>271.945</td>
<td>76.655</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 1</td>
<td>0.240</td>
<td>142.265</td>
<td>138.979</td>
<td>53.825</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 2</td>
<td>0.073</td>
<td>42.568</td>
<td>41.480</td>
<td>34.482</td>
<td>0.006</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 3</td>
<td>0.026</td>
<td>14.867</td>
<td>13.347</td>
<td>18.984</td>
<td>0.159</td>
<td>0.231</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 4</td>
<td>0.014</td>
<td>5.135</td>
<td>4.797</td>
<td>6.048</td>
<td>0.077</td>
<td>0.091</td>
<td></td>
</tr>
</tbody>
</table>

Note: *=trace test statistics and p-values based on the Bartlett small-sample correction.

Table 3: Rank determination tests (trace tests).

Japan, it indicates three cointegration relations. However, an inspection of the third cointegration relation and the number of roots of the companion matrix in Table 4 do not support a cointegration rank of $r = 3$. If the cointegration rank were three, the companion matrix would have only two roots close to unity. However, in the case of Japan, there are clearly three roots close to unity. Accordingly, we stay with a cointegration rank of two for each of the three models.

To check the assumptions of the standard I(1) approach, we applied several misspecification tests to the estimated cointegrated VAR models. The results of these tests are presented in Table 5. The multivariate LM test statistics for first and second order residual autocorrelation are not significant at the 5% level, so, impor-
Table 4: Modulus of the five largest roots of the companion matrix for different cointegration ranks.

<table>
<thead>
<tr>
<th>Country</th>
<th>Rank</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$\rho_3$</th>
<th>$\rho_4$</th>
<th>$\rho_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>$r = 1$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.364</td>
</tr>
<tr>
<td></td>
<td>$r = 2$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.375</td>
<td>0.375</td>
</tr>
<tr>
<td></td>
<td>$r = 3$</td>
<td>1</td>
<td>0.934</td>
<td>0.368</td>
<td>0.358</td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>$r = 1$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.380</td>
</tr>
<tr>
<td></td>
<td>$r = 2$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.453</td>
<td>0.453</td>
</tr>
<tr>
<td></td>
<td>$r = 3$</td>
<td>1</td>
<td>0.96</td>
<td>0.545</td>
<td>0.545</td>
<td></td>
</tr>
<tr>
<td>JP</td>
<td>$r = 1$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.627</td>
</tr>
<tr>
<td></td>
<td>$r = 2$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.742</td>
<td>0.522</td>
</tr>
<tr>
<td></td>
<td>$r = 3$</td>
<td>1</td>
<td>0.948</td>
<td>0.741</td>
<td>0.420</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Misspecification tests: p-values of the corresponding test statistics.

<table>
<thead>
<tr>
<th>Tests for autocorrelation:</th>
<th>US</th>
<th>UK</th>
<th>JP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM(1)</td>
<td>0.136</td>
<td>0.198</td>
<td>0.075</td>
</tr>
<tr>
<td>LM(2)</td>
<td>0.055</td>
<td>0.543</td>
<td>0.252</td>
</tr>
<tr>
<td>Test for Normality</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tests for ARCH:</th>
<th>US</th>
<th>UK</th>
<th>JP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM(1)</td>
<td>0.041</td>
<td>0.001</td>
<td>0.00</td>
</tr>
<tr>
<td>LM(2)</td>
<td>0.409</td>
<td>0.001</td>
<td>0.00</td>
</tr>
</tbody>
</table>

stantly, the property of no autocorrelation is not rejected. Table 5 additionally shows that multivariate normality is clearly violated. Since the univariate misspecification tests, which are not reported here, indicate that the rejection of normality results from excess kurtosis and not skewness, non-normality is a less serious problem for the estimation results. With respect to ARCH effects, we find that only for the US model the multivariate LM test does not reject the hypothesis of no ARCH effects on typical significance levels. However, as shown by Rahbek, Hansen, and Dennis (2002), the cointegration rank tests are robust against moderate residual ARCH effects. Additionally, we performed tests on parameter constancy. The results are available upon request and support the constancy of the parameters in the chosen reduced rank VAR models.

After having specified the VECMs and having checked all the assumption of the I(1) model, we obtain three models of the dynamics of prices, interest rates and exchange rates. The parameter estimates of these country-specific VECMs are presented in Tables 8, 9 and 10 in the Appendix. A general look at the three models shows
that they are not only well specified from an econometric point of view, but are also economically reasonable in the sense that in almost all cases the signs of the estimated coefficients are plausible.

3.3 Results: Hedge Ratios and Hedging Effectiveness

The VECMs that we have specified in the previous subsection can now be used to quantify the term structure of currency hedge ratios. The resulting variance minimizing hedge ratios $HR^*_t$ according to Equations (17) for different countries and different hedge horizons are shown in Table 6 and Figure 1.

<table>
<thead>
<tr>
<th></th>
<th>1 month</th>
<th>6 months</th>
<th>1 year</th>
<th>2 years</th>
<th>5 years</th>
<th>10 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>US $</td>
<td>0.97</td>
<td>0.93</td>
<td>0.89</td>
<td>0.83</td>
<td>0.64</td>
<td>0.34</td>
</tr>
<tr>
<td>UK £</td>
<td>0.99</td>
<td>0.96</td>
<td>0.92</td>
<td>0.86</td>
<td>0.71</td>
<td>0.53</td>
</tr>
<tr>
<td>JP ¥</td>
<td>0.99</td>
<td>0.96</td>
<td>0.93</td>
<td>0.87</td>
<td>0.73</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Table 6: Hedge ratios $HR^*_t$ for different hedge horizons.

Figure 1: The term structure of currency hedge ratios.

The table and the figure provide several interesting results. Firstly, the term structure of hedge ratios clearly decreases. Secondly, hedge ratios are still close to one for shorter hedge horizons of up to one year. Thirdly, the hedge ratios lie substantially below one for hedge horizons of five years or longer for all three currencies. For a hedge horizon of ten years, values drop down to about one third (US Dollar) or about one half (British Pound and Japanese Yen). Finally, for very long hedge
horizons, there can be clear discrepancies between the forward positions in the three currencies; i.e., different currencies should be hedged differently.

These results have important practical implications. Seen from one perspective, we could say that they indicate the existence of dependencies between revenues, costs and exchange rates over longer periods, which make long-term hedging with currency forwards less important. The quantitative effects of these dependencies or natural hedges can be substantial for exposures that lie several years in the futures. For shorter time periods, however, like the next one or two years, one should not rely too much on these natural hedges and use an almost full hedge with forward contracts.

Seen from another point of view, we could say that hedges lose attractiveness and effectiveness due to unhedgable risks, which result from uncertain revenues and costs. As these unhedgable risks get more and more important for longer hedge horizons, we should hedge less and less.

In order to get a better understanding of our results and their different interpretations, we have to take a closer look at the driving forces that lie behind them. Our first interpretation of the downward sloping term structure of hedge ratios, the importance of natural hedges, is closely linked to the observation that relevant risk factors enter into cointegration relations, which in turn make the correlation structure a function of the hedge horizon. Figure 2 shows these effects.

Going back to the forward positions in the real profit equation (10), we see that revenue risk \( \left( \frac{1}{1 + \tilde{\epsilon}_{f,t}} \right) \) and real exchange rate risk \( 1 + \tilde{u}_t \) are connected in a multiplicative way. Therefore, the firm’s hedging strategy crucially depends on the correlation between these two risk factors. A strong positive correlation implies that forwards are effectively more sensitive to changes in real exchange rates than real profits are. Accordingly, a variance minimizing strategy would require a lower usage of forward contracts if the correlation were higher. Figure 2 shows that such an effect is very relevant for our results. The crucial correlation between \( \left( \frac{1}{1 + \tilde{\epsilon}_{f,t}} \right) \) and \( 1 + \tilde{u}_t \) increases steadily with the hedge horizon for all three currencies, reaching values of 60 percent or more.

The second interpretation of the downward sloping term structure, the increasing importance of unhedgable risks, corresponds to the observation that prices follow I(2) processes, but exchange rates (and deviations from PPP) follow I(1) processes. The relative increase in variance of I(2) processes with the hedge horizon is much stronger than the relative increase in variance of I(1) processes. As Figure 3 shows for the risk factors of our model, we observe a linear function (I(1) process) in
contrast to an exponential one (I(2)). Therefore, both the proportion of hedgable risks and the hedge ratios should decrease with the hedge horizon.

In summary, the interplay of two effects – the different degrees of integration of different risk factors and the cointegration relations between them – drives our results on the downward sloping term structure of hedge ratios. This finding highlights the importance of capturing the integration and cointegration properties adequately. Also note that a movement of two countries towards PPP is not necessarily a prerequisite for a downward sloping term structure of hedge ratios. Even if the deviation from PPP is an I(1) process, we might still have price risks that increase even more with time and therefore lead to declining hedge ratios.

Our two explanations for a downward sloping term structure of hedge ratios, natural hedges and unhedgable risks, have quite different implications for the risk management strategies of firms. If the effects were completely driven by natural hedges, one
would not observe a strong decrease in hedging effectiveness with the hedge horizon. Hedging with forwards should be reduced for longer horizons, but the overall risk reduction would be sufficient. To the contrary, if unhedgable risks were the dominant reason for an increased underhedging, hedging effectiveness would deteriorate dramatically with the hedge horizon. In such a situation, financial hedging alone would not be sufficient to reduce risk and the firm should think about supplementary measures, like operational hedging.

In order to judge the quantitative importance of the two different reasons for a downward sloping term structure, we take a look at the hedging effectiveness. This hedging effectiveness can be measured by the percentage variance reduction of the

Figure 3: Variances of risk factors.
hedge, the Johnson measure,\textsuperscript{26} that is formally defined as

\[
JM = \text{Percentage in Variance Reduction} = \frac{\sigma^2_U - \sigma^2_H}{\sigma^2_U},
\]

where \(\sigma^2_U\) is the variance of the unhedged position and \(\sigma^2_H\) the variance of the hedged position.

To get a general impression of the hedging effectiveness that can be achieved, let us look at a firm that sells one unit of its product in each of the following ten years and consider the variance reduction of real profits over the total ten-years period. The second column of Table 7 provides the corresponding results for all three currencies. As we see, variance reduction is highest for the British Pound and lowest for the US Dollar, which might be due to the closer link between the United Kingdom and Germany as members of the European Union. Most interestingly, however, we see that hedging effectiveness is generally very high for all countries, achieving a risk reduction of 89% or more.\textsuperscript{27}

Even though the “average” variance reduction of all exposures that a firm faces over a ten-years period is high, it is instructive to check how effectively single exposures at certain times in the future can be hedged. Such maturity specific measures of hedging effectiveness are shown in the third to eights column of Table 7.

As we see, hedging effectiveness decreases with the hedge horizon. However, the decrease is much smaller than the decrease in hedge ratios. Take the results for the British Pound, for example. Hedge ratios decrease substantially from 0.99 (hedge horizon of one month) to 0.53 (hedge horizon of ten years), whereas the variance reduction decreases only slightly from 98% to 92%. This result shows that the forward hedge is still very effective and the correlation effect is the main explanation

\[\text{See Johnson (1960).}\]

\[\text{Note that a strategy that allows for a general cross hedging between different maturities and exploits all covariances between operating profits and forward contracts of all maturities improves the hedging effectiveness only marginally. The corresponding variance reductions are 88.9\% for the US, 97.9\% for the UK and 95.6\% for Japan. These results provide a further argument for the use of a maturity matched hedging strategy.}\]
for the low hedge ratios. The very low hedge ratio for the US Dollar at a ten years hedge horizon, however, can partly be explained by the increased importance of non-hedgable risks, as the clear drop in the hedging effectiveness shows. In this case, it might pay to look for alternatives to a pure financial hedge with currency forward contracts.

4 Conclusions and Outlook

This paper has analyzed the hedging of exchange rate risk at different hedge horizons. In an initial step, we derived variance minimizing currency hedge ratios for an exporting firm, taking uncertain revenues, costs and exchange rates into account. In a second step, the term structure of currency hedge ratios was quantified in an empirical study. Based on a cointegrated VAR model of prices and interest rates in two countries and the exchange rate, we simulated future price paths by means of a bootstrap algorithm. These price paths allowed us to quantify hedge ratios for different hedge horizons and the hedging effectiveness.

Our empirical study provided three major results. Firstly, it showed that a substantial underhedging of exchange rate risk for longer hedge horizons can be explained to a large extent by the interplay of prices and exchange rates: i.e., by the existence of natural hedges. Accordingly, although hedge ratios become quite low, hedging effectiveness is still high. This result holds irrespective of our finding that there is no mean reversion towards PPP. Secondly, it can be important to follow different hedging strategies for different currencies. In fact, the price and exchange rate dynamics captured by our VAR model imply a ten-years hedge ratio for the US Dollar as low as 0.34. For the British Pound and the Japanese Yen, the corresponding hedge ratios are still approximately 0.5. Thirdly, for short hedge horizons of up to one year differences between currencies are very small and hedge ratios are still close to one; i.e., firms can not rely on natural hedges for shorter hedge horizons.

The most important driving forces behind our results are the integration and cointegration properties of the risk factors that determine the hedge ratios, since the degree of integration strongly influences how a certain risk increases with the hedge horizon. Thus, our study highlights that decisions on longer-term hedging arrangements deserve a careful analysis of the integration properties of revenues, costs and exchange rates.

The analysis presented in this paper is only a first step towards an understanding of the impact of diversification effects between prices and exchange rates on a firm’s
hedging decision. In particular, we look at a firm with revenues and costs that grow in line with the price level (PPI) of the country. Therefore, we can call the resulting term structure of hedge ratios a kind of country benchmark. Of course, specific firms will generally differ from this benchmark and might even experience stronger effects on their hedging strategy. To check this conjecture would be an interesting extension of our study. One could use industry specific price indices for the implementation of the hedging strategies or even firm specific information if available.

Another open issue is the characterization of the term structure of hedge ratios for hedging criteria other than variance minimization. Under more general criterion functions, forward contracts will in general no longer be optimal hedging instruments. As shown by Moschini and Lapan (1995) and Brown and Toft (2002) in the context of specific models, some kinds of options should be added to forward positions. Moreover, one need not even restrict the set of possible hedging instruments to currency derivatives. Since interest rates are closely related to prices and exchange rates, interest rate derivatives are natural candidates to consider. For long hedge horizons, where the hedging effectiveness of currency forwards is relatively low, they could bring a significant improvement. In addition, inflation derivatives build a promising asset class for hedging long-term exposures.

Another interesting issue concerning the term structure of hedge ratios would be to look at countries with higher inflation rates than Germany, the US, the United Kingdom and Japan. If inflation rates are higher, we would expect that currencies would react more strongly, and the impact on hedging decisions could be higher. Finally, it would be interesting to understand what happens to a firm that hedges different exchange rate risks simultaneously. Since prices and currencies in different countries should be economically related, there could be additional natural hedges in this multi-country case.
References


Appendix

Figure 4: Overview of time series in levels.
Figure 5: Overview of time series: log PPI.
Figure 6: Overview of time series: log FX.
Figure 7: Overview of time series: interest rates.
Figure 8: Overview of the absolute purchasing power parity (a) PPP and (b) PPP, first difference.
Note: t-statistics in brackets. Significant test statistics are given in bold face.

Table 8: Long-run and short-run structure of the VECM model for the US.
Table 9: Long-run and short-run structure of the VECM model for the UK.

| β'   | ppp | Δ|f|,t | Δ|h|,t | i|,t | C(1982:08) | C(1985:04) |
|------|-----|---|---|----|----|----|-----|------------|------------|
| Beta(1) | 1.000 | -0.598 | 1.320 | 0.429 | -0.413 | 0.003 | 0.000 |
|       | [NA] | [-8.661] | [14.285] | [1.893] | [-1.324] | [3.284] | [NA] |
| Beta(2) | -1.026 | 1.000 | -0.690 | -0.099 | -0.236 | 0.000 | 0.002 |
|        | [-6.491] | [NA] | [-10.560] | [-0.534] | [-1.008] | [NA] | [3.747] |

<table>
<thead>
<tr>
<th>α</th>
<th>Alpha(1)</th>
<th>Alpha(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ</td>
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<tr>
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<td>Δi</td>
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<td>[0.896]</td>
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</table>

| II   | ppp | Δ|f|,t | Δ|h|,t | i|,t | C(1982:08) | C(1985:04) |
|------|-----|---|---|----|----|----|-----|------------|------------|
| Δ|ppp| | 0.003 | 0.000 |
|   |     | [0.583] | [-0.032] |
| Δ|f|,t| | 0.673 | -0.049 |
|   |     | [12.270] | [-14.584] |
| Δ|h|,t| | -0.257 | -0.263 |
|   |     | [-4.779] | [-10.610] |
| Δi|,t| | 0.003 | 0.000 |
|   |     | [0.606] | [-0.352] |
| Δi|h|,t| | 0.009 | -0.008 |
|   |     | [2.849] | [-2.384] |

| Γ1   | Δ|ppp| t-1 | Δ|f|,t-1 | Δ|h|,t-1 | Δi|,t-1 | Δi|h|,t-1 | CONSTANT |
|------|-----|----|----|----|----|----|----|-----|-----|-----|-----|--------|
| Δ|ppp| | 0.088 | -0.005 | 0.111 | -0.041 | 0.000 |
|   |     | [1.632] | [-1.061] | [1.82] | [0.754] | [0.071] |
| Δ|f|,t| | -0.104 | -0.009 | 0.782 | 0.608 | 0.011 |
|   |     | [-0.176] | [-0.171] | [1.402] | [14.124] |
| Δ|h|,t| | -1.753 | -0.079 | 1.129 | 1.114 | -0.000 |
|   |     | [-3.032] | [-1.612] | [2.067] | [1.408] |
| Δi|,t| | -0.077 | 0.000 | 0.214 | 0.097 | 0.000 |
|   |     | [1.483] | [0.043] | [4.357] | [0.285] |
| Δi|h|,t| | 0.132 | 0.000 | 0.050 | 0.320 | 0.000 |
|   |     | [-3.712] | [2.978] | [1.506] | [2.359] |

Note: t-statistics in brackets. Significant test statistics are given in bold face.
Table 10: Long-run and short-run structure of the VECM model for Japan.

<table>
<thead>
<tr>
<th>$\beta'$</th>
<th>$\Delta ppp_t$</th>
<th>$\Delta p_{f,t}$</th>
<th>$\Delta p_{h,t}$</th>
<th>$i_{f,t}$</th>
<th>$i_{h,t}$</th>
<th>C(1982:08)</th>
<th>C(2000:04)</th>
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<td>-1.256</td>
<td>0.105</td>
<td>-0.070</td>
<td>-0.027</td>
<td>-0.001</td>
<td>0.000</td>
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<td>[N.A.]</td>
<td>[-28.734]</td>
<td>[0.925]</td>
<td>[-0.261]</td>
<td>[-0.069]</td>
<td>[-3.365]</td>
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<tr>
<td>Beta(2)</td>
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<td>1.000</td>
<td>-0.496</td>
<td>0.012</td>
<td>0.325</td>
<td>0.000</td>
<td>0.001</td>
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<td>[N.A.]</td>
<td>[-6.407]</td>
<td>[0.062]</td>
<td>[1.184]</td>
<td>[N.A.]</td>
<td>[4.552]</td>
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<th>$\alpha$</th>
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<th>Alpha(2)</th>
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<tr>
<td>$\Delta^2 p_{f,t}$</td>
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<tr>
<td>$\Delta^2 p_{h,t}$</td>
<td>1.164</td>
<td>1.527</td>
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<tr>
<td>$\Delta i_{f,t}$</td>
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<td>0.008</td>
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<td>$\Delta i_{h,t}$</td>
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<th>$\Delta^2 p_{f,t-1}$</th>
<th>$\Delta^2 p_{h,t-1}$</th>
<th>$\Delta i_{f,t-1}$</th>
<th>$\Delta i_{h,t-1}$</th>
<th>CONSTANT</th>
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<td>[1.233]</td>
<td>[0.168]</td>
<td>[6.979]</td>
<td>[6.332]</td>
<td>[-1.938]</td>
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</tbody>
</table>

Note: t-statistics in brackets. Significant test statistics are given in bold face.