# The Risk Microstructure of Corporate Bonds: A Case Study from the German Corporate Bond Market*

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**Abstract**

This article presents joint econometric analysis of interest rate risk, issuer-specific risk (credit risk) and bond-specific risk (liquidity risk) in a reduced-form framework. We estimate issuer-specific and bond-specific risk from corporate bond data in the German market. We find that bond-specific risk plays a crucial role in the pricing of corporate bonds. We observe substantial differences between different bonds with respect to the relative influence of issuer-specific vs. bond-specific spread on the level and the volatility of the total spread. Issuer-specific risk exhibits strong autocorrelation and a strong impact of weekday effects, the level of the risk-free term structure and the debt to value ratio. Moreover, we can observe some impact of the stock market volatility, the respective stock's return and the distance to default. For the bond-specific risk we find strong autocorrelation, some impact of the stock market index, the stock market volatility, weekday effects and monthly effects as well as a very weak impact of the risk-free term structure and the specific stock's return. Altogether, the determinants of the spread components vary strongly between different bonds/issuers.



**Keywords:** Credit risk, Duffie/Singleton framework, Liquidity risk,
Markov chain Monte Carlo estimation.

**JEL:** C51, G12, E43

# 1 Introduction

Credit risk literature and industry measure the difference between risky bonds and risk-free bonds in the form of spreads. These spreads include several components, such as differences in credit risk, liquidity, taxation and other institutional differences (see e.g. Elton et al. (2001) or Collin-Dufresne et al. (2001)). We focus on the separation between the issuer-specific and the bond-specific spread component. Issuer-specific risk includes credit risk and other issuer-specific factors (like any issuer-specific liquidity). Bond-specific risk represents bond-specific liquidity, any component due to the respective bond features (e.g. seniority, registration requirements, collaterals or bond covenants) and other sources arising from market microstructure noise (see e.g. Campbell et al. (1996)).

A clear-cut separation between issuer-specific and bond-specific spread is a most relevant prerequisite for modeling each of these two types of risk. This in turn is essential for several reasons. One important field is the risk management of bond portfolios, the modeling of bond indexes and the valuation of bond index derivatives with more than one bond issued by the same issuer in the portfolio/index. The larger the part of the issuer-specific process, the higher the correlation between the total spreads of the bonds of the same issuer, thus the higher the risk of the complete portfolio/index and the higher the value of the respective index derivative. Also, a large impact of issuer-specific risk or a strong homogeneity of bond-specific processes show that in bond portfolio management the selection of the issuer is more important, while a large share of bond-specific risk as well as strong heterogeneity of bond-specific processes indicates that the bond selection is relevant, too. Only in the first case does working with one spread curve per issuer make sense. A correct split-up between issuer-specific risk and bond-specific risk is also important for mixed portfolios/indexes including stocks and bonds of the same issuer to aggregate the total issuer-specific risk over all bonds and stocks of the same issuer. Furthermore, if one wants to investigate specific accounting issues (e.g. the impact of earnings announcements or earnings management) by analyzing corporate bond spreads it makes sense to use only the issuer-specific part of the spread. Also, in corporate finance practice corporations that have only non-traded debt but no bonds outstanding frequently use information from traded bonds issued by other, similar firms to calculate their cost of debt. In this case a separation between issuer-specific (credit) risk and bond-specific (liquidity) risk is necessary in order to adjust for liquidity and other bond-specific differences. Another situation where this separation is necessary arises in event studies, e.g. studies that analyze how the spread reacts to rating changes: if changes in the issuer rating are investigated, one should only analyze the issuer-specific spread. By contrast, for changes in the bond rating the total spread has to be analyzed.

An adequate separation of issuer-specific and bond-specific spread enables correct identification of the patterns of the term structures of issuer-specific and bond-specific spreads in analogy to Driessen (2005) and Liu et al. (2006) as well as the respective process properties and determinants. Also, answering the questions 1) what part of a bond's risk is systematic vs. unsystematic; 2) whether issuer-specific/credit risk or bond-specific/liquidity risk is priced by the market; and 3) what is the market price of risk of the respective factors (in analogy to Chacko (2005), Jarrow et al. (2005), Liu et al. (2006), and Chen et al. (2007)) requires a correct separation between issuer-specific and bond-specific risk.

The main objective of this article is to model, separate and analyze interest rate risk, issuer-specific and bond-specific risk. Our model is based on the Lando (1998), Duffie and Singleton (1999) and Feldhütter and Lando (2008) framework. In contrast to previous literature that often uses a number of latent factors identical to or smaller than the number of bonds, we use for each issuer one latent issuer-specific factor and for each bond one latent bond-specific factor. This is a prerequisite for a correct dissection between issuer-specific and bond-specific risk. As a consequence the complete bond-specific component enters into the pricing equation. Although our model includes correlation between the risk-free term structure and issuer-specific risk, it permits sequential estimation of the risk-free term structure parameters and the issuer-specific as well as bond-specific components.

After setting up the model we use time series of German corporate bond prices in order to estimate the model parameters. The complexity of the model requires a more refined estimation procedure (e.g. due to the coupon effects pointed out by Diaz and Navarro (2002) one should avoid estimation from yields in order to obtain a correct bond-specific spread estimate; because of our model structure direct maximum likelihood techniques are not feasible). We solve the under-identification problem arising from the mismatch between data and latent processes that arises from the additional factor in our model, by data augmentation, thereby generalizing previous estimation approaches. We use a Bayesian analysis and

Markov Chain Monte Carlo estimation.

In general, bond-specific components create large differences in the spreads, even for bonds of the same issuer. Thus, bond-specific risk is substantial and priced by the market. Depending on the issuer, we obtain both upward and downward sloping term structures of the issuer-specific spread. The term structure of the bond-specific spread is flat. We investigate the properties of the issuer-specific and bond-specific processes and regress the estimates against variables hypothesized or identified in literature as determinants of the spread between risky and risk-free rates. In contrast to existing literature that investigates the determinants of the total spread, we analyze the determinants of each spread component alone. The issuer-specific spread exhibits strong autocorrelation and is strongly influenced by weekday effects, the level of the risk-free term structure and the debt to value ratio. In addition, we observe some impact of the stock market volatility, the respective stock's return and the distance to default on the issuer-specific spread. For the bond-specific risk we also observe strong autocorrelation, some impact of the stock market index, the stock market volatility, weekday effects and monthly effects and very weak impact of the risk-free term structure and the specific stock's return. We find that systematic risk is especially present with long-term bonds. For holders of mixed funds the relation between stock price dynamics and the spread components is particularly important. Also, for financial engineering purposes it is relevant to know the split-up between issuer-specific and bond-specific spread and the respective determinants instead of only knowing the determinants of the total spread. Equally, a firm can reduce its cost of capital by fixing the bond features in response to the determinants of the bond-specific spread, especially if the bond-specific spread accounts for a large part of the total spread. Finally, by means of principal components analysis we investigate if there are market-wide factors for the issuer-specific spread and the bond-specific spread. Our results suggest that using one spread per issuer, as is frequently done in literature and industry, is not sufficient.

This paper is organized as follows: Section 2 presents the model; Section 3 describes the data used; Section 4 outlines the estimation procedure; Section 5 presents the estimation results; and Section 6 concludes.

## 2 Model

We work in a frictionless and arbitrage-free market in continuous time $t$. On a filtered probability space, fulfilling the usual conditions, we consider the empirical probability measure $P$ and an equivalent martingale measure (risk-neutral measure) $Q$, respectively. We define as a "risk class" a homogeneous set of bonds with identical issuer-specific and bond-specific risk.

We consider one issuer with $j = 1, \ldots, J$ coupon bonds on the market, symbolizing by $U_j(t)$ the set of coupon dates for bond $j$ occurring between $t$ and maturity. Traded are risk-free zero-coupon bonds for all maturities and risky zero-coupon bonds for all maturities and all risk classes, all with a face value of 1.

As usual in the Duffie and Singleton (1999) model, default of an issuer occurs at the first event time of a counting process. The default intensity, that is the mean arrival rate of default conditional on all current information, is stochastic. Default results in a downward jump in the market price of the bond ("Fractional Recovery of Market Value Assumption"). In addition, following Jarrow et al. (2005) we assume that default event risk can be diversified.

In our model, all types of risk associated with a coupon bond $j$ are included in the prices of the zero-coupon bonds associated with the risk of this bond $j$. The time $t$ price of a zero-coupon bond with maturity $\tau$ associated with the risk of coupon bond $j$ is abbreviated by $v_j(t, \tau)$. The time $t$ price of the risky coupon bond $j$, $p_j(t)$, is a linear combination of its remaining cash flows $C_j(u)$ and the risky zero-coupon bond prices $v_j(t, u)$:

$$p_j(t) = \sum_{u \in U_j(t)} v_j(t, u) C_j(u) \ . \tag{1}$$

All bond prices satisfy the no-arbitrage condition. The risk-free term structure, the issuer-specific risk and the bond-specific risk are modeled by the following latent stochastic processes $X(t)$ under the risk neutral measure $Q$:

**Assumption 1.** As shown by literature (e.g. Litterman and Scheinkman (1991) and Dai and Singleton (2002)), three factors are necessary to model the default-free term structure dynamics. We therefore model the **risk-free rate** as a linear combination of three correlated factors, where the respective latent vector process $(X_{rf}(t))$ is given by $X_{rf}(t) := (X_1(t), X_2(t), X_3(t))^\top$. Based on recommendations in recent literature (see Tang and Xia (2007)), we model $(X_{rf}(t))$ as a member of the $\mathbb{A}_1(3)$ family introduced by Dai and Singleton (2000). From $(X_{rf}(t))$ we obtain the risk-free discount rate $R_{rf}(t) = \delta_{x,rf} X_{rf}(t)$, with $\delta_{x,rf} = (\delta_1, \delta_2, \delta_3)$, thus

$$R_{rf}(t) \quad = \quad \sum_{l=1}^{3} \delta_l X_l(t) \; . \tag{2}$$

**Assumption 2.** We model **issuer-specific risk** for each issuer with one latent process $(X_4(t))$ that is independent of $(X_1(t))$,$(X_2(t))$ and $(X_3(t))$. Since $X_4(t)$ is assumed to drive especially credit risk (and therefore enters into the default intensity), it should have a positive domain. Therefore, we model $(X_4(t))$ by means of a square root process with parameters $\alpha_4^Q$ (long-run mean), $\beta_4^Q$ (speed of mean reversion) and $\sigma_4$ (volatility). From the latent processes driving the risk-free segment and from $(X_4(t))$ we define $X_I(t) := (X_1(t), \ldots, X_4(t))^\top$. As the empirical results in numerous studies (see e.g. Longstaff and Schwartz (1995), Suhonen (1998), Duffee (1998), Duffee (1999), Düllmann et al. (2000) and Frühwirth and Sögner (2006)) raise arguments for correlation between the risk-free and the risky segment, particularly for correlation between credit risk and interest rate risk, we integrate correlation between the risk-free rate and the issuer-specific spread. In order to maintain a separate treatment in the estimation procedure of the risky and the risk-free components we apply the methodology developed in Lando (1998) and Duffie and Singleton (1999), where correlations are parsimoniously modeled by a scalar parameter $c$. The discount rate for a fictitious bond issued by issuer $I$ with zero bond-specific risk, $R_I(t)$, is defined as follows:

$$R_I(t) \quad := \quad R_{rf}(t) + X_4(t) - cR_{rf}(t) = (1-c) \sum_{l=1}^{3} \delta_l X_l(t) + X_4(t) \tag{3}$$

**Assumption 3.** To model **bond-specific risk** we use one latent Ornstein/Uhlenbeck process for each bond. This process is represented by $(X_{5,j}(t))$, where $j$ stands for the index of the corresponding bond ($j = 1, \ldots, J$) of issuer $I$. The parameters of each process $(X_{5,j}(t))$ are $\alpha_{5_j}^Q$, $\beta_{5_j}^Q$ and $\sigma_{5_j}$. $X_{5,j}$ is independent of all the other state variables, including the bond-specific factors for the other bonds. Note that by our specification $X_{5,j}(t) < 0$ is possible. Intuitively, $X_{5,j}(t) < 0$ may occur for bonds with higher liquidity than the average liquidity of all bonds of this issuer. The bond-specific discount rates, $R_j(t)$, are defined as follows:

$$R_j(t) \quad := \quad R_{rf}(t) + X_4(t) - cR_{rf}(t) + X_5(t) = (1-c)\sum_{l=1}^{3} \delta_l X_l(t) + X_4(t) + X_{5,j}(t) \ . \tag{4}$$

By this we are able to construct an affine term structure model where the issuer-specific and the bond-specific rates are correlated with the risk-free term structure *and* nevertheless a separate, sequential estimation of the risk-free term structure parameters and issuer-specific and bond-specific components is feasible.

For notational convenience we introduce $X_j(t) = (X_1(t), X_2(t), X_3(t), X_4(t), X_{5,j}(t))^\top$, $j = 1, ..., J$, $X(t) = (X_1(t), \ldots, X_3(t), X_4(t), X_{5,1}(t), \ldots, X_{5,J}(t))^\top$, and $\delta_x = (\delta_1, \delta_2, \delta_3, \delta_4, \delta_{5,1}, \ldots, \delta_{5,J})^\top$, with $\delta_1, \delta_2$ and $\delta_3$ being estimated from risk-free data, $\delta_4 = 1$ and $\delta_{5,j} = 1$ for $j = 1, \ldots, J$.

From Assumptions 1, 2 and 3 the vector process $X(t)$ is affine under $Q$ and of dimension $M = 4 + J$. It can be represented by

$$dX(t) \quad = \quad \beta^Q(\alpha^Q - X(t))dt + \Sigma\sqrt{S(t)}dW^Q(t) \, , \tag{5}$$

where $W^Q(t)$ is an $M$-dimensional Brownian motion under the equivalent martingale measure with independent components. $\beta^Q$ is a lower triangular $M \times M$ matrix. $\beta^Q$ includes in the (non-negative) diagonal the speeds of mean reversion $\operatorname{diag}\beta = (\beta^Q_{1,1}, \beta^Q_{2,2}, \beta^Q_{3,3}, \beta^Q_{4,4}, \beta^Q_{5,5_1}, \ldots, \beta^Q_{5,5_J})$. In addition, below the diagonal we have the elements $\beta^Q_{2,1}$, $\beta^Q_{3,1}$ and $\beta^Q_{3,2}$ that result from the $\mathbb{A}_1(3)$ setting. Since both the issuer-specific process and the bond-specific processes are independent of all other processes, we use a simplified notation for the elements in the diagonal: $\beta^Q_1{=}\beta^Q_{1,1}, \ldots, \beta^Q_3{=}\beta^Q_{3,3}$, $\beta^Q_4{=}\beta^Q_{4,4}$, $\beta^Q_{5_1}{=}\beta^Q_{5,5_1}, \ldots, \beta^Q_{5_J}{=}\beta^Q_{5,5_J}$. $\Sigma$ is a diagonal $M \times M$ matrix with non-negative elements $\operatorname{diag}\Sigma = (1, 1, 1, \sigma_4, \sigma_{5_1}, \ldots, \sigma_{5_J})$, $\alpha^Q$ is $M \times 1$ and includes the long-run means ($\alpha^Q_1$, $\alpha^Q_2$, $\alpha^Q_3$, $\alpha^Q_4$, $\alpha^Q_{5_1}$, $\ldots$, $\alpha^Q_{5_J}$). $S(t)$ is a diagonal matrix including the components

$$S_{ii}(t) \quad = \quad a_i + b_i^\top X(t) \ , \tag{6}$$

where $i$ represents the number of the respective factor, $a_i$ is a scalar and $b_i$ a vector of dimension $4{+}J$. In the following we use $b_{i(j)}$ for the j-th component of $b_i$. Consistent with the above assumptions we set $a_1 = 0$, $a_2, a_3 = 1$, $a_4{=}0$, $a_5, \ldots, a_{4+J} = 1$, $b_{1(1)} = 1$, $b_{2(1)}, b_{3(1)} = 0$ and $b_{4(4)} = 1$. All other components of $b_i$ are zero.

*Market Prices of Risk:* For the two parameters $\alpha$ and $\beta$ we employ extended affine market prices of risk $\Lambda(t)$ from Cheridito et al. (2007). Thus, by construction $(X(t))$ is an affine stochastic process also under $P$:

$$dX(t) \quad = \quad \beta^P(\alpha^P - X(t))dt + \Sigma\sqrt{S(t)}dW^P(t), \tag{7}$$

where $\alpha^P$, $\beta^P$ and $W^P$ have a structure analogous to $\alpha^Q$, $\beta^Q$ and $W^Q$ and $dW^Q(t) = dW^P(t) - \Lambda(t)$. By estimating $\beta$ and $\alpha$ under both measures $P$ and $Q$ the market price of risk parameters can be estimated implicitly which would allow to study how the market compensates investors for bearing different types of risk (see e.g. Driessen (2005) or Berndt et al. (2005)). Thus, an explicit estimation of the market price of risk parameters is not required (see Cheridito et al. (2007)).

Under the above assumptions, for each issuer the risky zero-coupon bond prices for bond $j$ are derived by the following well-known exponential-affine pricing formula:

$$v_j(t,T) \quad = \quad \mathbb{E}_t^Q\left[\exp\left\{-\int_t^T R_j(s)ds\right\}\right] = \exp\Big(A_j(T-t) - B_j(T-t)^\top X_j(t)\Big) \ , \tag{8}$$

where $E_t^Q$ is the expectation under $Q$, conditional on the information set at time $t$, and $A_j(T-t)$ and $B_j(T-t)$ are functions of the parameters (under $Q$) described above that can be found as solutions to Riccati equations (see Duffie and Kan (1996)). These risky zero-coupon bond prices enter into equation (1) yielding the risky coupon bond prices.

We emphasize that many models (see e.g. Duffie and Singleton (1997) or Duffee (1999)) assign – at least some part[1] of the bond-specific component – to an *iid* residual. In many empirical studies (see e.g. Duffee (2002), Duffie et al. (2003) or Cheridito et al. (2007)), however, the sample residual has shown to be highly autocorrelated. In contrast to the existing modeling literature, in our model by including bond-specific processes the bond prices are completely described by the model. Thus, the specification of an error term is not required, the "residual" enters into the pricing equation. This makes sense as e.g. Chacko (2005), Longstaff et al. (2005), Liu et al. (2006) or Chen et al. (2007) find that liquidity/bond-specific risk is rewarded by the market. Moreover, the "residual" is implicitly assigned an autoregressive structure which is in line with empirical literature.

Based on equations (3) and (4) we define the instantaneous total spread for bond $j$ at time $t$, $TSPR_j(t)$, as well as its two components, the instantaneous issuer-specific spread at time $t$, $ISPR(t)$, and the instantaneous bond-specific spread for bond $j$ at time $t$, $BSPR_j(t)$:

$$\begin{aligned} TSPR_j(t) &= R_j(t) - R_{rf}(t) = -c \sum_{l=1}^{3} \delta_l X_l(t) + X_4(t) + X_{5,j}(t) \ , \\ ISPR(t) &= R_I(t) - R_{rf}(t) = -c \sum_{l=1}^{3} \delta_l X_l(t) + X_4(t) \ , \\ BSPR_j(t) &= R_j(t) - R_I(t) = X_{5,j}(t) \ . \end{aligned} \tag{9}$$

Note that the parameter $c$ controls the correlation between the risk-free rate and the issuer-specific spread and the total spread. $c > 0$ implies a negative correlation and vice versa.

# 3 Data

The data used in our study consist of daily observations from January 6th, 2004 to August 31st, 2005. We use daily data in order to check for weekday effects as described later. Excluding holidays and weekends the observation period includes 426 days with data.

For the risk-free segment we use EURIBOR data for maturities of 1 month, 3 months and 6 months provided by the Deutsche Bundesbank under http://www.bundesbank.de. For maturities 1, 2, ..., 10 years we use swap rates (middle rates, semi-annually fixed rate vs. 6-month EURIBOR) from Datastream. First, we interpolate the swap rates to obtain the respective swap rates for maturities in between full years (i.e. 1.5 years, 2.5 years, ..., 9.5 years). Then, based on the standard assumption of no counterparty risk (see Liu et al. (2006), Feldhütter and Lando (2008) and several arguments in the latter paper) and by means of bootstrapping (see Liu et al. (2006)) we convert the time series of money market and swap rates into a time series of zero-coupon bond prices. Finally, we derive continuously compounded yields from these zero-coupon bond prices.

We use this data in spite of the credit risk involved in these interest rates (see e.g. Cossin and Pirotte (1998) or Feldhütter and Lando (2008)), for the following reasons: for the short end of the risk-free term structure we use EURIBOR data instead of bond data as, due to very low liquidity, estimates from government bond prices are known to be unreliable at the short end of the term structure. On the long end, we use swap rates instead of government bond prices due to the higher liquidity on the swap market compared to the bond market and in order to ensure consistency with respect to credit risk with the short end (homogeneous EURIBOR - swap market credit quality assumption). Use of this data is also in line with current literature (e.g. Duffie et al. (2003), Dewachter et al. (2004) and Feldhütter and Lando (2008)) showing that not the government bond curve but the swap curve is seen as the reference default-free curve and bringing further arguments for the use of swap data instead of bond data for the risk-free rate.

To obtain a one-to-one relation between observed data and latent processes, for our analysis we assume the spot rates with a maturity of 6 months, 2 years and 5 years to be observed without measurement error. This selection is a compromise between the one in Duffee (2002) and the one in Aït-Sahalia and Kimmel (2002). The remaining spot rates are assumed to be measured with error.

The default-risky coupon bond data set comprises 7 German Mark (DEM) or Euro (EUR) denominated fixed-rate senior unsecured bonds without sinking fund provisions or embedded options. We deem bonds issued by a financing subsidiary and guaranteed by the mother to be issued by the guaranteeing mother. From the Bloomberg database we extract for each issuer the rating history and for each bond the respective features. As regards rating, we use the long-term domestic issuer rating from S&P. All issuers selected have a stable rating. Neither the coarse rating nor the fine rating (reflected by - or +) changed during the observation period. 5 bonds have been issued by Bayerische Hypo- und Vereinsbank (HVB) with an A rating, 2 bonds have been issued by METRO with a BBB rating. All bonds were issued before the beginning of the observation period and have a maturity after the end of the observation period. Issuer, maturity, coupon rate and instrument code (ISIN) of all bonds are listed in Appendix B. All HVB bonds are without bond covenants. The METRO bonds have covenants included, namely a cross default pledge and a negative pledge.

For each bond and each trading day, we obtain closing prices (both clean and dirty prices) from the Datastream database with the prices of the HVB bonds originating from Munich stock exchange and those of the METRO bonds from Frankfurt stock exchange. To obtain a reliable database it is important to filter out non-transaction prices. If for a specific bond on a particular day there was no trade, Datastream in this market segment uses the same clean price as on the previous trading day. Therefore, we eliminate a price from our database if the corresponding clean price equals the clean price of the most recent trading day.

# 4 Model Estimation

Data are observed on a discrete grid with a step width of $\Delta$. We use $X_n$ for $X(t)$ observed at $t = n\Delta$, $n = 1, \ldots, N$. The data observed, $D$, includes three risk-free rates and $J$ bond prices for each issuer, resulting in a stacked vector, $\bar{P}_n$, of observables of dimension $3 + J$. On the other hand, the dimension of the latent vector process $X_n$ is of dimension $4 + J$. This results in an underidentification problem.

Duffie et al. (2003) tackle this problem with the strong assumption that one of the bonds observed is a *benchmark bond* without any bond-specific risk. Under this assumption the dimension of the vector of latent processes can be reduced by one, such that the dimensions match. As a result, simulated maximum likelihood can be used to estimate the parameters. The drawback of this methodology is that the estimation results are not invariant with respect to the choice of the benchmark bond. The likelihoods and thereby the estimates of the benchmark bond spread and the relative bond-specific spreads depend on the benchmark bond selected.

Therefore, we decide to waive the assumption of a benchmark bond. Instead, we solve the dimension problem described above by data augmentation (see Tanner and Wong (1987)), where the set of unknown parameters is augmented by the "artificial state variables" $X_{4,n}$, $n = 1, ..., N$. Including $X_{4,n}$ into $\bar{P}_n$ results in the vector $P_n$ matching the dimension of $X_n$, such that $X_n$ can be identified from the (augmented) vector of observations. For details the reader is referred to Appendix A.

This approach generalizes the Duffie et al. (2003) approach because it allows identification and estimation of issuer-specific components and bond-specific components of all bonds. Thus, with this methodology it is possible to split up the benchmark bond spread (in the Duffie et al. (2003) terminology) into an issuer-specific and a bond-specific component and the estimation results do not depend on the choice of the benchmark bond. With our methodology it is also possible to find out if a specific bond is appropriate as a benchmark bond and to identify the bond that is best suitable as a benchmark bond with the Duffie et al. (2003) approach.

As Frühwirth et al. (2006) observe in experiments with simulated data that implied state maximum likelihood can be unstable and Bayesian estimation based on *Markov Chain Monte Carlo* (MCMC) methods improves the quality of estimation, we apply Markov Chain Monte Carlo (MCMC) estimation. For the latent processes' transition densities see Appendix A.

By construction, there is only one risk-free term structure, holding for all issuers analyzed. In a first step, the risk-free term structure process is estimated from the risk-free data described in Section 3. From Collin-Dufresne et al. (2008) one knows that the risk-free term structure model results in 14 identifiable parameters: $\delta_1$, $\delta_2$, $\delta_3$, $\alpha_1^P$, $\alpha_1^Q$, $\beta_1^P$, $\beta_1^Q$, $\beta_{2,1}^P$, $\beta_2^P$, $\beta_2^Q$, $\beta_{3,1}^Q$, $\beta_{3,2}^Q$, $\beta_3^P$, $\beta_3^Q$. We estimate these parameters using the MCMC methodology. From these 14 parameters and the data we obtain estimates of $(X_{1,n}, X_{2,n}, X_{3,n})$.[2] The remaining procedure is performed with these fixed estimates. In a second step, given the risk-free term structure parameters we estimate the remaining parameters – including $c$ – of the risky term structures issuer by issuer. The overall parameter vector for one issuer is denoted by $\psi$, where $\psi = (\alpha_4^P, \alpha_4^Q, \beta_4^P, \beta_4^Q, \sigma_4, \alpha_{5_1}^P, ..., \alpha_{5_J}^P, \alpha_{5_1}^Q, ..., \alpha_{5_J}^Q, \beta_{5_1}^P, ..., \beta_{5_J}^P, \beta_{5_1}^Q, ..., \beta_{5_J}^Q, \sigma_{5_1}, ..., \sigma_{5_J}, c)$. Then the posterior distribution of $\psi$ and the latent process $(X_{4,n})$ are estimated by means of Markov Chain Monte Carlo (MCMC) estimation. A more detailed description of the MCMC parameter estimation algorithm is provided in Appendix A.

Note that the estimation of the latent vector processes $(X_n)$ is a byproduct of our MCMC algorithm. Estimates of these processes are important from an economic point of view, since these estimates provide us with the necessary information required to separate interest rate risk, issuer-specific and bond-specific risk and in order to find out the respective determinants.

# 5 Empirical Results

## 5.1 Estimated Spread Process Parameters

From the MCMC output we estimate the spreads defined in equations (9) by taking the multivariate posterior median of the MCMC samples of the processes $(X_{1,n})$, $(X_{2,n})$, $(X_{3,n})$, $(X_{4,n})$ and $(X_{5,j,n})$, $j = 1, 2, \dots, J$, (see Collin-Dufresne et al. (2008)). In the rest of the paper we follow the usual convention to express estimates by the symbol $\hat{}$.

*Stationarity:* By restricting parameters (all $\beta_{i,j} > 0$ and Feller condition), we ensure model parameters corresponding to stationary processes. Nevertheless, the standard augmented Dickey-Fuller test (trend and constant) rejects the zero hypothesis of non-stationarity for $\hat{X}_{5,1}$ and $\hat{X}_{5,2}$ for HVB on a 1% level and for METRO's $\hat{X}_{5,2}$ and HVB's $\hat{X}_{5,3}$ on a 5% level. For all other processes the test statistics are more or less close to the 10% critical level, but the zero hypothesis of non-stationarity is not rejected even for a significance level of 10%.

Table 5 presents estimates of the model parameters (median and, in order to see the dispersion, two quantiles) concerning the issuer-specific and bond-specific segments. We observe the following results:

From Table 5 we see an estimate $\hat{c} = 0.10$ for METRO and $\hat{c} = 0.62$ for HVB, both implying a negative correlation between the risk-free rate and the issuer-specific and total spreads. This is consistent with structural credit risk models and with the results of empirical literature (Longstaff and Schwartz (1995), Duffee (1998), Düllmann et al. (2000), and Frühwirth and Sögner (2006)). Comparing the two issuers, we observe that the dependence on the risk-free term structure is far more pronounced for HVB, which is plausible as HVB is a financial institution where one would expect a stronger influence of the interest rate environment than for the retailer METRO.

Furthermore, we see from Table 5 that for both issuers the long-run means, $\alpha^P$, are higher for the issuer-specific processes than for the bond-specific processes. By contrast, the mean reversion speed $\beta^P$ is far higher for the bond-specific processes than for the issuer-specific processes. This is in contrast to Liu et al.

(2006) who find for the US market that under the empirical measure the credit risk component is rapidly mean reverting while the liquidity component displays a high degree of persistence. The volatilities $\sigma$ are higher for the issuer-specific spread than for the bond-specific spread. This is consistent with Liu et al. (2006), who obtain high volatilities for the credit risk component.

Comparing the two issuers, we can observe that under the empirical measure the long-run mean of the issuer-specific component is far higher for HVB than for METRO and that the volatilities of the processes are higher for HVB than for METRO. Both effects are surprising as HVB has a better rating than METRO. We will further comment on this later in this section. Except with the first HVB bond, the persistence is higher ($\beta^P$ is lower) for HVB bonds than for the METRO bonds. Comparing across the bond sample of the same issuer, we can observe under the $P$ measure that the higher the maturity of the respective bond, the higher the long-run mean and the smaller the mean reversion speed of the bond-specific process.

Table 5 shows estimates of the long-run mean and the mean reversion speed under both probability measures. The differences between the parameters under the two measures can be attributed to the market prices of risk. Since we use extended affine market prices of risk, both mean and mean reversion speed under $P$ and $Q$ are affected (see Cheridito et al. (2007)). These effects are observed with both firms, for the issuer-specific and the bond-specific factors, respectively. Comparing the parameters under $Q$ with those under $P$ allows to investigate if/how the market prices the respective source of risk (see Jarrow et al. (2005)). We observe smaller (for HVB) and larger (for METRO) long-run mean parameters $\alpha$ under the $Q$-measure than under the $P$-measure for the issuer-specific component. For the bond-specific component we see that for both issuers $\alpha$ under the $Q$-measure exceeds $\alpha$ under the $P$-measure. We observe that the mean reversion speed is far smaller under $Q$, which implies that under the equivalent martingale measure the processes are more persistent than under $P$. These differences are significant. Similar effects are observed by Cheridito et al. (2007). This result also supports Duffee (1999), who finds that the default risk process is mean reverting under the $P$ measure and mean averting under $Q$. Also, it is interesting to see that under the $P$ measure the issuer-specific process has a higher persistence (lower $\beta$) than the bond-specific processes, while under the $Q$ measure the opposite is true. Moreover, under the $P$ measure the issuer-specific process has a higher long-run mean than the bond-specific process, while under the $Q$ measure it is the other way round.

For comparison, Liu et al. (2006) find under the empirical measure that the liquidity process is very persistent, while the default intensity process is rapidly mean reverting. We obtain stronger mean reversion for the issuer-specific process than for the bond-specific processes under the $Q$ measure but the opposite is true under the $P$ measure. From Table 5 we observe for both issuers and all processes that the speed of mean reversion is much larger under $P$ than under $Q$.

[Table 1 about here.]

Table 2 provides, based on the process parameters described, the spread estimates (means and medians, respectively, aggregated over the sample medians at all points of time) of the total spreads and the individual components (issuer-specific component $\hat{X}_4$, issuer-specific spread $\widehat{ISPR}$ and bond-specific component $\widehat{BSPR}_j = \hat{X}_{5,j}$) as well as their maximums, minimums and standard deviations over all points in time in terms of basis points.

First, we see from the MEAN column that all spreads are positive except the bond-specific spread of the first HVB bond which may be due to above-average liquidity. The mean issuer-specific spread is 49 basis points for HVB and 4 basis points for METRO. The spread for HVB seems plausible, however that for METRO too small compared to the spreads derived in literature as will be explained later. The mean bond-specific spread is between -5 and +33 basis points for HVB and between 14 and 21 basis points for METRO. Therefore, one can see substantial differences between the bond-specific spreads even of the same issuer.

Comparing the mean total spreads of different bonds from the same issuer we can observe that the total spread is an increasing function of maturity for both issuers which is due to the mean bond-specific spreads that show the same pattern. We see from the SD column that the standard deviation of the bond-specific spread as a function of the time to maturity shows a U-shaped pattern while the standard deviation of the total spread increases with maturity. This apparent contradiction can be explained as follows: Even though, by construction of the model, the processes $(X_4)$ and $(X_{5,j})$ are assumed to be independent, we can observe correlations between the estimated latent processes $(\hat{X}_4)$ and $(\hat{X}_{5,j})$: We observe negative (positive) correlation between $X_4$ and the bond-specific processes of bonds with short (long) maturities. This explains a reduction of the total spread volatility at the short end and a relative increase of total spread volatility on the long end of the term structure, turning the U-shaped pattern into an increasing term structure of volatilities.

In a next step we compare the two issuers by means of Table 2: in contrast to common intuition, the issuer-specific spread of the BBB issuer METRO is smaller than that of the A rated issuer HVB and as a consequence the total spreads of the METRO bonds are smaller than those of the HVB bonds. To make sure that this is not due to our estimation procedure we perform a stability analysis approximating the total spread by the difference between the yield to maturity of the respective bond and the risk-free spot rate with the time to maturity matched to that of the bond. We symbolize this crude proxy for bond $j$ and time step $n$ by $TSPR_{j,n}^{proxy}$. The last column of Table 2 presents the mean of $TSPR_{j,n}^{proxy}$ over all points in time, $MTSPR^{proxy}$. Note that the difference between the the total spread and the proxy of the total spread is due to the fact that the proxy is a non-instantaneous spread (thereby assuming a flat term structure), while the total spread in our model is an instantaneous spread based on the modeling of the term structure evolution. Moreover, the proxy includes the coupon effects pointed out by Diaz and Navarro (2002). The figures in the last column of Table 2 show that these results (higher spread for HVB than for METRO) are not due to a particular estimation procedure but that this is a real phenomenon existing in the data. Comparisons with literature (e.g. Liu et al. (2006) and Feldhütter and Lando (2008)) show that the HVB spreads seems reasonable while the METRO spreads are surprisingly low. There are several potential reasons for this: first, debt covenants are embedded in the METRO bonds which certainly reduces the METRO spreads. Although we believe that debt covenants are unlikely to be so powerful to cause the full extent of this phenomenon, this certainly explains part of the puzzle. A second part could be explained by the different industry: E.g. Altman and Kishore (1996) find that retail traders have a recovery rate that is about 10 percentage points higher than that of financial institutions. This should translate c.p. into smaller spreads for retailers than for financial institutions. A third partial explanation can be seen from Table 5: The volatility parameters are higher for HVB than for METRO which also should translate into higher spreads for HVB compared to METRO. This suggests that the ratings do not really capture properly the order of risk.

A comparison of the means and the medians in Table 2 shows that the distribution of the estimates is close to symmetric. This finding is confirmed looking at the maximum and minimum value in the MAX and MIN columns. We can relate Table 2 to literature: Liu et al. (2006) find that the default component is larger, while the liquidity component is slightly more volatile. In our case for HVB the issuer-specific spread exceeds the bond-specific spread, while for METRO the bond-specific spreads are higher than the issuer-specific spread. Concerning the order of the processes regarding volatility, we observe from column $SD$ that both issuer-specific and bond-specific risk vary significantly over time (compared to their respective means) and that the issuer-specific process is more volatile than the bond-specific process.

In a next step, we want to investigate the non-instantaneous spreads of the bond-specific and the issuer-specific component. Splitting up $A_j(T-t)$ and $B_j(T-t)$ in equation (8) into the first three components (risk-free segment), the fourth component (issuer component) and the fifth component (bond-specific component) and plugging in the parameter estimates (under $Q$) listed in Table 5 and the corresponding means of the estimates of $\widehat{X}_{4,n}$ and $\widehat{X}_{5,j,n} = \widehat{BSPR}_{j,n}$ (see Table 2) gives the non-instantaneous spreads for maturity $T-t$ for the issuer component and the bond-specific component, conditional on $X_n$. By varying the maturity we obtain the term structure of the respective spread.

Concerning the issuer-specific component, $X_4$, we observe an upward sloping term structure for METRO and a downward sloping term structure for HVB. Especially for METRO the slope of this term structure is small. The small, positive slope for METRO is in line with Diaz and Navarro (2002) and Liu et al. (2006) who detect for the swap market a rather flat term structure of default premia.

For the bond-specific factor, $X_5$, the term structure of the bond-specific factors is nearly flat. This adds to ambiguous results from the literature: Liu et al. (2006) obtain for the liquidity factor an upward sloping term structure while Janosi et al. (2002), Diaz and Navarro (2002) and Driessen (2005) observe a downward sloping term structure.

Table 3 shows the relative impact of issuer-specific and bond-specific spread on the total spread and its volatility. We see that for HVB most of the spread can be attributed to the issuer-specific component, while the opposite is true for METRO. As regards the volatility, with the exception of the first HVB bond, most of the volatility comes from the issuer-specific spread. Comparing the bonds of an issuer, we can observe for both issuers that the longer the maturity of a bond the higher the percentage share of the spread attributable to bond-specific risk. As already pointed out, the findings in Table 3 are relevant for the risk management of bond portfolios, the modeling of bond indexes and the valuation of bond index derivatives with more than one bond issued by the same issuer in the portfolio/index. As for HVB the issuer-specific process is more dominant, the correlation between the total spreads of the HVB bonds c.p. is higher than that of the METRO bonds (where the bond-specific processes are more dominant).

[Table 2 about here.]

The following subsection provides a more in-depth analysis of the properties and determinants of the two types of spreads.

## 5.2 Determinants of the Spreads

The goal of this subsection is to find out the drivers of the two types of spreads presented. To this end, we present plausible candidates in Section 5.2.1. In Section 5.2.2 we show the estimation results.

### 5.2.1 Candidates for Determinants of the Spreads

The candidates used in our analysis as explanatory variables for the issuer-specific spread and the bond-specific spread are the default-free term structure level, the default-free term structure slope (also referred to as "term spread"), the returns of the stock market index and of the respective stock, the volatility of the stock market, the market value debt ratio, the distance to default, lagged terms and market anomalies like weekday effects and monthly effects. In the following paragraphs we shall discuss the economic plausibility together with related literature for the candidates used in the regression analysis.

We include as explanatory variables the default-free term structure level and slope to check if the dependence of the spread processes on the default-free term structure can be fully captured by the parameter $c$. As indicated by several articles (see e.g. Litterman and Scheinkman (1991) or Duffee (1998)), most of the variation in the default-free term structure can be captured by its level and its slope.[3] Our proxy for the default-free term structure level is the one year spot rate (denoted as $RFLEVEL$ in the regression models). Our proxy for the default-free term structure slope (symbolized by $RFSLOPE$ in the regression models) is the difference between the ten year spot rate and the one year spot rate.

Elton et al. (2001) introduce equity factors into bond spread analysis. Other literature (e.g. Silva et al. (2003)) also shows that there is a link between the stock market and the corporate bond market. As a result, parts of the literature (e.g. Jarrow and Turnbull (2000) or Janosi et al. (2002)) include a stock market index into default intensity models. For these reasons, we want to check if there is a link between the respective spreads and the stock market index. We use a time series of the DAX 30 Xetra Performance Index, extracted from Datastream. Since stock indexes are known to be non-stationary, we use index returns, abbreviated by $DAXR_n$ (see e.g. Janosi et al. (2002) for an analogous procedure). An above average economic development reflected by above average rising stock prices ($DAXR_n$ above "mean" return) should reduce the credit risk perceived by the market participants, by this reducing especially the (credit risk related) issuer-specific spread. Instead of assuming a direct causality one could alternatively think of a common factor (like business cycle or general market sentiment) that has an impact on both the stock market and the corporate bond spreads. Additionally, following Collin-Dufresne et al. (2001), to receive a more disaggregated picture, we include returns of the respective stock, $STR_n$, based on a similar argument as with the $DAXR$ but with a presumably closer link between the spread of an issuer and the stock price of this issuer. The stock price time series are taken from Datastream which uses data from the Xetra system.

Motivated by the results of Collin-Dufresne et al. (2001), Berndt et al. (2005) and Pan and Singleton (2008), we add as a potential determinant a volatility index from the German stock market, namely the VDAX. VDAX data also comes from Datastream.

A further candidate is the market value debt ratio (*debt to value ratio*)

$$DVR_n = \frac{D_n}{S_n + D_n} \ , \tag{10}$$

where $S_n$ is the market capitalization at stock exchange and $D_n$ is the market value of a firm's debt. To obtain $S_n$ we use the daily time series of the market capitalization that we receive as the product of stock price (as described above) and number of shares (taken from Bloomberg). Since the difference between book and market values with debt is far smaller than with equity and the market value of debt is far harder to observe or compute, we follow in using book values for $D_n$ (see Martell (2008) for the same methodology). For the dates at the end of each quarter we take the book value of debt from the quarterly balance sheets. For all other dates, we derive $D_n$ by linear interpolation.

In Merton type models and in industry practice the distance to default is frequently used to describe the conditional probability of default. Intuitively, the distance to default is the number of standard deviations of annual asset growth by which the firm's expected assets at a given maturity exceed a measure of book liabilities. Technically, the distance to default (symbolized below by $DD$) is derived by an iterative procedure that matches both market value of equity and equity volatility to the figures that can be observed in the market (for details see Crosbie and Bohn (2003)).

*Seasonalities/Anomalies:*

Finally, we check for weekday effects and monthly effects, in issuer-specific and bond-specific spreads.

Numerous authors, e.g. French (1980) and Keim and Stambaugh (1984), analyze weekday effects in stock markets. Most of these studies show significant negative (or at least compared to the other days significantly lower) Monday returns. Recent studies based on US data (for an overview see Pettengill (2003)) show that over time especially for big firms Monday returns have become positive, sometimes even exceeding the returns on the other weekdays (reversing the "traditional" Monday effect). Several articles (e.g. Gibbons and Hess (1981), Flannery and Protopapadakis (1988) and Johnston et al. (1991)) find weekday effects in the fixed-income segment, as well. Our goal is to add empirical evidence to the different, in part contradicting, pieces of literature and to extend the literature on weekday effects in the corporate bond market. In our study, following French (1980) the day of the week effects are measured relative to Monday. The corresponding dummy variables for the remaining work-days of a week will be called TUES, WEDN, THUR, FRID. E.g. TUES is 1 on Tuesday and 0 on all other days. The Monday returns are part of the intercept.

In addition, we want to check for "'turn-of-the-month effects"' as Compton and Kunkel (2000) and McConnell and Xu (2008) find significant turn-of-the-month effects for Treasury bills, investment grade corporate bonds and high-yield corporate bonds. For this purpose we use dummy variables for the beginning and for the end of each month, with the dummy SMON being 1 on the first three trading days of each month and 0 for the other days and the variable EMON being 1 on the last three trading days of each month and 0 else. [4]

We check the stationarity of the variables used in our regression model: In Subsection 5.1 we already described the results of the stationarity tests for the response variables, $X_4$, and $X_{5,j}$ respectively, where the zero hypothesis of non-stationarity cannot be rejected for most components, even though the parameter restrictions for stationarity are fulfilled. With the augmented Dickey-Fuller test we also check the stationarity of the explanatory variables: The stock market returns (DAX returns) and the respective stock's returns (for both HVB and METRO) are stationary. The stock market volatility index (VDAX) and the debt to value ratio of METRO are stationary at a 10% level but not at the usual 5% level. The level of the risk-free term structure, the slope of the risk-free term structure, the debt to value ratio of HVB and the distance to default for both issuers turn out to be non-stationary. As a result we replace the time series of the level and slope of the risk-free term structure, of the VDAX, of the debt to value ratio and of the distance to default by their first differences. Theses first differences are abbreviated by $\Delta\hat{X}_{4,n} = \hat{X}_{4,n} - \hat{X}_{4,n-1}$, etc. These first differences are stationary. Thus, the regressions in Section 5.2.2 will be performed with stationary data.

Finally, due to the models or results of Duffee (1998), Duffie et al. (2003) and Frühwirth and Sögner (2006), we also investigate if the spreads in our model show an autoregressive pattern. To this end, we regress both the first differences of issuer-specific and bond-specific spread on the respective first differences of the spread of the previous trading day, $\Delta\widehat{X}_{n-1}$, and the trading day before, $\Delta\widehat{X}_{n-2}$.

### 5.2.2 Results

The purpose of this subsection is to identify the drivers of issuer-specific and bond-specific components. Table 1 presents the results of the regression analysis (t-values in parentheses) for the issuer-specific and bond-specific component, with the regression setting:[5]

$$
\begin{aligned}
\Delta\widehat{X}_{4,n} &= \gamma_0+\gamma_1\Delta RFLEVEL_n+\gamma_2\Delta RFSLOPE_n+\gamma_3 DAXR_n+\gamma_4\Delta VDAX_n+\gamma_5 STR_n\\
&\quad+\gamma_6\Delta DVR_n+\gamma_7\Delta DD_n+\gamma_8\Delta\widehat{X}_{4,n-1}+\gamma_9\Delta\widehat{X}_{4,n-2}\\
&\quad+\gamma_{10}TUES_n+\gamma_{11}WEDN_n+\gamma_{12}THUR_n+\gamma_{13}FRID_n+\gamma_{14}SMON_n+\gamma_{15}EMON_n+\varepsilon_{I,n}\ ,\\
\Delta\widehat{X}_{5,j,n} &= \gamma_0+\gamma_1\Delta RFLEVEL_n+\gamma_2\Delta RFSLOPE_n+\gamma_3 DAXR_n+\gamma_4\Delta VDAX_n+\gamma_5 STR_n\\
&\quad+\gamma_6\Delta DVR_n+\gamma_7\Delta DD_n+\gamma_8\Delta\widehat{X}_{5,j,n-1}+\gamma_9\Delta\widehat{X}_{5,j,n-2}\\
&\quad+\gamma_{10}TUES_n+\gamma_{11}WEDN_n+\gamma_{12}THUR_n+\gamma_{13}FRID_n+\gamma_{14}SMON_n+\gamma_{15}EMON_n+\varepsilon_{j,n}\ .
\end{aligned}
$$

*Autocorrelation:*

In Table 1, we observe highly significant first order autocorrelation coefficients for almost all factors estimated (see $\hat{\gamma}_8$). Also significant second order autocorrelation for the first METRO bond-specific factor, the first and second HVB bond-specific factors and for the issuer-specific factor of HVB can be observed (see $\hat{\gamma}_9$). The fact that the first order coefficient is significant is consistent with our model assumptions 2 and 3. The significance of the second order coefficient indicates that some remaining parts of the factor dynamics are not fully matched by a first order autoregressive setting. The strong persistence is in line with e.g. Driessen (2005) and Feldhütter and Lando (2008). We point out that using the Newey/West methodology instead of integrating lagged variables does not change the results.

*Risk-Free Term Structure:*

Since the parameter $c$ was used to model the correlation between the risk-free term structure and the spreads, $\hat{\gamma}_1$ and $\hat{\gamma}_2$ only measure the incremental effect that is left after the link between the risk-free segment and the risky segment has been established via the parameter $c$. If the model would perfectly fit the data, there should remain no significant dependence of $\Delta\hat{X}_{4,n}$ and $\Delta\hat{X}_{5,j,n}$ on $\Delta RFLEVEL_n$ (see $\hat{\gamma}_1$) and $\Delta RFSLOPE_n$ (see $\hat{\gamma}_2$). Thus, a significant regression parameter tells us that the parameter $c$ is not able to reflect the full correlation between the risky and the risk-free segment. Due to the positive $\hat{c}$ and our definitions of the spreads in equations (9), a significant $\hat{\gamma}_1$ can be interpreted as follows: In the context of the issuer-specific spread a positive (negative) parameter value $\hat{\gamma}_1$ indicates that the model overestimates (underestimates) the impact of the risk-free term structure. In the context of the bond-

Determinants of Default and Liquidity Risk

| | | $\hat{\gamma}_0$ | $\hat{\gamma}_1$ | $\hat{\gamma}_2$ | $\hat{\gamma}_3$ | $\hat{\gamma}_4$ | $\hat{\gamma}_5$ | $\hat{\gamma}_6$ | $\hat{\gamma}_7$ | $\hat{\gamma}_8$ | $\hat{\gamma}_9$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $X_4$ | -0.00005 | 0.18886 *** | 0.06869 | -0.00161 | -1.3E-06 | -0.00228 ** | 0.13160 *** | -0.00020 ** | -0.33775 *** | -0.29899 *** |
| | | ( -1.32418 ) | ( 2.72241 ) | ( 1.61982 ) | ( -0.85312 ) | ( -0.05483 ) | ( -2.41016 ) | ( 3.48196 ) | ( -2.23623 ) | ( -7.13700 ) | ( -5.26060 ) |
| H | $X_{5,1}$ | 0.00002 | 0.03609 | -0.02278 | 0.00188 | 0.00007 ** | 0.00010 | -0.08397 | -0.00016 | -0.42402 *** | -0.19857 *** |
| | | ( 0.28326 ) | ( 0.41723 ) | ( -0.38980 ) | ( 0.63889 ) | ( 1.97882 ) | ( 0.07006 ) | ( -1.42291 ) | ( -1.17742 ) | ( -8.82592 ) | ( -4.12824 ) |
| V | $X_{5,2}$ | 0.00003 * | -0.00773 | -0.01471 | -0.00028 | -3.3E-06 | 0.00109 *** | 0.00188 | 0.00006 | -0.17652 *** | -0.15891 *** |
| | | ( 1.82823 ) | ( -0.34818 ) | ( -0.96879 ) | ( -0.37401 ) | ( -0.35335 ) | ( 2.85447 ) | ( 0.12192 ) | ( 1.58911 ) | ( -3.67023 ) | ( -3.23771 ) |
| B | $X_{5,3}$ | 6.0E-07 | -0.00150 | -0.00106 | -0.00013 | -2.6E-06 | 0.00014 | -0.00136 | -0.00001 | -0.06831 | -0.07035 |
| | | ( 0.07866 ) | ( -0.14025 ) | ( -0.14859 ) | ( -0.36502 ) | ( -0.59942 ) | ( 0.76871 ) | ( -0.18972 ) | ( -0.51281 ) | ( -1.35266 ) | ( -1.41749) |
| | $X_{5,4}$ | -3.8E-06 | 0.02437 | 0.01167 | -0.00128 | -0.00001 | -0.00112 | -0.00081 | -0.00001 | -0.30654 | -0.11168 |
| | | ( -0.80660 ) | ( 0.27330 ) | ( 0.43610 ) | ( -0.08210 ) | ( -0.10940 ) | ( -0.00280 ) | ( -0.95710 ) | ( -0.82600 ) | ( -0.00000 ) | ( -0.02680 ) |
| | $X_{5,5}$ | -0.00003 ** | 0.02233 | 0.02052 | -0.00138 ** | -3.6E-07 | -0.00009 | -0.01952 | 0.00001 | -0.28028 *** | -0.05180 |
| | | ( -2.35447 ) | ( 1.08933 ) | ( 1.46908 ) | ( -2.00246 ) | ( -0.04351 ) | ( -0.26926 ) | ( -1.41510 ) | ( 0.25842 ) | ( -5.63186 ) | ( -1.02111 ) |
| M | $X_4$ | -0.00007 *** | 0.01725 | -0.00171 | 0.00076 | 0.00002 ** | 0.00176 | 0.00065 | 0.00120 | -0.10475 ** | 0.03359 |
| E | | ( -5.31953 ) | ( 0.91865 ) | ( -0.13507 ) | ( 1.17799 ) | ( 2.37695 ) | ( 1.17013 ) | ( 0.33064 ) | ( 1.57530 ) | ( -2.15047 ) | ( 0.69548 ) |
| T | $X_{5,1}$ | 0.00008 *** | -0.04778 ** | -0.03315 ** | 0.00044 | -0.00001 | 0.00154 | -0.00015 | 0.00080 | -0.36121 *** | -0.18139 *** |
| R | | ( 5.29638 ) | ( -2.23051 ) | ( -2.29794 ) | ( 0.60251 ) | ( -0.98777 ) | ( 0.89175 ) | ( -0.06557 ) | ( 0.92168 ) | ( -7.40014 ) | ( -3.71172 ) |
| O | $X_{5,2}$ | 0.00003 ** | 0.03061* | 0.00892 | -0.00035 ** | 0.00001 ** | -0.00141 | 0.00050 | -0.00059 | -0.12322 ** | -0.01131 |
| | | ( 2.16298 ) | ( 1.88452 ) | ( 0.81514 ) | ( -2.00246 ) | ( 2.14034 ) | ( -1.07713 ) | ( 0.29142 ) | ( -0.88514 ) | ( -2.49348 ) | ( -0.22547 ) |

Day of the Week and Monthly Effects

| | | $\hat{\gamma}_{10}$ | $\hat{\gamma}_{11}$ | $\hat{\gamma}_{12}$ | $\hat{\gamma}_{13}$ | $\hat{\gamma}_{14}$ | $\hat{\gamma}_{15}$ | $R^2$ |
|---|---|---|---|---|---|---|---|---|
| | $X_4$ | -0.00003 | -0.00003 | 0.00007 | 0.00009 * | 0.00003 | 0.00001 | 18.7% |
| | | ( -0.48180 ) | ( -0.57598 ) | ( 1.34881 ) | ( 1.66117 ) | ( 0.57597 ) | ( 0.11048 ) | |
| H | $X_{5,1}$ | -0.00003 | -0.00012 | 0.00016 * | -4.0E-06 | -0.00012 | 0.00002 | 20.7% |
| | | ( -0.37876 ) | ( -1.39047 ) | ( 1.83947 ) | ( -0.04511 ) | ( -1.52168 ) | ( 0.31144 ) | |
| V | $X_{5,2}$ | -0.00005 ** | -0.00003 | -0.00001 | -0.00002 | -2.61E-6 | -0.00002 | 9.2% |
| | | ( -2.47497 ) | ( -1.28118 ) | ( -0.35195 ) | ( -1.09662 ) | ( -0.13042 ) | ( -0.97004 ) | |
| B | $X_{5,3}$ | -0.00001 | -0.00001 | 8.3E-07 | -0.00001 | 0.00002 ** | 0.00001 | 3.7% |
| | | ( -1.01427 ) | ( -0.99180 ) | ( 0.08106 ) | ( -0.69802 ) | ( 2.37128 ) | ( 0.94391 ) | |
| | $X_{5,4}$ | -0.00002 | -0.00002 | 0.00003 | 5.7E-07 | 0.00001 | -0.00001 | 15.3% |
| | | ( -0.26160 ) | ( -0.27510 ) | ( 0.21930 ) | ( 0.98130 ) | ( -0.44840 ) | ( -0.50370 ) | |
| | $X_{5,5}$ | 0.00002 | 0.00003 * | 0.00002 | 0.00002 | 0.00004 ** | 0.00001 | 9.8% |
| | | ( 0.75877 ) | ( 1.64975 ) | ( 1.13991 ) | ( 1.18830 ) | ( 2.22897 ) | ( 0.39896 ) | |
| M | $X_4$ | 0.00004 ** | 0.00008 *** | 0.00011 *** | 0.00007 *** | -0.00003 | 0.00002 | 14.1% |
| E | | ( 2.20249 ) | ( 4.24142 ) | ( 6.22000 ) | ( 3.93806 ) | ( -1.60702 ) | ( 1.08773 ) | |
| T | $X_{5,1}$ | -0.00007 *** | -0.00015 *** | -0.00008 *** | -0.00007 *** | -0.00002 | -0.00004 * | 23.3% |
| R | | ( -3.23649 ) | ( -7.17444 ) | ( -3.60305 ) | ( -3.08564 ) | ( -1.11387 ) | ( -1.87302 ) | |
| O | $X_{5,2}$ | -0.00001 | -0.00003 ** | -0.00005*** | -0.00002 | 0.00001 | -0.00002 | 8.3% |
| | | ( -0.90653 ) | ( -2.18660 ) | ( -3.37995 ) | ( -1.25723 ) | ( 0.73675 ) | ( -1.19202 ) | |

**Table 1:** Regression estimates (and t-values in parenthesis) of the issuer-specific components $\Delta\hat{X}_4$ and the bond-specific components $\Delta\hat{X}_{5,j}$. Day-of-the-week effects are measured relative to Monday. $\Delta\widehat{X}_{4,n} = \gamma_0 + \gamma_1\Delta RFLEVEL_n + \gamma_2\Delta RFSLOPE_n + \gamma_3 DAXR_n + \gamma_4\Delta VDAX_n + \gamma_5 STR_n + \gamma_6\Delta DVR_n + \gamma_7\Delta DD_n + \gamma_8\Delta\widehat{X}_{4,n-1} + \gamma_9\Delta\widehat{X}_{4,n-2} + \gamma_{10}TUES_n + \gamma_{11}WEDN_n + \gamma_{12}THUR_n + \gamma_{13}FRID_n + \gamma_{14}SMON_n + \gamma_{15}EMON_n + \varepsilon_{I,n}$ and $\Delta\widehat{X}_{5,j,n} = \gamma_0 + \gamma_1\Delta RFLEVEL_n + \gamma_2\Delta RFSLOPE_n + \gamma_3 DAXR_n + \gamma_4\Delta VDAX_n + \gamma_5 STR_n + \gamma_6\Delta DVR_n + \gamma_7\Delta DD_n + \gamma_8\Delta\widehat{X}_{5,j,n-1} + \gamma_9\Delta\widehat{X}_{5,j,n-2} + \gamma_{10}TUES_n + \gamma_{11}WEDN_n + \gamma_{12}THUR_n + \gamma_{13}FRID_n + \gamma_{14}SMON_n + \gamma_{15}EMON_n + \varepsilon_{j,n}$ . ***, **, * denote that the coefficient is statistically significant at the 1%, 5%, and 10% level.

specific spread a significant $\hat{\gamma}_1$ shows that there is a bond-specific correlation effect, in addition to the issuer-specific effect included in $c$. This effect could be captured by an additional bond-specific correlation parameter. Significant parameter estimates $\hat{\gamma}_2$ suggest that if there is room for further extensions of the model, the slope would be a plausible candidate to be integrated into the model.

The regression results in Table 1 show that in most cases the parameter $c$ absorbs the impact of the term structure (including any effects of expected future inflation) quite well as there is a significant impact of neither the level nor the slope of the risk-free term structure. However, for the issuer-specific component of HVB we can observe a significant positive impact of $\Delta RFLEVEL$, i.e. the model overestimates the impact of the risk-free term structure. Also, for the bond-specific components of METRO we observe a significant impact of $\Delta RFLEVEL$ and $\Delta RFSLOPE$ showing a bond-specific correlation effect for these bonds.

*Stock Market Variables:*

From Table 1 (see $\hat{\gamma}_3$) we see that the DAX returns are significantly negatively related to the bond-specific components of the long-term bonds (second METRO bond and fifth HVB bond). From $\hat{\gamma}_5$ we observe a highly significant relation between the stock return and the issuer-specific factor (negative) and the second bond-specific factor (positive) of HVB. For the volatility index (see $\hat{\gamma}_4$) we obtain a significant positive influence for the first HVB bond-specific spread, the second METRO bond-specific spread and the METRO issuer-specific spread.

First, it is interesting to see that for some bonds it is the bond-specific spreads that are linked to the stock market variables and not the issuer-specific spread. So, there seems to be a bond-specific relation between stock market variables and spreads. Also, it is interesting to see that, depending on the bond/issuer, in some cases the returns of the specific stock are significant, in other cases the DAX return is significant and for some bonds it is the stock market volatility. Thus, the influence of the equity market variables strongly depends on the issuer or even on the bond. The significant impact of the stock index returns on the bond-specific spreads of the long-term bonds shows that especially the long-term bonds are subject to systematic risk. As regards the signs, a negative relation between DAX returns/the specific stock's returns and the spread are intuitive. An increase in the general stock market or in the specific stock price goes hand in hand with a decrease in the risk perceived by the market participants. Moreover, it is plausible that an increase in the volatility on the stock market is related to an increase in the spreads. In all significant cases, except the relation between stock return and bond-specific spread of the second HVB bond, the signs meet the intuition. Concerning the relation between the stock return and the bond-specific spread of the second HVB bond one has to mention that when adding the parameter for the issuer-specific spread and that for the bond-specific spread, we obtain a negative net impact of the stock return on the total spread, which is in line with common intuition.

*Default Proxies:*

From Table 1 (see $\hat{\gamma}_6$ and $\hat{\gamma}_7$) we observe that changes in the debt to value ratio, $\Delta DVR$, and the distance to default, $\Delta DD$, have only highly significant influence on the issuer-specific factor of HVB; for all other components no significant impact can be observed. Several things are worth mentioning here. First, it is intuitive that the two ratios that represent/include the capital structure of the firm have no impact on the bond-specific spreads but only on the issuer-specific spread (of HVB). Second, it is interesting to see that these ratios have an impact on HVB's (A rating) but not on METRO's (BBB rating) issuer-specific spread, as one would rather expect that default proxies are more relevant for issuers with lower rating. This counterintuitive result, however, is consistent with our findings on the relative order of the total spreads between the METRO bonds and the HVB bonds discussed in Section 5.1. A potential explanation is that, irrespective of the rating, the market is more sensitive to default proxies for banks than for retailers. Third, it is interesting to see that for HVB none of the two related ratios (debt to value ratio and distance to default) is sufficient alone as both of them are significant. Fourth, the negative sign of the estimated parameter with the distance to default and the positive sign with the debt to value ratio for HVB meet our expectations. The higher the distance to default and the smaller the debt to value ratio, the lower the credit risk and therefore the smaller the issuer-specific spread. *Seasonalities/Anomalies:*

From the regression parameters $\hat{\gamma}_{10}$, ..., $\hat{\gamma}_{13}$ in Table 1 it can be seen that there are hardly any weekday effects for HVB while there are significant weekday effects for METRO. Abraham and Ikenberry (1994) and Chatterjee and Maniam (1997) show for stock markets that weekday effects are more pronounced for small firms than for big firms. As HVB has about 15 times the balance sheet total of METRO, our results extend their findings to the corporate bond market. As regards the signs of the variables for METRO, the results are mixed. For METRO's issuer-specific component all weekday parameters (Tuesday, Wednesday, Thursday and Friday) have a significant positive impact, i.e. there is a traditional Monday effect. For the bond-specific factors of METRO, almost all weekday parameters show a significantly negative impact, i.e. we can observe an inverse Monday effect. The lower significance levels with the second METRO bond (insignificant for Tuesday and Friday, significant but smaller t-values for Wednesday and Thursday) are compatible with Flannery and Protopapadakis (1988) and Adrangi and Ghazanfari (1997) who suggest that the strength of the (traditional) Monday effect is increasing with maturity.

We can observe some monthly effects, namely a significantly higher return with some of the bond-specific components at the beginning of the month (third and fifth HVB bonds) and a lower return with the bond-specific component of the first METRO bond at the end of the month – a further example that shows that the behavior of spreads cannot be generalized over all bonds.

The importance of seasonalities can also be judged from the change in $R^2$ resulting from an inclusion of seasonalities. For the HVB regressions the $R^2$ grows by between 1.8 and 3 percentage points, while for the METRO regressions the growth in $R^2$ is between 3.7 and 10.9 percentage points.

While the HVB issuer-specific spread is mainly related to the level of the risk-free term structure, the HVB stock return and default proxies and influenced by autocorrelation, the METRO issuer-specific spread is particularly driven by weekday effects, stock market volatility and autocorrelation. A look at the HVB bond-specific spreads illustrates the differences between bonds of the same issuer: the first HVB bond is linked to the stock market volatility, the second HVB bond to the HVB stock return and the fifth HVB bond to the DAX. This shows, that a disaggregated view distinguishing between different industries, issuer-specific spread vs. bond-specific spread and different bonds of the same issuer is required.

### 5.3 Market-Wide Factors and One-Factor Issuer Models

In the previous subsection we derived estimates of the two processes $X_4$ for HVB and METRO as well as estimates of the five $X_{5,j}$ processes for HVB and the two for METRO. Based on these individual components, we check for common factors by means of principal components analyses ($PCA$).[6]

First, to obtain information whether to create a joint model for the whole market consisting of the two issuers, we perform a principal components analysis including all processes (6 HVB processes + 3 METRO processes). Such a market-wide model is required to find out if there is a market-wide factor (e.g. Driessen (2005) uses one market-wide factor and one issuer-specific factor). The PCA shows a percentage of 69.7% for a one factor setting. Therefore, using one common factor for the whole market alone leads to a significant loss in explanatory power.

In a next step, we focus on the bond-specific processes on the market. In the PCA we analyze the seven bond-specific processes of both issuers together, in order to see if there is a market-wide bond-specific factor, that could be interpreted e.g. as a market-wide liquidity factor. The PCA shows that the first factor explains 73.9% of the variance. This suggests that the bond-specific influence should not be reduced to one factor.

Next, we focus on the two issuer-specific components on the market. By this, we can find out the modeling cost of using one market-wide issuer-specific factor (instead of one issuer-specific factor for each individual issuer), complemented by the respective bond-specific processes. We observe that one market-wide issuer component explains 89.4% of the two issuer-specific factors. This provides an argument for a model with one ”market wide” issuer-specific factor.

Finally, consistent with Section 2, we investigate at what cost spreads can be calculated on an issuer-specific basis (”spread per issuer”, one-factor issuer model) as often done in literature and industry. For this purpose, we analyze, for each issuer separately, the universe of all bond-specific and issuer-specific processes (6 processes for HVB and alternatively 3 processes for METRO). The explanatory power of the first component shows at what cost our model can be reduced to a one-factor issuer model. We observe that for HVB 77.7% and for METRO 80.9% can be explained by a one-factor issuer model. Thus, a model with only one factor per issuer seems to be too simple when demanding an explanatory power beyond 81%.

# 6 Conclusions

In this article we dissect the corporate bond spread into an issuer-specific and a bond-specific component for a sample of German corporate bonds issued by HVB and METRO. Using data augmentation we are able to separate issuer-specific and bond-specific components without having to specify in advance a benchmark bond that is free of any bond-specific risk. With this methodology it is possible to check if the benchmark bond assumption made by Duffie et al. (2003) is justified and if it is, which bond should be used as benchmark bond.

We show that the bond-specific spread is highly relevant: first, a significant part of the total spread is represented by the bond-specific spread. Second, bond-specific components differ substantially even between bonds of the same issuer. Consistent with this, the results of the issuer-specific principal components analysis imply that the standard procedure used in literature and industry to assume one factor for all bonds of the same issuer (as is implicitly done when deriving issuer spread curves) may be insufficient. The relevance of the bond-specific spread has several implications: first, due to the bond-specific spread the correlations between the bonds of the same issuer are no longer perfect. Also, if one wants to extract correlations between different issuers from bond data, it is necessary to only use the issuer-specific part of the spread. In addition, when obtaining the cost of capital from the bond market as done in corporate finance, adjustments for bond-specifics (e.g. liquidity) should be performed.

We provide important economic insight on the determinants of issuer-specific and bond-specific components with implications for future credit/liquidity risk modeling. Our study shows that the issuer-specific component should be modeled separately from the bond-specific component. The estimates of the correlation parameter $c$ imply a negative correlation between risk-free rates and spreads which should be included in credit risk models. Although our correlation specification is not always able to capture the full relation between the risk-free term structure and the spreads, overall it absorbs the relation between the risk-free and credit-risky market quite well. We find that in future credit/liquidity risk modeling especially for long-term bonds modeling systematic risk should be taken into consideration. We investigate typical default proxies such as the distance to default and the debt to value ratio. Both are significant for the HVB issuer-specific spread (even though the debt to value ratio is included in the distance to default) but insignificant for the METRO issuer-specific spread. Therefore, the question whether to include these default proxies into modeling of the issuer-specific spread seems to depend on the issuer or at least the industry. Moreover, our results with respect to weekday effects may suggest possible trading strategies, especially with bonds from small issuers, paying attention, however, to transaction costs.

One of the main drawbacks of our approach is that it is computationally very intensive which currently is a problem especially for issuers with many bonds. This, however, will improve over time as computing capacities are increasing. Extensions of our methodology are straightforward, given sufficient computing power to perform higher dimensional density approximations as well as a sufficient number of MCMC steps. The methodology developed in this article allows to build cascades of factors, e.g. rating-specific factors on an upper (more aggregate) level, industry-specific factors on a lower (more disaggregated) level, issuer-specific factors on a third level and bond-specific factors on the lowest level. An analysis of this kind is hardly possible with the approaches currently used in literature. Even more generally, our methodology can be transferred to all situations where extra securities are necessary to complete the model and to perform parameter estimation.

As the differentiation between issuer-specific and bond-specific spread is only in its infancy, numerous other interesting research questions arise, once the split-up has been successfully completed. Given our results, it would be interesting to compare different credit risk measures in greater detail, starting with various traditional balance sheet ratios, over the Altman Z score to the distance to default by investigating which of the credit risk measures has the strongest link to the issuer-specific spread and quantifying the loss of information from using simpler measures. Similarly, the impact of earnings announcements has been intensively analyzed for stock markets (see e.g. Gajewski (1999) or Liu et al. (2003)) or other markets (see Donders et al. (2000) for the options market). An investigation of the impact of earnings announcements on corporate bond spreads, especially the issuer-specific spread, promises to be an interesting field of research. Another field that can benefit from a proper split between issuer-specific and bond-specific spread is behavioral finance. One issue worth investigating is the analysis of the impact of the weather on the two components issuer-specific and bond-specific spread (see Pardo and Valor (2003) or Goetzmann and Zhu (2005) for methodologies linking the stock market development to weather). Finally, if one wants to analyze the impact of the issuers' advertising intensity at sports or cultural events on the corporate bond market (see Fehle et al. (2005) for a stock market related study), there should be an impact only on the issuer-specific part of the spread.

# A Estimation Methodology

## A.1 Transition Densities

We consider a problem with discretely sampled data, generated by diffusion processes. We have equidistant time gaps $\Delta = t_n - t_{n-1}$ and $n = 1, \ldots, N$. As stated in the text already, the measurements of the continuous-time stochastic process $(X(t))$ and the corresponding transformations $X_{rf}(t)$, $R_j(t)$, and $p_j(t)$ at $t_n$ are symbolized by $X_n$, $X_{rf,n}$, $R_{j,n}$, and $p_{j,n}$. From our assumptions in Section 2 the transition density $\pi(X_n \mid X_{n-1}; \psi)$ corresponds to

$$\begin{aligned} \pi(X_n \mid X_{n-1}; \psi) &= \pi(X_{1,n}, X_{2,n}, X_{3,n} \mid X_{1,n-1}, X_{2,n-1}, X_{3,n-1}; \psi) \\ &\quad \cdot \pi(X_{4,n} \mid X_{4,n-1}; \psi) \prod_{j=1}^{J} \pi(X_{5,j,n} \mid X_{5,j,n-1}; \psi) \ . \end{aligned} \tag{11}$$

where $\psi$ are the model parameters. We approximate all non-Gaussian transition densities by an Euler approximation. For any one-to-one transformation $P_n = F(X_n)$, the transition density of $P_n$ is derived via the density transformation formula $\pi(P_n|P_{n-1}; \psi) = \pi(X_n|X_{n-1}; \psi)\frac{1}{det|JF(X_n)|}$ where $\det |JF(X_n)|$ is the determinant of the Jacobian of the function $F(X_n)$. This requires the Jacobian of the transformation $F(X_n)$ to be of full rank.

## A.2 Augmentation of the Parameter Space

As described in the text, our goal is the estimation of the parameters without having to exogenously determine a benchmark bond. The main problem that occurs without defining a benchmark bond is that the number of bonds is too small compared to the number of latent stochastic processes. We solve this by augmentation of the parameter space:

*Risk-free segment:* As pointed out in the text, we assume the spot rates with a maturity of 6 month, 2 years and 5 years to be observed without measurement error. The remaining spot rates are assumed to be measured with error. Thus, in the risk-free segment with three latent processes and three time series of data the dimensions match.

*Risky segment:* By Assumptions 2 and 3, estimating the issuer-specific process and $J$ bond-specific processes demands for a joint density of dimension $1 + J$. In the risky data, we have $J$ time series of coupon bond prices.

Thus, a function $\bar{F}(X_n)$ mapping from $X_n \in \mathrm{R}^M$ to $\bar{P}_n \in \mathrm{R}^L$ cannot be one-to-one, with $M = L + 1 > L$ and $L = 3 + J$ for the current application. Due to this lack of a one-to-one relation between the latent factors and the data, the transition densities of the bond prices cannot be calculated by using the change of variables formula.Therefore, we apply *data augmentation* (see Tanner and Wong (1987)) and add the entire time series of an *artificial bond* to the parameter space. Thus, we augment the set of parameters by $\tilde{X}_{4,n}$, $n = 1, \ldots, N$, which is a one-to-one transformation (parameterization) of $X_{4,n}$; this transformation will be abbreviated by $g(X_{4,n})$. By this the vector of augmented interest rate data and bond price data $P_n$ includes three risk-free yields, the artificial bond $\tilde{X}_{4,n}$ and the risky coupon bond prices $p_j$, $j = 1, \ldots, J$. As a consequence, $P_n \in \mathrm{R}^M$, such that the transformation $P_n = F(X_n)$ is one-to-one.

A well-known fact with MCMC methods is that the parameterization of latent variables has an important impact on the convergence properties of the sampler (see e.g. Papaspiliopoulos et al. (2003) or Roberts et al. (2004)). For this reason we performed simulation experiments, which support $\tilde{X}_{4,n} = X_{4,n}$.

## A.3 MCMC Estimation

As already noted, the complex model structure makes a direct application of maximum likelihood infeasible. Therefore, we apply Bayesian simulation methods to estimate the posterior distribution of the model parameters. The simulation methods concerned with this task are Markov Chain Monte Carlo methods (see Robert and Casella (1999)). Using $D$ for the data observed, by means of the Bayes theorem the posterior distribution of a parameter $\theta$, $\pi(\theta|D)$ is proportional to the likelihood $f(D|\theta)$ times the prior $\pi(\theta)$, i.e. $\pi(\theta|D) \propto f(D|\theta)\pi(\theta)$. If there were no missing data, the interest rate and bond price data $\bar{P} = (\bar{P}_1, \ldots, \bar{P}_n, \ldots, \bar{P}_N)$, $n = 1, \ldots, N$, where $N$ is the number of periods considered, would correspond to $D$. Since some prices are not observed due to no trade or measurement errors, $\bar{P}$ consists of actually observed prices and missing values $D^{miss}$, such that $D = \bar{P} \setminus D^{miss}$.

*Data Augmentation and Missing Values:* We have already highlighted the augmentation of the parameter space by $X_4$. To perform a full Bayesian analysis we additionally include the starting values of the latent processes, $X_0$, and $D^{miss}$. Thus with the augmented set of parameters we obtain $\theta = (X_4, X_0, D^{miss}, \psi)$, where $\psi$ are all remaining model parameters. By the Bayes theorem the a-posteriori distribution fulfills

$$
\begin{aligned}
\pi(\theta|D) &\propto f(D|\theta)\pi(\theta) \\
&\propto f(D|D^{miss}, X_4, X_0; \psi) f(D^{miss}|X_4, X_0; \psi)\pi(X_4|X_0; \psi)\pi(X_0|\psi)\pi(\psi) \\
&\propto f(\bar{P}|X_4, X_0; \psi)\pi(X_4|X_0; \psi)\pi(X_0|\psi)\pi(\psi) \\
&\propto f(P|X_0; \psi)\pi(X_0|\psi)\pi(\psi) \ . \qquad (12)
\end{aligned}
$$

where $P = (P_1, \ldots, P_N)$ and $P_n = (\bar{P}_n, \tilde{X}_{4,n})^\top$. The likelihood $f(P|X_0; \psi)$ can be described by $f(P|X_0; \psi) = \pi(P_1|X_0; \psi) \prod_{n=2}^{N} \pi(P_n|P_{n-1}; \psi)$ , where $\pi(P_n|P_{n-1}; \psi)$ is derived by equation (11) and the density transformation formula. Since $P_0 = F(X_0)$ we get $\pi(P_1|X_0; \psi) = \pi(P_1|P_0; \psi)$. $\pi(X_0|\psi)$ and $\pi(\psi)$ are the priors of the initial value of $X$ and the unknown parameters of the stochastic processes, $\psi$. These priors are chosen by the econometrician. We choose a Gamma distribution for $\pi(X_0|\psi) = \pi(X_0)$, i.e. the prior of the starting values is independent of $\psi$.

We use relatively uninformative priors for $\pi(X_0)$ and $\pi(\psi)$, conditional on the restriction of parameters fulfilling the Feller condition of a stationary limit distribution. We do not use the stationary limit distribution for the prior on $X_0$ but apply a less informative distribution in order to prevent a strong impact of the prior on the estimation output.

*MCMC:* Since all conditional distributions are well-defined, Markov Chain Monte Carlo methods can be applied. By decomposing an updating sweep $m$ into updating steps (which usually result from the structure of the joint density of the model), we construct an ergodic Markov chain $(\theta^{[m]})$. In an application of MCMC, the updating procedure is repeated until the Markov chain has reached its invariant distribution. In the current application the autocorrelation of the sample paths are high. To cope with this, we generate runs with 2,000,000 simulation sweeps (500,000 burn-in steps). For more detailed information on Markov Chain Monte Carlo methods the reader is referred to Robert and Casella (1999) and Albert and Chib (2003). Note that the structure of the model allows us to split up the estimation into an estimation of the risk-free segment and an estimation of the risky segment.

[Table 3 about here.]

[Table 4 about here.]

## Notes

[1]Since in these studies the mean of the residual is usually set to zero, some part of our bond-specific factor may enter into the issuer-specific component considered in these studies.

[2]In analogy to Collin-Dufresne et al. (2008) we use the multivariate median from the risk-free posterior resulting from the Bayesian simulation.

[3]Assuming the expectations theory and the Fisher thesis allows a further possible interpretation of the impact of level and slope on the spread processes. With these two parities holding, the nominal long-term spot rate is some average of the current nominal short-term rate and the expected future nominal short-term rates and therefore an average of current and future real short-term rates and current and future inflation rates. Thus, the impact of the level could be caused by an impact of the real short-term spot rate and the short-term inflation rate and the impact of the slope could be due to the expected future short-term real spot rates and the expected future inflation rates (see Oertmann et al. (2000), p. 466, and Silva et al. (2003), p. 212, for similar arguments).

[4]As Ariel (1987) and Pettengill and Jordan (1988) observe that the returns are significantly higher in the first half of the month, we also investigate this issue with our dataset. However, as we do not find these effects in the data, we do not include the dummy variables for the first vs. second half of the month in the presentation of our results.

[5]In order to cope with any heteroscedasticity we used White robust standard errors, in addition. Our results are robust with respect to the selection of standard setting vs. White robust setting.

[6]The reader should note that this principal components analysis does not fully capture heterogeneities in the prices, since this analysis is performed with the instantaneous issuer-specific and bond-specific components. As some of the estimated parameters significantly differ from each other (see Table 5) the ongoing analysis cannot fully reflect differences in the term structure of non-instantaneous spreads.

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| | MEAN | MEDIAN | MAX | MIN | SD | $MTSPR^{proxy}$ |
|---|---|---|---|---|---|---|
| | | | HVB | | | |
| $\widehat{X}_4$ | 180.20 | 181.55 | 233.30 | 129.30 | 24.18 | |
| $\widehat{ISPR}$ | 49.16 | 49.28 | 61.17 | 45.32 | 21.61 | |
| $\widehat{BSPR}_1$ | -5.23 | -6.66 | 30.35 | -45.28 | 11.66 | |
| $\widehat{BSPR}_2$ | 11.17 | 10.47 | 20.54 | 2.89 | 4.26 | |
| $\widehat{BSPR}_3$ | 17.55 | 17.75 | 22.25 | 12.26 | 1.99 | |
| $\widehat{BSPR}_4$ | 28.89 | 33.38 | 44.89 | 10.36 | 9.82 | |
| $\widehat{BSPR}_5$ | 32.65 | 40.32 | 51.94 | 8.10 | 13.25 | |
| $\widehat{TSPR}_1$ | 43.93 | 42.62 | 91.52 | 0.04 | 19.21 | 42.5 |
| $\widehat{TSPR}_2$ | 60.33 | 58.87 | 115.25 | 2.13 | 25.14 | 22.1 |
| $\widehat{TSPR}_3$ | 66.72 | 65.80 | 125.98 | 1.27 | 29.15 | 37.7 |
| $\widehat{TSPR}_4$ | 78.05 | 78.94 | 146.88 | 0.87 | 36.92 | 45.9 |
| $\widehat{TSPR}_5$ | 81.81 | 85.96 | 150.21 | 0.15 | 39.93 | 37.3 |
| | | | METRO | | | |
| $\widehat{X}_4$ | 26.12 | 22.25 | 60.81 | 5.59 | 13.84 | |
| $\widehat{ISPR}$ | 4.42 | 0.87 | 26.73 | -2.94 | 9.66 | |
| $\widehat{BSPR}_1$ | 14.15 | 14.58 | 21.28 | 3.90 | 3.57 | |
| $\widehat{BSPR}_2$ | 21.30 | 21.41 | 30.67 | 13.61 | 3.65 | |
| $\widehat{TSPR}_1$ | 18.57 | 15.45 | 48.01 | 0.96 | 13.23 | 42.3 |
| $\widehat{TSPR}_2$ | 25.71 | 21.98 | 70.00 | 2.89 | 15.82 | 31.7 |

**Table 2:** Descriptive statistics of issuer-specific components, issuer-specific and bond-specific spreads and total spreads (in basis points) estimated from the MCMC output (2,000,000 MCMC steps, 500,000 burn-in steps). The last column provides the mean total spread approximations derived by subtracting for each point in time from the yield to maturity of this bond the risk-free rate for the same maturity and after that taking the mean over all points in time.

| | Percentage Share | | | |
|---|---|---|---|---|
| | MEAN | | SD | |
| | ISPR | BSPR | ISPR | BSPR |
| $HVB_1$ | 111.9% | -11.9% | 39.3% | 60.7% |
| $HVB_2$ | 81.5% | 18.5% | 83.1% | 16.9% |
| $HVB_3$ | 73.7% | 26.3% | 93.2% | 6.8% |
| $HVB_4$ | 63.0% | 37.0% | 73.4% | 26.6% |
| $HVB_5$ | 60.1% | 39.9% | 66.8% | 33.2% |
| $METRO_1$ | 23.8% | 76.2% | 73.0% | 27.0% |
| $METRO_2$ | 17.2% | 82.8% | 76.9% | 23.1% |

**Table 3:** Percentage influence of issuer-specific and bond-specific spread on mean and standard deviation of the total spread - Spreads estimated from the MCMC output (2,000,000 MCMC steps, 500,000 burn-in steps).

## HVB bonds (A bank):

| Bond# | Abbreviation | Issuer | Maturity (MM/DD/YYYY) | Coupon (p.a.) | ISIN |
|---|---|---|---|---|---|
| 1 | HVB 1 | Bayerische Hypo- und Vereinsbank | 02/13/2006 | 4.75% | DE0002515590 |
| 2 | HVB 2 | Bayerische Hypo- und Vereinsbank | 01/08/2007 | 4.5% | DE0002516416 |
| 3 | HVB 3 | Bayerische Hypo- und Vereinsbank | 08/11/2008 | 3.875% | DE0008087834 |
| 4 | HVB 4 | Bayerische Hypo- und Vereinsbank | 11/26/2010 | 5.75% | DE0002515566 |
| 5 | HVB 5 | Bayerische Hypo- und Vereinsbank | 03/27/2012 | 5.625% | DE0002516556 |

## METRO bonds (BBB non-bank):

| Bond# | Abbreviation | Issuer | Maturity (MM/DD/YYYY) | Coupon (p.a.) | ISIN |
|---|---|---|---|---|---|
| 6 | METRO 1 | METRO Finance BV | 03/09/2006 | 5.75% | DE0006111909 |
| 7 | METRO 2 | METRO AG | 02/13/2008 | 5.125% | DE0002017217 |

**Table 4:** List of Bonds Used for Estimation. Note that he METRO 1 bond has been issued by METRO Finance BV, however guaranteed by METRO AG. Therefore, we assume the same issuer-specific risk for these two bonds.

| | HVB | | | METRO | | |
|---|---|---|---|---|---|---|
| | Estimate | Q(2.5%) | Q(97.5%) | Estimate | Q(2.5%) | Q(97.5%) |
| $\alpha_4^P$ | 0.0231 | 0.0078 | 0.0283 | 0.0029 | 0.0020 | 0.0040 |
| $\beta_4^P$ | 1.0459 | 0.5793 | 6.2715 | 1.0667 | 1.0249 | 3.3863 |
| $\beta_4^Q$ | 0.0740 | 0.0652 | 0.0948 | 0.0841 | 0.0024 | 0.2380 |
| $\sigma_4$ | 0.0420 | 0.0399 | 0.0472 | 0.0362 | 0.0294 | 0.0464 |
| $\alpha_4^Q$ | 0.0122 | 0.0123 | 0.0116 | 0.0119 | 0.2083 | 0.0055 |
| $\alpha_{5_1}^P$ | -0.0008 | -0.0030 | 0.0009 | 0.0016 | -9.0 e-5 | 0.0025 |
| $\alpha_{5_2}^P$ | 0.0011 | -0.0013 | 0.0024 | 0.0019 | 0.0003 | 0.0033 |
| $\alpha_{5_3}^P$ | 0.0015 | -0.0007 | 0.0035 | | | |
| $\alpha_{5_4}^P$ | 0.0026 | 0.0003 | 0.0047 | | | |
| $\alpha_{5_5}^P$ | 0.0032 | 0.0005 | 0.0049 | | | |
| $\beta_{5_1}^P$ | 16.9229 | 10.4117 | 16.9402 | 13.0021 | 10.6085 | 16.9124 |
| $\beta_{5_2}^P$ | 8.5705 | 6.9549 | 16.8276 | 11.7238 | 5.3699 | 12.7878 |
| $\beta_{5_3}^P$ | 7.5553 | 5.1297 | 15.4210 | | | |
| $\beta_{5_4}^P$ | 5.7124 | 5.0276 | 8.0029 | | | |
| $\beta_{5_5}^P$ | 5.0389 | 5.0084 | 6.2893 | | | |
| $\beta_{5_1}^Q$ | 3.5 e-6 | 2.9 e-6 | 0.0035 | 0.0029 | 4.3 e-5 | 0.0062 |
| $\beta_{5_2}^Q$ | 0.0015 | 2.9 e-5 | 0.0039 | 0.0007 | 2.1 e-05 | 0.0028 |
| $\beta_{5_3}^Q$ | 0.0005 | 1.8 e-5 | 0.0024 | | | |
| $\beta_{5_4}^Q$ | 0.0003 | 1.1 e-5 | 0.0014 | | | |
| $\beta_{5_5}^Q$ | 0.0003 | 7.9 e-6 | 0.0012 | | | |
| $\alpha_{5_1}^Q$ | 8.2857 | 0.1931 | 0.0223 | 0.0690 | 0.1744 | 0.1129 |
| $\alpha_{5_2}^Q$ | 0.1333 | 0.1966 | 0.1538 | 0.5714 | 1.8571 | 0.3214 |
| $\alpha_{5_3}^Q$ | 2.6000 | 16.6667 | 0.5833 | | | |
| $\alpha_{5_4}^Q$ | 5.6667 | 109.0909 | 1.3571 | | | |
| $\alpha_{5_5}^Q$ | 6.0000 | 164.5570 | 1.6667 | | | |
| $\sigma_{5_1}$ | 0.0093 | 0.0092 | 0.0119 | 0.0024 | 0.0022 | 0.0028 |
| $\sigma_{5_2}$ | 0.0022 | 0.0021 | 0.0026 | 0.0016 | 0.0014 | 0.0019 |
| $\sigma_{5_3}$ | 0.0011 | 0.0008 | 0.0014 | | | |
| $\sigma_{5_4}$ | 0.0024 | 0.0022 | 0.0026 | | | |
| $\sigma_{5_5}$ | 0.0022 | 0.0021 | 0.0025 | | | |
| $c$ | 0.6188 | 0.5314 | 0.6631 | 0.1025 | 0.0748 | 0.1429 |

**Table 5:** Parameter estimates taken from the multivariate posterior median and the 2.5% and 97.5% quantiles Q(2.5%) and Q(97.5%). (2,000,000 MCMC steps; burn-in phase 500,000 steps)