How to find plausible, severe, and useful stress scenarios

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Abstract

1 Introduction

The current regulatory framework of the Basel Committee on Banking Supervision [2005] requires banks to perform stress tests which meet three requirements.

1. Plausibility of stress scenarios: “Quantitative criteria should identify plausible stress scenarios to which banks could be exposed.” (par. 718 (LXXIX))

2. Severity of stress scenarios: “(...) a bank should also develop its own stress tests which it identifies as most adverse based on the characteristics of its portfolio” (par. 718 (LXXXIII))

3. Suggestiveness of risk reducing action: “Qualitative criteria should emphasise that two major goals of stress testing are to evaluate the capacity of the banks capital to absorb potential large losses and to identify steps the bank can take to reduce its risk and conserve capital.” (par. 718 (LXXIX))

How do we find stress scenarios which are the same time plausible, severe, and suggestive for the design of risk reducing action? Our paper gives a systematic answer to this question. We suggest a method that can be implemented for a wide class of stress testing problems usually encountered in practice. We illustrate the method and the issues in the context of an example: stress tests for a portfolio of adjustable rate loans in home and foreign currency.

The quality of a stress test crucially depends on the definition of stress scenarios. Defining stress scenarios is a thought experiment. It is a counterfactual exercise where a risk manager tries to imagine what adverse or even catastrophic
events might strike his portfolio. Such a thought experiment is prone to two major pitfalls: consideration of implausible scenarios and neglect of plausible scenarios. Thinking about scenarios requires to imagine situations that have not yet occurred but might occur in the future. The bias towards historical experience can lead to the risk of ignoring plausible but harmful scenarios which did not yet happen in history. This creates a dangerous blind spot. If the imagination of a stress tester puts excessive weight on very implausible scenarios management face an embarrassing decision: should one react to alarming results of highly implausible stress scenarios? Our method allows a precise trade off between plausibility and severity. In this way we can ensure that in a model of portfolio risk, no harmful but plausible scenarios are missed. Furthermore, our stress test method suggests ways to reduce risk if desired.

We analyse the problem of finding extreme but plausible scenarios in a classical quantitative risk management framework. A portfolio of financial instruments, say a portfolio of loans, is given. The value of each loan at some given horizon in the future is described by the realization of certain risk factors. In the case of a loan portfolio, for example, these risk factors will comprise the macroeconomic environment (because of its impact on the payment ability and thus on the solvency of borrowers), market factors like interest rates (or exchange rates in the case of foreign currency loans) but also idiosyncratic factors that influence a borrower’s solvency. The uncertainty about the realization of risk factors is described by a risk factor distribution that is estimated from historical data. Plausibility is captured by specifying how far we go into the tails of the distribution in our search for stress scenarios. The severity of scenarios is maximised by systematically searching for the worst case, the maximum portfolio loss, in a risk factor region of given plausibility.

This general idea of looking at extreme scenarios has been formulated in the literature before. It is informally discussed by Čihák [2004, 2007]). More formally the idea is discussed in Studer [1999, 1997] and in Breuer and Krenn [1999]. This literature leaves however two open issues that seem technical at first sight but are of great practical relevance: The problem of partial scenarios and the problem of dimensional dependence of maximum loss.

The partial scenario problem comes from the situation that a portfolio may depend on many risk factors but the modellers are interested in stressing not all but only a few factors at a time. For example in a loan portfolio we are often interested in stress scenarios for particular variables: A certain move in the exchange rate, or a particular drop in GDP. How do we deal with the other risk factors consistently? Do we leave them at their last observed value, at some average value, should we condition on the stressed macro factor and if so how? We show that the way to deal with the partial scenario problem that maximises plausibility is to set the non stressed systematic risk factors to their conditional expectation for the given the value of the stressed factors. We show furthermore that this has the same plausibility than the computationally more intensive full loss simulation from the conditional stress distribution as in Bonti et al. [2005].

If we look for maximum loss in a risk factor region of given plausibility we want the maximum loss to be independent from the inclusion of irrelevant risk factors or risk factors that are highly correlated with factors already included in the analysis. The plausibility measures that were used in the previous literature (see Studer [1999, 1997]) suggested to define plausibility regions as regions with a given probability mass. This definition of plausibility has an undesirable
property, known as the problem of dimensional dependence of maximum loss. To get a more intuitive understanding of the problem consider an example from Breuer [2008]. We have a bond portfolio with risk factors consisting of two yield curves in 10 foreign currencies. One risk manager chooses to model the yield curve with seven maturity buckets and another risk manager uses 15 buckets. In this case the first risk manager uses 150 risk factors in his analysis and the second manager uses 310. As plausibility region both of them choose an ellipsoid of mass 95%. Breuer [2008] shows that the second risk manager will calculate a maximum loss that is 1.4 times higher than the maximum loss calculated by first risk manager. This is problematic because both of them look at the same portfolio and use the same plausibility level. We suggest an approach to define plausibility that does not have this problem.

The paper is organised as follows. We first define a quantitative measure of plausibility and explain why it is not subject to the dimensional dependence problem. We discuss how to deal with the problem of partial scenarios and explain the technique of worst case analysis. We finally analyse an example portfolio of foreign currency loans that illustrates the practical applicability as well as the potential improvement compared to a standard stress testing procedure.

2 Finding scenarios that are plausible, severe, and suggestive of counter-action

We consider the problem of stress testing a loan portfolio. The value of each position in the portfolio depends on \( n \) systematic risk factors \( r = (r_1, \ldots, r_n) \) and on \( m \) idiosyncratic risk factors \( \epsilon_1, \ldots, \epsilon_m \). In our approach we have to restrict the distribution of the systematic risk factors \( r \) to a class called the elliptical distributions. For the definition and some basic facts about elliptical distributions we refer to the standard work of Fang et al. [1987]. For our purpose it is enough to note that the standard distributions used in classical risk management problems are in fact from this class. We denote the covariance matrix and expectations of the distribution of \( r \) by \( \text{Cov} \) and \( \mu \). The distribution of the idiosyncratic risk factors may be arbitrary.

2.1 Plausible scenarios

In a stress test of a loan portfolio we imagine extreme realizations of one of more of the systematic risk factors. How would we quantify the plausibility of this thought experiment?

An intuitive approach could be to compare the extreme realization of a risk factor to its average. Intuitively the further we are away from this average value, the less plausible the stress scenario becomes. A naive approach to quantify this idea is to define a region of plausibility as points that are say two standard deviations away from the average. Then measure how many standard deviations the extreme realization is away from the average and use the distribution function, say the normal distribution, to compute the probability that the extreme realization belongs to the set of plausible scenarios. This approach is naive because it implicitly assumed that all the observations are distributed spherically around the average. If this is not the case, because of correlations, the probability of a
point belonging to the plausible set does not only depend on the distance from the average but also from the direction. If the plausible region is an ellipsoid, as it would be for instance with a multivariate normal distribution, in those directions where the ellipsoid has a short axis the test point must be closer, while in those directions where the axis is long the test point can be further away from the center. Mathematically, the ellipsoid that best represents the set’s probability distribution can be estimated by building the covariance matrix of the samples.

A statistical concept that formalises this idea is the so called Mahalanobis distance given by

\[
\text{Maha}(r) := \sqrt{\left(r - \mu\right)^T \cdot \text{Cov}^{-1} \cdot \left(r - \mu\right)},
\]

The Mahalanobis distance is simply the distance of the test point from the center of mass divided by the width of the ellipsoid in the direction of the test point. Intuitively, Maha\((r)\) can be interpreted as the number of standard deviations of the multivariate move from \(\mu\) to \(r\). Maha takes into account the correlation structure between the risk factors.

In contrast to the previous literature we define plausibility directly in terms of Maha\((r)\): A high value of Maha implies a low plausibility of the scenario \(r\). Earlier work defined plausibility in terms of the probability mass of the ellipsoid of all scenarios of equal or lower Maha, see Studer [1999, 1997] or Breuer and Krenn [1999]. This approach creates the problem of dimensional dependence. If one defined plausibility in terms of the ellipsoid containing of some fixed probability mass, then the maximum loss would depend on the number of systematic risk factors, which is to some degree arbitrary. In our approach this problem does not occur.

### 2.2 Partial scenarios

Typically portfolios are modelled with hundreds or thousands of risk factors. Stress scenarios involving the full plethora of risk factors are hardly tractable numerically and overwhelmingly complex to interpret. A feasible answer to this problem is to design partial stress scenarios, which involve only a handful of risk factors. How should the other risk factors be treated?

Kupiec [1998] discussed four different ways to deal with the risk factors not fixed by some partial scenario:

(A) The other systematic risk factors remain at their last observed value.

(B) The other macro risk factors take their unconditional expectation value.

(C) The other systematic risk factors take their conditional expected value given the values of the fixed risk factors. Denote by \(r_C\) the resulting vector of values of the systematic risk factors.

(D) The other systematic factors are not fixed but distributed according to the conditional distribution given the values of the fixed risk factors. Denote by \(r_D\) the vector of values of the fixed systematic risk factors.

Our first result suggests a choice between these alternatives based on our concept of plausibility. The result says that the specification of partial scenarios as in method (C) or (D) both maximise plausibility. In the literature on stress
testing of loan portfolios Bonti et al. [2005] have suggested to use method (D). This is indeed an approach that maximises plausibility. From our result we learn that we can achieve an equivalent plausibility by using the computationally more efficient approach (C).

We state this result more formally in the following

**Proposition 1.** Assume the distribution of systematic risk factors is elliptical with density strictly decreasing as a function of Maha. Then:

1. \( \text{Maha}(r_C) = \text{Maha}(r_D) \).

2. This is the maximal plausibility which can be achieved among all macro scenarios which agree on the fixed risk factors.

A proof of this proposition is in the working paper Breuer et al. [2008b]. This proposition is of high practical relevance. It is the basis of partial scenario analysis. It implies that two choices of macro stress distributions are preferable, namely (C) or (D). Assigning to the non-fixed risk factors other values than the conditional expected values given the fixed risk factors leads to less plausible macro stress scenarios.

### 2.3 Severe Scenarios

An important disadvantage of stress testing with hand-picked scenarios is the danger to ignore harmful but plausible scenarios. This may create an illusion of safety. A way to overcome this disadvantage is to search systematically for those macro scenarios in some plausible admissibility domain which are most harmful to the portfolio. By searching systematically over admissible domains of plausible macro scenarios one can be sure not to ignore any harmful but plausible scenarios. This is our approach to construct a stress test: find the relevant scenarios which are most harmful yet above some minimal plausibility threshold. This problem can be formulated as an optimisation problem which can be solved numerically by using an algorithm of Pistovčák and Breuer [2004].

The admissibility domain is determined by our concept of plausibility. It contains all scenarios with \( \text{Maha}(r) \) below a threshold \( k \):

\[
\text{Ell}_k := \{ r : \text{Maha}(r) \leq k \}.
\]

Geometrically this domain is an ellipsoid whose shape is determined by the covariance matrix of the systematic risk factors:

Partial scenarios do not specify a unique portfolio value but just a distribution, namely the distribution of portfolio values conditional on the values of the values of the risk factors fixed by the scenario. In order to measure the severity of scenarios one needs to quantify the severity of the corresponding conditional portfolio value distribution. In this paper we use the expectation value, although other risk measures could be used as well. Thus we call a partial scenario severe if it has a low conditional expected profit (CEP). To sum up, our stress testing method amounts to solving the following optimisation problem

\[
\min_{r \in \text{Ell}_k} \text{CEP}(r).
\]
The difference between the lowest CEP in the admissibility domain and the CEP in the expected scenario is the maximum expected loss in the admissibility domain. This concept of maximum loss overcomes the problem of dimensional dependence which we mentioned in the introduction. Maximum expected loss over the admissibility domain $\mathcal{E}_k$ is not affected by excluding or including macro risk factors which are irrelevant to the portfolio value Breuer [2008].

What is the advantage of this worst case search over standard stress testing? First, it achieves a controlled trade-off between plausibility and severity of scenarios. If we want to get more severe scenarios, we choose a higher $k$ and get less plausible worst case scenarios. If we want to get more plausible scenarios, we choose a lower $k$ and get less severe worst case scenarios. Second, it overcomes the historic bias by considering all scenarios which are plausible enough. In this way we can be sure not to miss scenarios which are plausible but did not yet happen in history. Thirdly, worst case scenarios reflect portfolio specific dangers. What is a worst case scenario for one portfolio might be a harmless scenario to another portfolio. This is not taken into account by standard stress testing. Portfolio specific dangers suggest possible counter-action to reduce risk if desired.

2.4 Scenarios Suggesting Risk Reducing Action

Risk reducing action is suggested by identifying the key risk factors which contribute most to the expected loss in the worst case scenario. We define key risk factors as the risk factors with the highest Maximum Loss Contribution (MLC). The loss contribution (LC) of risk factor $i$ to the loss in some scenario $\mathbf{r}$ is

$$\text{LC}(i, \mathbf{r}) := \frac{\text{CEP}(\mathbf{r}) - \text{CEP}(\mu_1, \ldots, \mu_{i-1}, r_i, \mu_{i+1}, \ldots, \mu_n)}{\text{CEP}(\mathbf{\mu}) - \text{CEP}(\mathbf{r})},$$

if $\text{CEP}(\mathbf{r}) \neq \text{CEP}(\mathbf{\mu})$. LC$(i, \mathbf{r})$ is the loss if risk factor $i$ takes the value it has in scenario $\mathbf{r}$, and the other risk factors take their expected values $\mathbf{\mu}$, as a percentage of the loss in scenario $\mathbf{r}$. In particular, one can consider the worst case scenario, $\mathbf{r} = r_{\text{WC}}$. In this case the loss contribution of some risk factor $i$ can be called the Maximum Loss Contribution:

$$\text{MLC}(i) := \text{LC}(i, r_{\text{WC}}).$$

MLC$(i)$ is the loss if risk factor $i$ takes its worst case value and the other risk factors take their expected values, as a percentage of MaxLoss.

The Maximum Loss Contributions of the macro risk factors in general do not add up to 100%. Sometimes the sum is larger, sometimes it is smaller. If this sum is equal to one, the loss in the scenario is exactly equal to the sum of losses from individual risk factor moves. This happens if and only if the risk factors do not interact:

**Proposition 2.** Assume CEP as a function of the macro risk factors has continuous second order derivatives. The loss contributions of the risk factors add up to 100% for all scenarios $\mathbf{r}$,

$$\sum_{i=1}^{n} \text{LC}(i, \mathbf{r}) = 1$$
if and only if $CEP$ is of the form

$$CEP(r_1, \ldots, r_n) = \sum_{i=1}^{n} g_i(r_i).$$

This is the case if and only if all cross derivatives of $CEP$

$$\frac{\partial^2 CEP(r)}{\partial r_i \partial r_j} = 0$$

vanish identically for $i \neq j$.

For the proof we refer to the working paper of Breuer et al. [2008b]. This characterization has a substantial practical relevance. The sum of loss contributions measures the role of interaction of systematic risk for the portfolio value. If the sum is larger than one the interaction between risk factors is positive. The total loss in the scenario is smaller than the sum of losses from individual risk factor moves.

Most dangerous is the case of negative interaction between risk factors. If the sum is smaller than one the total loss in the scenario is larger than the sum of losses from individual risk factor moves. The harm of the scenario cannot be fully explained by individual risk factor moves. The simultaneous move of some risk factors causes harm on top of the single risk factor moves. In this case it will be necessary to consider Maximum Loss Contributions not of single risk factor moves but of pairs or even of larger groups of risk factors.

A consequence of this insight outside of the stress testing problem is that it reveals a weakness in current regulatory thinking. Analyzing portfolio risk along the categories market and credit risk, and determining risk capital based on the aggregation of these separately calculated risk numbers may in fact underestimate the true portfolio risk because it ignores the risks stemming from simultaneous moves in market and credit risk factors. For a detailed discussion of this problem see Breuer et al. [2008a].

Possible risk reducing action can be designed with knowledge of the key risk factors. One strategy could be to buy hedges which pay off exactly when the key risk factors take their worst case value. Another, more comprehensive but also more expensive strategy is to buy hedges which neutralise the harm done not just by the worst case moves of the key risk factors but by all moves of the key risk factors. For the example of the foreign currency loan portfolios discussed in the next section, this strategy is demonstrated in Breuer et al. [2008c].

3 Application: Stress testing a portfolio of foreign currency loans

We now illustrate the concepts and their quantitative significance now in an example: a stress test for a portfolio of adjustable rate loans in home or foreign currency (CHF) to 100 borrowers in the rating class B+, corresponding to a default probability of $p_1 = 2\%$, or in rating class BBB+, corresponding to a default probability of $p_1 = 0.1\%$. At time 0, in order to receive the home currency amount $l = e 10000$ the customer of a foreign currency loan takes a loan of $le(0)$ units in a foreign currency, where $e(0)$ is the home currency value
of the foreign currency at time 0. The bank borrows \( lc(0) \) units of the foreign currency on the interbank market. After one period, at time 1, which we take to be one year, all the loans expire and the bank repays the foreign currency at the interbank market with an interest rate \( r_f \), e.g. LIBOR, and it receives from the customer a home currency amount which is exchanged at the rate \( e(1) \) to the foreign currency amount covering repayment of the principal plus interest rolled over from four quarters, plus a spread \( s \). So the borrower’s payment obligation to the bank at time 1 in home currency is

\[
\begin{align*}
    o_f &= l(1 + r_f) E + s_f l E \\
    o_h &= l(1 + r_h) + s_h l
\end{align*}
\]  

for the foreign resp. home currency loan. The first term on the right hand side is the part of the payment which the bank uses to repay its own loan on the interbank market. The second term is profits remaining with the bank. For all loans in the portfolio we assume they expire at time 1. The model can be extended to a multi period setting allowing for loans not maturing at the same time and requiring payments at intermediate times.

In order to evaluate idiosyncratic and systematic risk of a portfolio of such loans we use a one-period structural model specifying default frequencies and losses given default endogenously. For details of the model we refer to Breuer et al. [2008c]. The basic structure of the model is given by the payment obligation distribution (4) and a lognormal payment ability distribution, which involves lognormally distributed idiosyncratic changes and an additional dependence of the mean one future GDP changes. (Pesaran et al. [2005a] use a model of this type for the returns of firm value.) Each customer defaults in case their payment ability at the expiry of the loan is smaller than their payment obligation. In case of default the borrower pays what he is able to pay. The difference to the payment obligation first is lost profit and then loss for the bank.

The spread \( s_h \) resp. \( s_f \) and the variance of the idiosyncratic payment ability changes are determined jointly in a calibration procedure. The first calibration condition ensures that the model default probability coincides with the default probability determined in some external rating procedure. The second calibration condition ensures that expected profit from each loan reaches some target level of \( €160 \), which amounts to a return of 20% on a regulatory capital of 8%. Both calibration conditions depend on the spread \( s \) and the variance of the idiosyncratic payment ability changes.

The systematic risk factors entering the portfolio valuation are GDP, the home interest rate \( r_h \) and the foreign interest rate \( r_f \), and the exchange rate change \( E \). The probability law driving these risk factors is modelled by a time series model that takes account of economic interaction between countries and regions. Estimating the parameters of this model we can simulate scenarios for the systematic risk factors. For details of this model, known in the literature as GVAR model see Pesaran et al. [2001], Pesaran et al. [2005b], Garrett et al. [2006], and Dees et al. [2007].

The profit distribution was calculated in a Monte Carlo simulation by generating 100 000 scenario paths of four steps each. The resulting distribution of risk factors after the last quarter, which is not normal, was used to estimate the covariance matrix of 1yr macro risk factor changes. In each macro scenario defaults of the customers were determined by 100 draws from the idiosyncratic
changes in the payment ability distribution. From these we evaluated the profit distribution at the one year time horizon.

3.1 Hand-picked vs. systematic stress tests

Let us compare the severeness of the hand-picked scenario “GDP shrinks by 3%”, which is a $5.42\sigma$ event, to the worst case scenario of the same plausibility. Conditional expected profits for the standard scenario ‘GDP -3% and other risk factors at their conditional expected value’, and of worst case scenarios of the same plausibility are as follows.

<table>
<thead>
<tr>
<th>scenario</th>
<th>Maha</th>
<th>CEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>foreign B+</td>
<td></td>
<td></td>
</tr>
<tr>
<td>expected</td>
<td>0</td>
<td>16 001</td>
</tr>
<tr>
<td>GDP -3%</td>
<td>5.42</td>
<td>15 950</td>
</tr>
<tr>
<td>worst case</td>
<td>5.42</td>
<td>-98 101</td>
</tr>
<tr>
<td>foreign BBB+</td>
<td></td>
<td></td>
</tr>
<tr>
<td>expected</td>
<td>0</td>
<td>15 999</td>
</tr>
<tr>
<td>GDP -3%</td>
<td>5.42</td>
<td>15 870</td>
</tr>
<tr>
<td>worst case</td>
<td>5.42</td>
<td>-95 591</td>
</tr>
<tr>
<td>home B+</td>
<td></td>
<td></td>
</tr>
<tr>
<td>expected</td>
<td>0</td>
<td>16 000</td>
</tr>
<tr>
<td>GDP -3%</td>
<td>5.42</td>
<td>14 249</td>
</tr>
<tr>
<td>worst case</td>
<td>5.42</td>
<td>13 291</td>
</tr>
<tr>
<td>home BBB+</td>
<td></td>
<td></td>
</tr>
<tr>
<td>expected</td>
<td>0</td>
<td>16 000</td>
</tr>
<tr>
<td>GDP -3%</td>
<td>5.42</td>
<td>15 811</td>
</tr>
<tr>
<td>worst case</td>
<td>5.42</td>
<td>15 626</td>
</tr>
</tbody>
</table>

We observe that for all portfolios the conditional expected profits are considerably lower in the worst case scenarios than in the hand-picked GDP scenario. This is evidence of the danger which lies relying solely on hand-picked scenarios. Expected profits in this rather extreme hand-picked GDP scenario are only moderately lower, namely by amounts between €129 and €1 751 on a loan portfolio worth €1m giving an unconditional expected profit of €16 000. These moderate profit reductions in such an extreme scenario might provide a feeling of safety. But this in an illusion. There are other scenarios out there which are equally plausible but much more harmful. There are scenarios which reduce expected profit by amounts between €374 resp. €2 709 for the home currency loans, and by €114 101 resp. €111 591 for the foreign currency loan portfolios. These huge losses of roughly 11% are higher than the total regulatory capital of 8% for the loan portfolio.
3.2 Key risk factors and risk reducing actions

What is a worst case scenario for one portfolio might be a harmless scenario to another portfolio. This is not taken into account by standard stress testing. Stress testing is relevant only if the choice of scenario takes into account the portfolio. In a systematic way this is done by worst case search.

Key risk factors are ones with highest maximum loss contributions (MLC). The worst case scenarios, together with the MLC for each risk factor are given in Table 1, for different sizes of the admissibility domain. For each scenario the risk factor with the highest MLC are printed in bold face.

- For the foreign currency loan portfolio the exchange rate is clearly the key risk factor. This becomes apparent from Table 1. In the worst case scenario the FX rate alone contributes between 59% and 100% of the losses in the worst case scenarios, the other risk factors contribute less than 1%. This indicates that the FX rate is the key risk factor of the foreign currency loan portfolio.

- For the home portfolio, GDP is the key risk factor, but the home interest rate is also relevant. The moves in GDP alone contribute between 46% and 73% of the losses in the worst case scenarios.

- There is another interesting effect. The dependence of expected profits of foreign currency loans on the CHF/€ rate is not only non-linear, but also not monotone. For the BBB+ FX loan portfolio (bottom left plots in Figure 1), focusing on changes smaller than $4\sigma$ it becomes evident that a small increase in the exchange rate has a positive influence on the portfolio value, but large increases have a very strong negative influence. Correspondingly, in Table 1, if we restrict ourselves to small moves (Mahalanobis smaller than $4\sigma$) the worst case scenario is in the direction of increasing exchange rates, but if we allow larger moves the worst case scenario is in the direction of decreasing exchange rates. This effect also shows up in the worst macro scenarios of Table 1. The reason for this non-monotonicity is that a small decrease in the FX rate increases the EUR value of spread payments received. For larger moves of the FX rate this positive effect is outweighed by the increases in defaults due to the increased payment obligations of customers. For the bad quality B+ portfolios the positive effect of a small FX rate decrease persists only up to a maximal Mahalanobis radius of $k = 2$. 


Table 1: Systematic macro stress tests of the home and foreign currency loan portfolios. We search for the macro scenarios with the worst expectation value of the profit distribution, under the condition that macro scenarios lie in elliptical admissibility domain of maximal Mahalonobis radius $k$. Macro scenarios are specified by the macro risk factors GDP, exchange rate, and interest rates. We give the absolute values of these risk factors in the worst case scenario, as well as their change in standard deviations, and their Maximum Loss Contributions MLC. For the key risk factors MLC is printed in bold face.

<table>
<thead>
<tr>
<th>Worst Macro Scenario</th>
<th>max. Maha</th>
<th>abs. GDP</th>
<th>stdv</th>
<th>MLC</th>
<th>abs. home IR</th>
<th>stdv</th>
<th>MLC</th>
<th>abs. foreign IR</th>
<th>stdv</th>
<th>MLC</th>
<th>abs. CHF/€</th>
<th>stdv</th>
<th>MLC</th>
<th>CEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>foreign B+</td>
<td>1</td>
<td>231.73</td>
<td>-0.07</td>
<td>0.6%</td>
<td>0.022</td>
<td>0.04</td>
<td>0.8%</td>
<td>1.587</td>
<td>1</td>
<td>100.0%</td>
<td>15 959</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>231.64</td>
<td>-0.14</td>
<td>0.5%</td>
<td>0.022</td>
<td>0.04</td>
<td>0.4%</td>
<td>1.646</td>
<td>2</td>
<td>100.0%</td>
<td>15 400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>231.08</td>
<td>-0.11</td>
<td>0.1%</td>
<td>0.035</td>
<td>0.03</td>
<td>0.8%</td>
<td>1.363</td>
<td>-2.81</td>
<td>59.7%</td>
<td>1 631</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>230.98</td>
<td>-0.1</td>
<td>0.1%</td>
<td>0.039</td>
<td>1.17</td>
<td>0.4%</td>
<td>1.306</td>
<td>-3.78</td>
<td>65.3%</td>
<td>-26 084</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>230.85</td>
<td>-0.09</td>
<td>0.0%</td>
<td>0.042</td>
<td>1.39</td>
<td>0.2%</td>
<td>1.249</td>
<td>-4.76</td>
<td>71.2%</td>
<td>-73 257</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>230.86</td>
<td>-0.03</td>
<td>0.0%</td>
<td>0.046</td>
<td>1.59</td>
<td>0.2%</td>
<td>1.191</td>
<td>-5.74</td>
<td>77.0%</td>
<td>-136 000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>foreign BBB+</td>
<td>1</td>
<td>231.75</td>
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<td>0.0%</td>
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The diagnosis that the FX rate is the key risk factor for the foreign currency loans and GDP is the key risk factor for the home currency loans is confirmed by the right and left hand plots in Figure 1, which show the expected profits in dependence of single macro risk factor moves, keeping the other macro risk factors fixed at their expected values. Note the different scales of the two plots. Expected losses of the FX loan are considerably larger than for the home currency loan. This plot also shows that expected profits of both loan types depend non-linearly on the relevant risk factors. The profiles of expected profits in Figure 1 resemble those of short options. A home currency loan behaves like a short put on GDP together with a short call on the home interest rate. A foreign currency loan behaves largely like a short call on the FX rate.

One could ask why the effort to search for worst case scenarios is necessary to identify key risk factors. Wouldn’t it be easier to read the key risk factors from the plots in Figure 1? This would be true if losses from moves in different risk factors added up. But for certain kinds of portfolios the worst case is a simultaneous move of several risk factors—and the loss in this worst case might be considerably worse than adding up the losses resulting from moves in single risk factors. This is the message of Proposition 2. The effects of simultaneous moves are not reflected in Figure 1, but they do show up in the worst case scenario.

As an example, consider a B+ home currency loan and assume we are restricting ourselves to moves with Maha smaller than \( k = 6 \). From Table 1 we see that the MLC of the two risk factors sum up to 62.0% + 9.9% = 71.9%, which is considerably lower than 100%. This indicates that the loss of a joint move is considerably larger than sum of losses of individual risk factor moves. This is not reflected in Fig 1, which only displays the effects of single risk factor moves.

The same occurs for foreign currency loans. They show a dangerous interaction of market and credit risk. The exchange rate has a MLC of 65.3%, the interest rate has a MLC of 0.4%, GDP 0.1%, which is a total of 65.8% instead of 100%. Single risk factor moves leave about 35% of the maximum loss unexplained. The reason is that adverse exchange rate moves drive up payment obligations. This increases default probabilities and losses given default.

The identification of key risk factors suggests risk reducing counter-actions. Knowing that the exchange rate is the key risk factor for FX loans, one can plot the behaviour of CEP in dependence of exchange rate moves, as in the left hand plots of Fig 1. Breuer et al. [2008c] show how FX derivatives can be used to construct hedges reducing the exchange rate risk of foreign currency loans. It turns out that FX options can be used to virtually eliminate the dependence of expected loss on exchange rates—at some fixed level of interest rates and other macroeconomic factors. But the hedge is not perfect: Firstly it cannot fully remove dependence of expected losses on exchange rates at other levels of interest rates, and secondly it can bring to zero only the expectation but not the variance of losses caused by adverse exchange rate moves.

4 Conclusion

We introduce the technique of worst case search to macro stress testing. Among the macroeconomic scenarios satisfying some plausibility constraint we deter-
mined the worst case scenario which causes the most harmful loss in loan portfolios. This method has three advantages over traditional macro stress testing: First, it ensures that no harmful scenarios are missed and therefore prevents a false sense of safety which may result when considering only standard stress scenarios. Second, it does not analyse scenarios which are too implausible and would therefore jeopardise the credibility of stress analysis. Third, it allows for a portfolio specific identification of key risk factors. 

Another lesson from this paper relates to the use of partial stress scenarios specifying the values of some but not all risk factors: The plausibility of partial scenarios is maximised if we set the remaining risk factors to their conditional expected values.

In order to carve out the basic insights we presented the approach in the most basic framework. For practical purposes the framework has to be generalised to a multi-period setup, requiring scenario paths instead of one step scenarios. Admissibility domains also have to be defined for scenario paths instead of one step scenarios. In a multi period setup one can analyse portfolios of loans maturing at different times and requiring payments at intermediate times. The
computational burden in the multi-period framework is by far heavier.

References


