

Estimating the Capacity of an Equity Strategy

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Hans-Christian Lüdemann, PhD

State Street Associates

140 Mt. Auburn Street

Cambridge, MA 02138

617 234-9490

hluedemann@statestreet.com

William Kinlaw, CFA

State Street Associates

140 Mt. Auburn Street

Cambridge, MA 02138

617 234-9496

wbkinlaw@statestreet.com

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Introduction

Empirical findings indicate that the alpha generated by active management strategies decreases as assets under management (AUM) increase. This occurs because transaction costs increase with trade size. It is in the best interest of both managers and clients to close active strategies to new investment before transaction costs outpace alpha. This gives rise to the concept of product capacity. A product's threshold capacity is defined as the maximum AUM at which it is able to meet its stated performance objective. We present a methodology to analyze the relationship between AUM and annual transaction cost for a specific investment strategy. The nature of this relationship provides insight into the capacity of the strategy.

This article is organized as follows. We begin with a review of the literature on the topic. Next, we introduce the concept of implementation shortfall and describe its components. Then we outline our methodology for simulating portfolio turnover and explain how we calibrate our simulations using the historical track record of a specific strategy. Finally, we present results.

Review of the Literature

In the only empirical study that we have encountered, Allen [2007] analyzes the relationship between AUM and performance based on a large database of institutional investment products maintained by Callan Associates. The database includes 20 years of historical performance for over 5,000 products and is adjusted by the author for back-fill bias. His objective is to assess the usefulness of AUM to plan sponsors as a criterion for selecting managers rather than to offer techniques to estimate capacity for specific products.

The author finds that products in the smallest AUM quartile outperform products in the largest AUM quartile for 10 out of 11 equity styles and 2 out of 3 fixed income styles. The largest (and statistically significant) outperformance is found in what he terms the "capacity constrained" asset classes: small-cap equities and high-yield fixed income. Interestingly, he finds that this relationship is reversed for investment-grade

fixed income, suggesting that size — and the associated resources and expertise — provide an advantage in more liquid markets.

After adjusting for risk, it is evident that most of the performance advantage enjoyed by the smaller funds in his database is accompanied by higher active risk. However, small-cap equity and high-yield fixed income products remain the notable exceptions: the smaller AUM portfolios in these asset classes exhibit information ratios that are generally higher than or equal to the larger AUM portfolios. The author concludes that there is a statistically significant negative relationship between AUM and performance in capacity-constrained asset classes.

In what is perhaps the earliest capacity study, Perold and Salomon [1991] present a theoretical construct for estimating the capacity of an active equity strategy. They argue that the alpha-maximizing AUM level is zero, since *percent* alpha begins to decay as soon as the strategy moves from a paper portfolio to implementation. The authors focus on dollar wealth-maximization (percent alpha times AUM) as a criterion for determining the right amount of AUM. By this definition, capacity depends on three things: the quality of research (gross alpha), turnover, and market impact.

The authors devote considerable attention to the manager incentives associated with specific fee structures. A flat base fee encourages the manager to maximize his or her own income by increasing AUM far beyond the wealth-maximizing point. Fees that decline with account size exacerbate this effect. They advocate performance-based fees as the best way to align manager and client interests.

The authors present a simplified illustration of how a manager might estimate wealth-maximizing capacity. They assume that market impact is a linear function of trade size, that the portfolio includes 100 positions each with the same wealth-maximizing trade size, and that turnover is 100% per year. They then compute wealth-maximizing AUM formulaically for a range of gross alpha and market impact assumptions. Intuitively, high (low) gross alpha and low (high) market impact lead to the largest (smallest) capacity estimates. In general, their capacity estimates are relatively low. For gross alphas of 1 to 3% and “reasonable” market impact assumptions, they find that wealth-maximizing AUM is only a few hundred million dollars.

Vangelisti [2006] and Kahn and Shaffer [2005] outline backtest approaches to estimate capacity for quantitative investment models. Vangelisti proposes three definitions for capacity. Threshold capacity is the amount of AUM beyond which a strategy is no longer able to achieve its stated investment return objective, wealth-maximizing capacity is the AUM that maximizes the dollar amount of wealth created, and terminal capacity is the AUM that reduces net alpha to zero. Threshold capacity is of greatest interest to managers who charge a base fee whereas wealth-maximizing capacity is of greatest interest to managers who charge a performance fee.

He then introduces two methodologies for estimating the threshold capacity of a strategy: one based on transaction costs and one based on the size of the positions being accumulated in the portfolio (breadth). Both methodologies rely on backtests and are hence applicable only to quantitative models.

To illustrate the two methodologies, he runs backtests of an emerging markets stock model over the period 1993 through 2004. The model incorporates a monthly optimization trading off alpha, the risk of deviating from the benchmark along a number of dimensions, and the cost of trading. The transaction costs assumptions are 20 basis points for commissions and a linear market impact function of 30 basis points for each 10% of ADV traded.

In the transaction costs approach to capacity estimation, the author runs a backtest assuming \$1 billion in AUM and measures the net alpha of the strategy. He then increases AUM in \$1 billion increments to produce net alpha results for all AUM levels between \$1 billion and \$50 billion. Having identified the relationship between net alpha and AUM, he can then estimate wealth-maximizing capacity, threshold capacity, and break-even capacity. Because the portfolio construction process takes transaction costs into account, the break-even capacity is infinite — the strategy will converge to an index fund as AUM approach infinity.

In the breadth approach to capacity estimation, he notes that many investors face constraints on the percentage ownership of stocks in the portfolio. For example, investment policies may state that a product cannot hold more than 1% of the free float of any given stock. As AUM increase (or holdings become more concentrated), these constraints force the fund manager to resort to second- and third-best ideas. To evaluate

the impact of percent ownership constraints on capacity, the author runs backtests across a range of concentration levels and percent ownership constraints. For each combination of these two assumptions, he identifies the implied capacity of the product.

In another backtest approach, Kahn and Shaffer introduce a framework to analyze the threshold capacity of an optimized, quantitative equity strategy based on its expected net alpha. They compute net alpha as the difference between two quantities: expected gross alpha and expected costs.

Expected gross alpha is defined as the product of the intrinsic IR (the maximum IR achievable absent costs or constraints), the product's active risk level, and the transfer coefficient. The transfer coefficient is a value between 0 and 100% that reflects any degradation in pre-cost performance due to deviation from optimal mean-variance weights. The authors impose a long-only constraint and incorporate a transaction cost penalty into their optimization, resulting in transfer coefficients below 100%. They model the transfer coefficient as a function of turnover — as trading constraints are relaxed and turnover is allowed to increase, the transfer coefficient also increases.

Expected costs are defined as turnover times transaction costs, where transaction costs are themselves a function of both AUM and turnover. Rising turnover thus influences net alpha in two competing ways: it increases gross alpha (via the transfer coefficient) but it also increases transaction costs.

To estimate capacity, the authors first solve for the turnover level that maximizes net alpha at a given level of AUM. They then record this net alpha and repeat the procedure for a wide range of AUM levels to identify the relationship between net alpha and AUM. This enables them to estimate the threshold capacity of the product. (As AUM rises, the alpha-maximizing turnover level falls at a decreasing rate, reflecting the increased cost of trading).

The authors find that net alpha decays slowly with growing AUM and conclude that capacity is quite sensitive to the intrinsic IR and cost assumptions. As a result, they suggest that capacity cannot be estimated precisely, but that inaccuracies in the capacity estimate have little impact on net alpha.

Our contribution is threefold. First, we introduce a methodology for estimating product capacity that accommodates non-quantitative strategies; i.e. strategies that are not

amenable to backtests. Second, we explicitly model the opportunity cost component of implementation shortfall. Finally, our approach recognizes the stochastic quality of implementation shortfall, which is the primary determinant of product capacity.

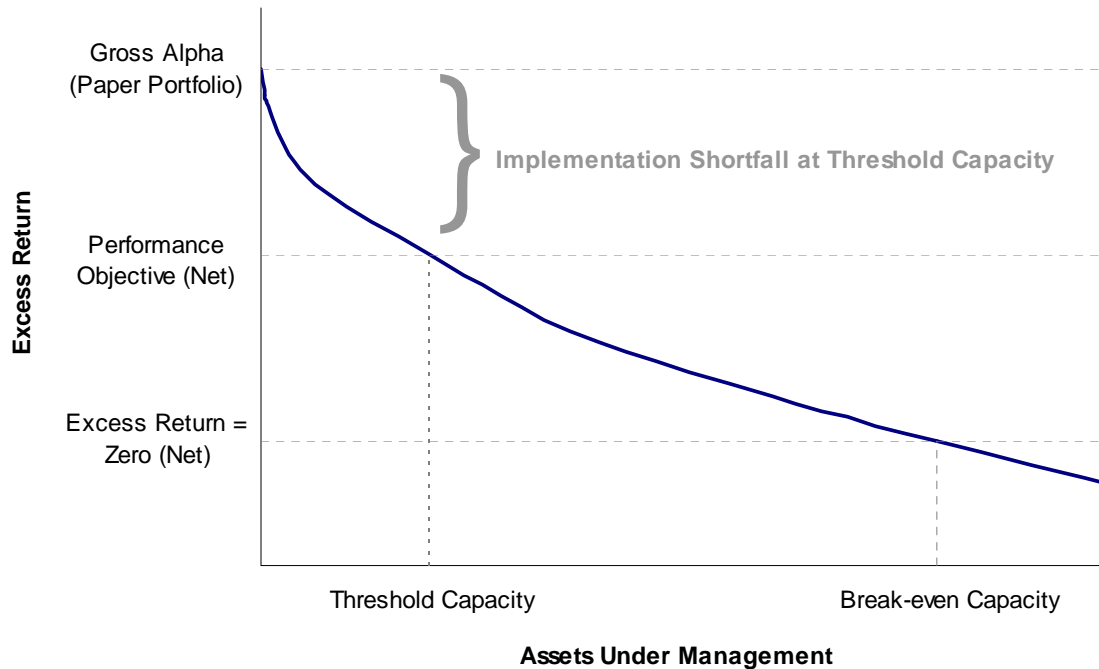
Implementation Shortfall

An active management strategy consists of a time-varying combination of security holdings selected to offer an excess return. Managers face a variety of explicit costs as they implement their strategies. These include commissions, bid-ask spreads, taxes, and foreign exchange costs; they also face implicit costs consisting of market impact and opportunity cost. Market impact refers to price movement that occurs in response to the purchase and sale of securities (i.e. the manager “moving the market”) whereas opportunity cost refers to price movements caused by external factors. Collectively, these implicit and explicit costs are known as implementation shortfall.

The notion of implementation shortfall was introduced by Perold [1988], although Kritzman, Myrgren and Page [2006] provide what is perhaps the most succinct definition: implementation shortfall is the difference between the calculated performance of a paper portfolio and the realized performance of an identical real portfolio based on actual market transactions. Perold’s key innovation was to extend the traditional perception of transaction costs to incorporate the opportunity cost of delaying the sale of a security intended for disposition or the purchase of a security targeted for acquisition.

As discussed in the previous section, empirical findings suggest that the implementation shortfall associated with active management increases as AUM increase. All else equal, an investor can therefore expect her strategy’s net alpha to decrease as its AUM increase. We present this relationship graphically in Figure 1, below.

Figure 1: Alpha Erodes as AUM Increase



In this article, we employ a simulation methodology to estimate the relationship between AUM and implementation shortfall. To apply this methodology, we are required to specify a cost model. We quote explicit costs in basis points for each security or for groups of securities. We model implicit costs (market impact and opportunity costs) as follows.

For a given security i , we model market impact as a function of trade size and average daily volume, as shown in Equation 1. This market impact model is commonly found in both the academic and practitioner literature.¹

Equation 1: Market Impact Cost Model

$$\text{market impact}_i = k \sqrt{\frac{\text{trade size}_i}{\text{average daily volume}_i * \text{trade time}}}$$

¹ See, for example: Loeb [1983], Hasbrouck [1991], Grinold and Kahn [1999], Hasbrouck and Seppi [2001], Hisata and Yamai [2002], Lillo, Farmer, and Montegna [2002], [2003], Plerou et al. [2002], [2004], Gabaix et al. [2003], [2005], Kissell and Glantz [2003], Lim and Coggins [2003], Lo, Petrov, and Wierzbicki [2003], Farmer and Lillo [2004], Brown et al. [2004], Almgren et al. [2005].

The coefficient k is a scaling factor that Kritzman, Myrgren and Page estimate empirically from a large sample of securities trades; it is equal to 0.0079758.² Opportunity cost is the residual of total shortfall minus explicit costs and market impact (as defined in Equation 1), and on average it should approach 0.

Perold and Solomon caution that market impact estimates gleaned from actual executions suffer from “severe selection bias” because active managers actively avoid executing trades that will incur high market impact. Because our database comes from a transition manager, as opposed to an active equity manager, we expect that this effect biases our data to a lesser degree. Indeed, a transition manager is provided with an explicit target portfolio and has little flexibility to substitute one trade for another.

For average daily volume, we take the average number of shares traded over the 10 preceding business days. The trade time is expressed in days, and is a proxy for the aggressiveness of the manager’s trading style. Short trade times are associated with higher market impact costs and lower opportunity costs, whereas the opposite is true for longer trade times. To gauge the impact of trading aggressiveness on our capacity estimates, we may present results for a range of trade time assumptions.

Managers also face the risk that securities targeted for sale will drift down in price or that securities targeted for purchase will drift up in price over the period of time that it takes to execute the trade. The realization of this risk is opportunity cost, and it is a function of the daily return volatility of the securities in question. We employ Monte Carlo simulation to model opportunity cost. For a given security i , our model is presented formulaically in Equation 2.

Equation 2: Opportunity Cost Model

$$\text{opportunity cost} \sim N(\mu, \sigma_i^2)$$

$$\sigma_i = \sqrt{\text{trade time} * \text{stock price volatility}_i}$$

² Our database includes 4,415 equity transitions performed by the world’s leading transition manager, which occurred during the period from January, 2001 through December, 2007. The sample encompasses 2,996,386 buy and sell transactions covering 27,587 securities across 10 sectors and 82 countries. The total value traded exceeds \$2 trillion.

We measure the daily return volatility of each security for the 20 days preceding each trade. We adjust this calculation for the trade time assumption, that is, for the number of days we assume it takes the manager to complete the trade. The fact that a manager chooses to purchase or sell a security implies assumptions about the future evolution of the security's price. An active manager will sell a security if the price is expected to decline and purchase a security if the price is expected to increase. Therefore, a manager who waits to purchase a security may lose some of the expected rise in value, while a manager who waits to sell a security may realize less than when he first decided to sell. Both effects may contribute a nonzero mean in the opportunity cost term in Equation 2, particularly for strategies with a rapid rate of alpha decay, and, by extension, high turnover. However, this term is less likely to be important for value strategies with low turnover and long holding periods.

Equation 3 presents our combined implementation shortfall model. The mean of the distribution is equal to the sum of commissions, spreads, and expected market impact whereas the standard deviation is equal to the expected stock price volatility times the square root of trade time. For commissions and spreads we assume a combined value of 12 basis points. In this case, we do not include an additional term to reflect a positive expected mean opportunity cost.

Equation 3: Implementation Shortfall Model

$$\text{Implementation Shortfall} \sim N(\mu, \sigma^2)$$

$$\mu = \text{commissions} + \text{spread} + k \sqrt{\frac{\text{trade size}}{\text{average daily volume} * \text{trade time}}}$$

$$\sigma = \sqrt{\text{trade time} * \text{stock price volatility}}$$

This model is applied to individual securities at the trade level. These trade-level costs can be aggregated cross-sectionally to compute the cost of portfolio turnover on a given day, and then through time to estimate annualized transaction costs. To estimate capacity, we simulate trades that mirror the actual turnover profile of a specific strategy

under consideration. We then apply our cost model to these simulated trades to estimate the relationship between AUM and implementation shortfall. In the next section, we describe our simulation methodology.

Methodology

Kahn and Shaffer and Vangelisti estimate the capacity of quantitative strategies by performing a series of optimized backtests at varying levels of AUM and examining the impact on net alpha. This approach requires access to a model that generates holdings for the strategy ex post, which does not exist for a discretionary strategy and may not be readily accessible in either case.

Our methodology does not require that security selection be determined by a quantitative model; it can hence be applied to discretionary products that cannot be backtested. Specifically, we analyze the historical track record of executed trades associated with a specific strategy. We then employ a simulation approach to generate 25,000 days of random turnover across the strategy's benchmark universe.³ We calibrate our simulations such that the synthetic turnover reflects the empirical track record in several key respects. For each day in the simulation, we then estimate implementation shortfall as a function of turnover and AUM. The nature of this relationship provides insight into the capacity of the product. Equation 4 summarizes the assumed relationship between net alpha, gross alpha, turnover, and AUM.

Equation 4: Net Alpha as a Function of Gross Alpha and Implementation Shortfall

$$\alpha_{\text{net}} = \alpha_{\text{gross}} - \text{Implementation Shortfall}(\text{turnover}, \text{AUM})$$

The value of α_{gross} can be specified directly based on paper portfolio results or on forecasts of future performance. Implementation shortfall is estimated via simulation. Our objective is to identify the level of AUM that sets α_{net} equal to the stated investment performance objective; this level of AUM is our estimate of threshold capacity.

³ We select a data frequency of one day and a sample size of 25,000 for illustration purposes. We could choose to use other period lengths or sample sizes.

Our methodology enables us to produce distributions of annualized implementation shortfall for various assumptions and across a wide range of AUM levels. Our approach can be summarized in two broad steps. First, we construct a simulated track record that replicates the strategy's characteristic turnover pattern (as observed from the historical data). Next, for each level of AUM we:

- > scale these simulated trades to reflect the desired AUM level, and
- > model the implementation shortfall these rescaled, simulated trades would have incurred.

We then examine the relationship between net alpha and AUM to estimate the capacity of the strategy. More specifically, we proceed as follows.

STEP 1: For each day in the historical track record, compute the turnover for each stock in the strategy and for the overall portfolio, expressed as a percentage of AUM. This value may be zero; on some days, no stocks trade at all.

STEP 2: Select a day at random, with replacement, from the historical track record.

STEP 3: Select the first stock that was traded on the day selected in STEP 2. Note the amount of the stock that was traded (as a percentage of AUM) and its market capitalization quintile within the benchmark universe.

STEP 4: Select a stock at random from the benchmark universe that falls in the same market capitalization quintile. Assign the realized turnover (as a percentage of AUM) to this stock and set it aside.

STEP 5: Repeat STEPs 3 through 4 until a replicating trade has been assigned to each actual trade that occurred on that day. This set of trades represents the first day in the simulation.

STEP 6: Repeat STEPs 2 through 5 another 24,999 times.

STEP 7: Assuming a specific AUM level, compute the implementation shortfall for each of the replicated trades (per Equation 3) and sum these values within each simulated day to construct a series of 25,000 daily shortfall values for the overall portfolio.

STEP 8: Construct a distribution of 25,000 annualized shortfall values by repeatedly drawing at random 252 observations (with replacement) from the sample constructed in STEP 7.

STEP 9: Assume a new level of AUM. Repeat STEPS 7 through 9 until annualized shortfall distributions have been produced across a wide range of AUM levels.

The sample of trades produced in STEPS 1 through 6 comprises a simulated track record. By construction, the simulated track record reflects the empirical track record in the following key respects⁴:

- > the distribution of trade sizes as a fraction of AUM reflects the distribution exhibited in the empirical sample,
- > the number of stocks across which turnover is distributed reflects the distribution exhibited in the empirical sample.
- > the distribution of turnover across market capitalization quintiles reflects the distribution exhibited in the empirical sample.

Next, in STEPS 7 through 9, we apply our cost model to this simulated track record to estimate annualized implementation shortfall as a function of AUM. The output of this process can be summarized in a chart like the one presented in Figure 1.

In the next section, we apply this methodology to a specific investment strategy and present results.

⁴ Here, we have chosen to replicate trades in a way that captures the distribution of the underlying strategy's trades across market capitalization quintiles. Depending on the specific case, one might wish to capture other characteristics, such as geographic region or style.

Data

We use constituent-level data on U.S. small capitalization (S&P 600), mid capitalization (S&P 400), and large capitalization (S&P 500) indices as well as their pure growth and pure value sub-indices. For each constituent of these indices, we download the following data from Datastream at a daily frequency from October 2005 through April 2008: price index, share price, daily volume traded in number of shares, and market capitalization.

We use the price index to compute the daily price return of each security and we multiply the share price and daily volume (expressed in number of shares) to compute the daily dollar volume traded. This enables us to compute the security price volatility and average daily volume for use in Equation 3.

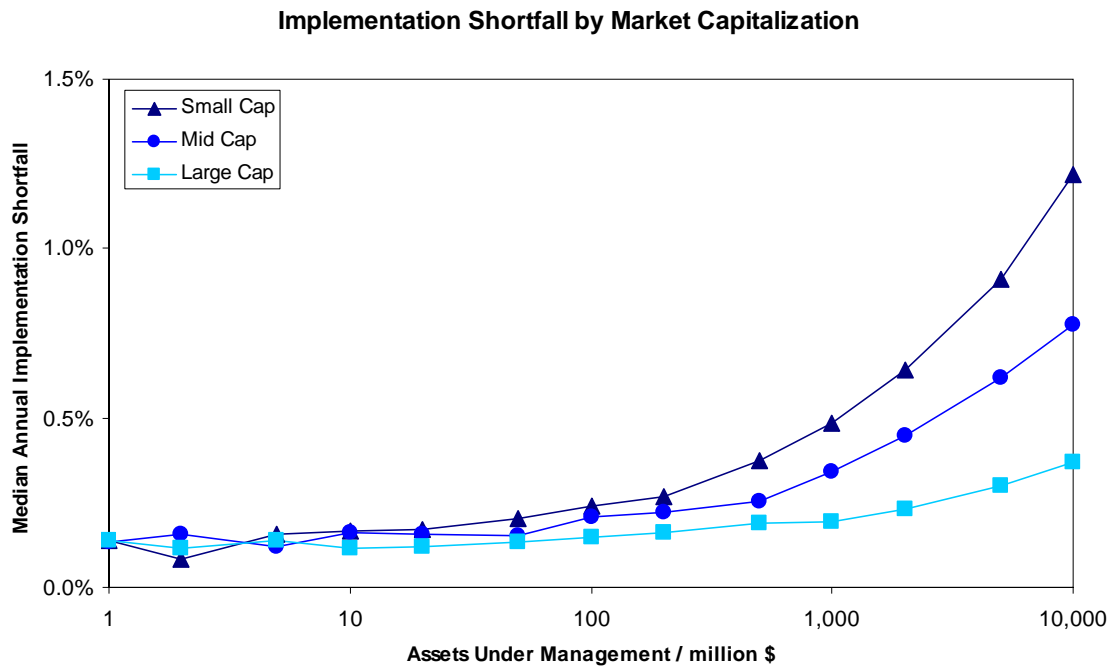
Results

We analyze the sensitivity of a strategy's annual implementation shortfall to various parameters. For each universe (as defined by market capitalization and style) and trade size, we simulate 25,000 trades as described above, with the following modification: in the absence of a specific (publicly available) manager turnover record to calibrate our simulation, we assume that all trades are of equal size and are distributed uniformly across days. We further assume that the probability of a given stock being selected is proportional to the market capitalization of that stock.

We begin by simulating a strategy with an annual turnover of 50%. For such a strategy, the sum of the values of all trades in a given year is 100% of AUM. Assuming all trades have an equal size of 0.20% of AUM, the sum of 500 randomly selected simulated trades adds up to the assumed annual turnover. In order to arrive at 25,000 estimates for annual implementation shortfall, we then draw 25,000 random samples of size 500 from the simulated cost vector (of length 25,000) and sum the cost values for each sample.

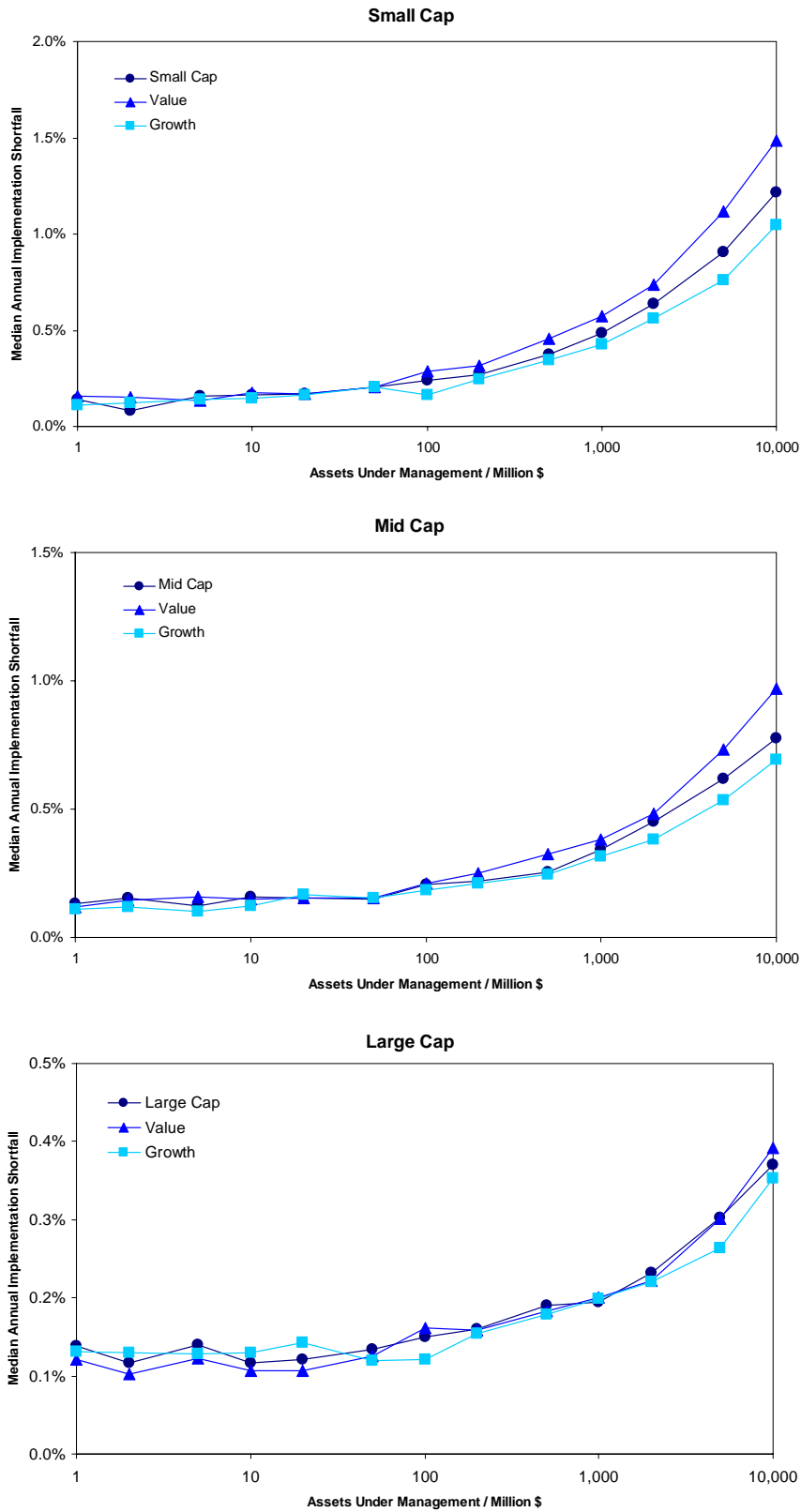
The results of this simulation for the three market capitalization universes are shown in Figure 2 below. We present the median of the simulated annual implementation shortfall values expressed as a fraction of assets under management.

Figure 2



We observe that for the same dollar amount traded, implementation shortfall increases as market capitalization decreases, an intuitive result. In Figure 3 below, we show the results of a simulation comparing the pure growth and pure value sub-universes of the small, mid, and large capitalization universes respectively.

Figure 3



Comparing the different styles for the small and mid capitalization universes, we observe that expected implementation shortfall is smaller for growth stocks than for value stocks. This effect is not readily apparent for large capitalization stocks. The following facts may help explain this phenomenon.

Growth stocks tend to have higher market capitalizations than value stocks. As of December 31, 2007, the median market capitalization for a small capitalization growth stock is 74% larger than for a small capitalization value stock. For mid capitalization and large capitalization stocks, these numbers are 50% and 28%, respectively.

In addition, growth stocks change hands more quickly than value stocks. Dividing market capitalization by dollar volume traded for each security on each day reveals that the median number of days for the entire market capitalization of a stock to change hands is 100, 100, and 126 for pure growth stocks in the small, mid, and large capitalization universes, respectively. The corresponding numbers for pure value stocks are 155, 146, and 155 days, respectively.

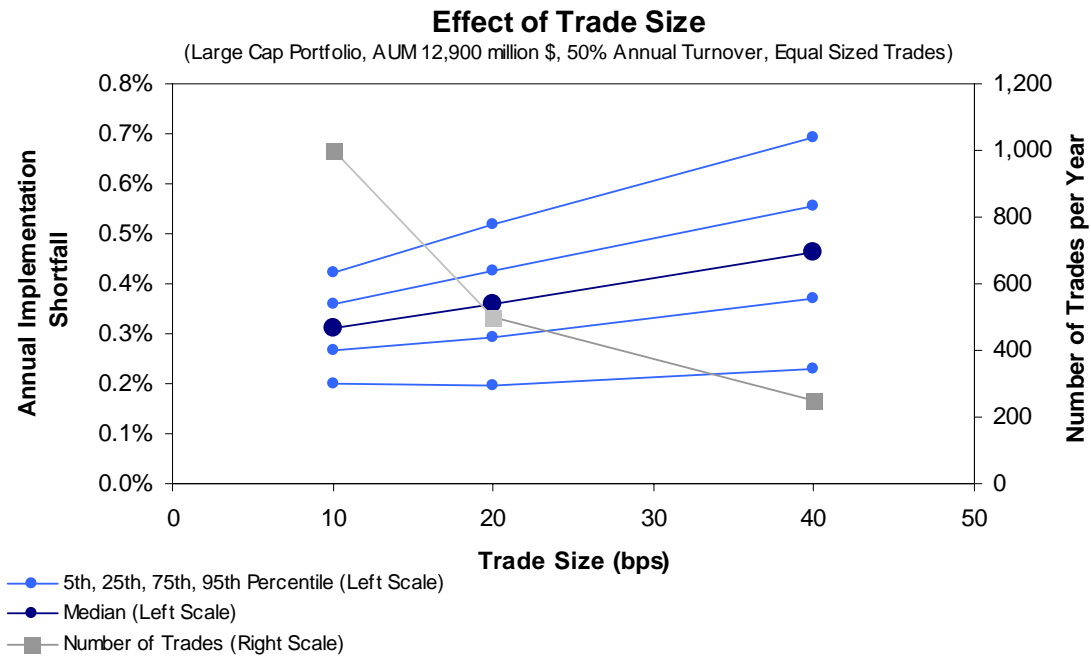
In addition to its market capitalization and style segment, a strategy's implementation shortfall is determined by its total turnover and the distribution of that turnover across trade sizes. All else equal, implementation shortfall increases linearly with turnover. The annual turnover of a typical value fund is smaller than that of a typical growth fund (their approximate median values are 40% and 70%, respectively).⁵ The contribution of the individual trade size is less obvious. We find empirically that the trade size distribution of actual turnover records exhibits positive skewness: it contains a large fraction of small trades and fewer large trades.

We explore this effect for a large cap strategy with assets under management of 0.1% of the market capitalization of the universe (12.9 billion USD). We simulate three different strategies, each with the same level of AUM, the same percentage turnover of 50%, and equal trade sizes across all trades. The strategies differ from each other only in the size of the individual trades. As percentage turnover is held constant, the total number of trades varies from strategy to strategy. We show the results of this simulation in Figure 4 below. In addition to the median, we show the 5th, 25th, 75th, and 95th

⁵ Based on an analysis of over 1,000 funds from each category in the Morningstar fund database.

percentile of the annual implementation shortfall distribution. On the right-hand axis of the graph, we show the number of trades per year corresponding to a given trade size.

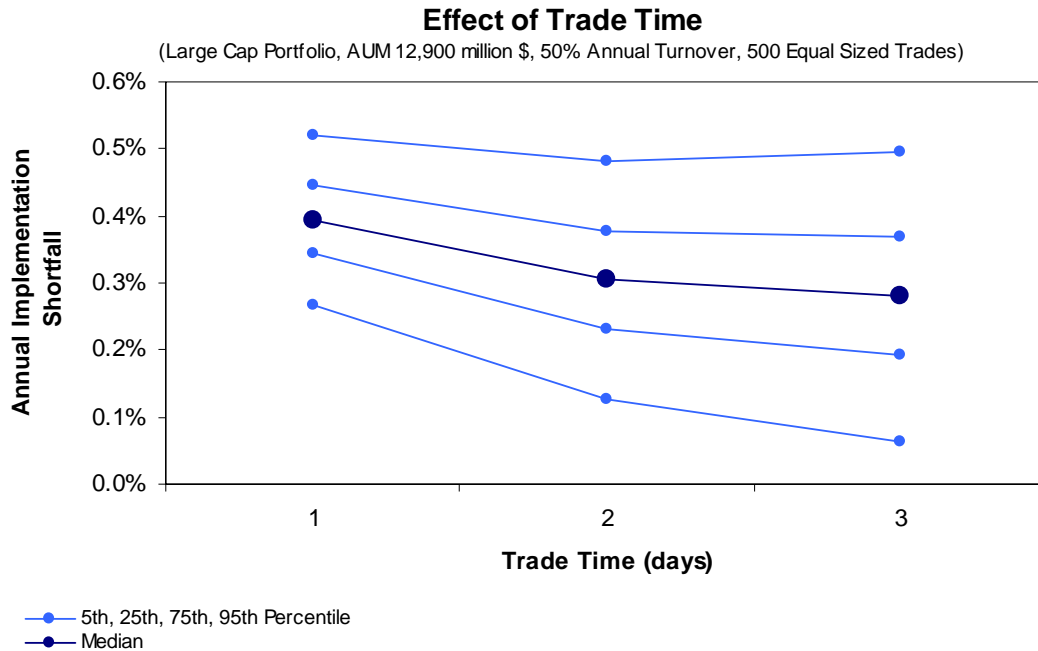
Figure 4



All else equal, increasing the trade size increases median implementation shortfall as each individual trade constitutes a larger fraction of a given security's average daily volume. Increasing trade size and thus decreasing the number of trades per year (at constant turnover) also increases the range of the distribution: an annual implementation shortfall constitutes a smaller sample of cost numbers from the same population, resulting in more dispersion in outcomes and a larger standard deviation.

As AUM increase, or as a strategy calls for implementing larger individual trades, a manager may choose to take steps to decrease market impact by spreading out a given trade over a few days. The results of the corresponding simulation are shown in Figure 5 below.

Figure 5



As Figure 5 shows, one result of an increase in trade time is a decrease in the median of the annualized implementation shortfall distribution, caused by the reduction in market impact of each trade. Another consequence is an increase in the range of the distribution as the longer time to implement the trade means that a security's price will be subject to larger fluctuations over the longer trade time due to opportunity cost.

To analyze the capacity of a given active strategy in practice, we use that manager's actual historical turnover record. The simplified examples above illustrate the sensitivity of implementation shortfall (and by extension, capacity) to various parameters. Given an assumption for the strategy's gross alpha, we can now estimate the relationship between excess return and AUM. This is illustrated in Figure 6 below for the mid capitalization value strategy presented in Figure 3. We assume that the strategy has a gross alpha (on paper) of 1.5% p.a. and a performance objective of 1.0% p.a..

Figure 6

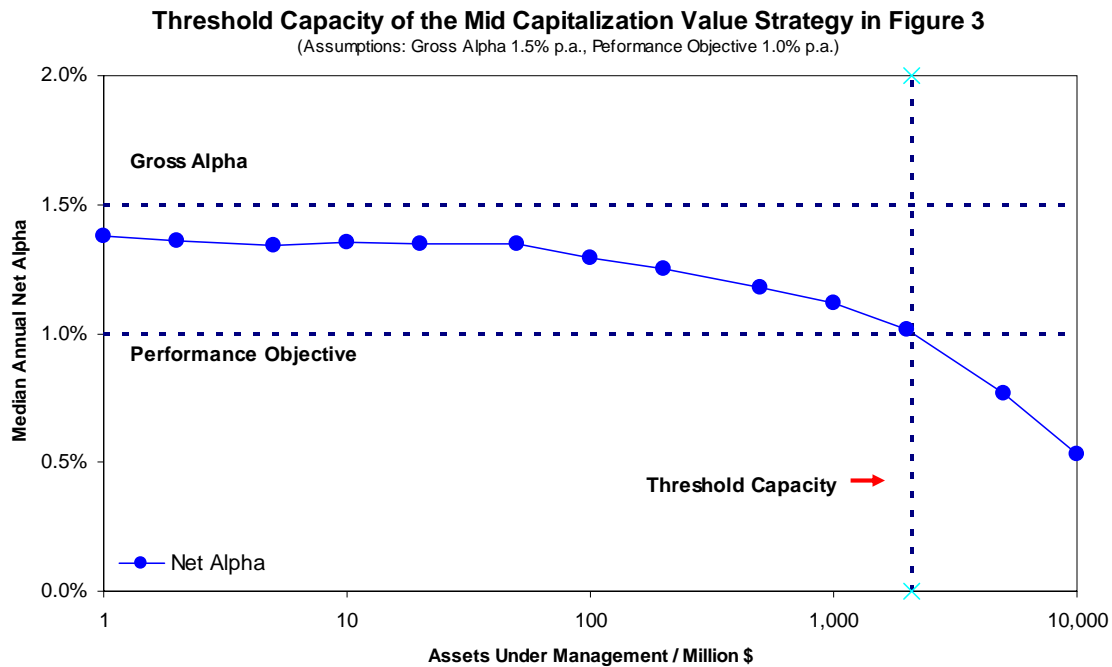


Figure 6 indicates that we would estimate a threshold capacity of slightly in excess of \$2 billion for this strategy.

Conclusions

We present a simulation methodology to analyze the relationship between AUM and annual transaction cost for a specific investment strategy. Understanding this relationship enables us to estimate the capacity of the strategy. Unlike previous approaches to capacity estimation, our methodology accommodates non-quantitative strategies that cannot be backtested. We explicitly model both the market impact and the opportunity cost components of implementation shortfall. Our approach recognizes the stochastic quality of implementation shortfall, which is the primary determinant of product capacity. We illustrate our approach using a simplified set of assumptions for an active management strategy. All else equal, we find that value strategies have lower capacity than growth strategies in the small and mid capitalization segments. We also illustrate the sensitivity of capacity to trading aggressiveness and the trade size distribution.

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