# „Country Risk-Indicator. An Option Based Evaluation“

## Implicit default probabilities of foreign USD bonds

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## 1 Motivation

The interest rates of USD bonds issued by risky debtor nations such as Brazil are much higher than those issued by solvent countries such as the USA.  The spread - the difference between the risky interest rate and the secure interest rate - is widely interpreted as a measure of the risk or the probability that the debtor nation will not be able to repay the debt. Usually high spread indicates high risk. But the measurement of default risk of a particular bond is only valid in relation to the spreads of other debtor nations. There can only be made qualitative statements as *„nation A is more risky than nation B“*.

In the ideal world of risk neutrality the spread is equal to the expected value of losses of one unit of credit. Thus, if the loss distribution is known it is possible to compute the default probability directly using the spread. However, risk neutral markets do not exist in the world. Thus, the spread includes the aspect of expected losses as well as an additional risk premium. In order to compute the default probabilities in the risk averse markets, the loss distributions and the utility functions of the participants in the markets must be known. But this is a rather unrealistic assumption as the assumption of risk neutrality is. That means that an exact measurement of default risk based solely on the spreads is not possible in a direct way.

In this paper we use an option pricing model in combination with basic economic data of a debtor nation to develop a model for computing the default risk. Our model clearly shows that high spreads do not necessarily mean high default risks.

# 2 The Model

## 2.1 Concept

The Arbitrage Pricing Theory predicts that if it were possible to create secure portfolios the earnings of those secure portfolios would be equal to the riskless interest rate. If there would exist an insurance against default of one unit of risiky bond, it is clear that a portfolio including both, one risky bond and one insurance contract of the above type, would generate the same earnings as the riskless bond would do – otherwise arbitrage would be possible. In the case of a zero bond this would mean that the difference of the bond prices would have to be equal to the price of the insurance contract $P_{ins}$ against the default risk. This is satisfied by equation (1):

$$B^{zero}_{riskless} - B^{zero}_{risky} = P_{ins} \tag{1}$$

When a nation is able to repay its debt only when its central bank owns a sufficent amount of foreign currency reserves, a put option on the reserves of the foreign currency K can be used to insure the investor against default of a risky bond. As we are interested only in repay abilities at the expiration date of the risky bond, we consider here an European style put option having the same expiration date t* as the risky bond. The strike price S of the put option must be the cumulative repayment requirement of the debtor nation until the expiration date of the bond.[1] We write Put($K^{t*}$, S) for the value of this option. It insures the sum of all foreign debts of the nation with same or earlier expiration date against default but the buyer of one bond owns only a part $\alpha$ of the foreign debts. Therefore, a buyer has to hold only the part $\alpha$ of the put option. Now equation (1) can be written as

$$B^{zero}_{riskless} - B^{zero}_{risky} = \alpha \cdot \mathrm{Put}\left(K^{t*}, S\right) \quad \text{with} \quad \alpha = \frac{B^{zero}_{risky} \cdot \left(1 + r^{eff}_{risky}\right)}{\sum \mathrm{Debts}^{t \le t*}} \tag{2}$$

[1] The strike price should be increased by the minimum reserves the nation needs for survival (for imports of oil, food, medicine) as the debtor nation will not spend all of its reserves for credit repayment in the case of default.

The price of the put option is computed by the put option formula[2] which is analogeous to the call formula of the Black-Scholes model:

$$\mathrm{Put} = e^{-i\cdot t}\cdot S\cdot N\left(\frac{\ln\left(\frac{S}{K^0}\right)-\left(i-\frac{\sigma^2}{2}\right)\cdot t}{\sigma\sqrt{t}}\right)-K^0\cdot N\left(\frac{\ln\left(\frac{S}{K^0}\right)-\left(i+\frac{\sigma^2}{2}\right)\cdot t}{\sigma\sqrt{t}}\right) \tag{3}$$

with $S$ = sum of the foreign debts with same or ealier expiration[3]
$K^0$ = foreign currency reserves of a debtor nation
$i$ = interest rate (in logarithmic form) of the riskless bond with the same maturity
$t$ = time until maturity of the bond in years
$\sigma$ = implicit volatility of the foreign currency of a debtor nation (endogeneously determinated by relation (3))

Now we have two formulas to compute the price of the put option: the market price of equation (2) and the price of the option model of equation (3). Since the price of the put option in equations (2) and (3) is the same, we can use these equations to compute the implicit volatility $\sigma$ of the debtor nation´s foreign currency reserves. Since the inverse function of the density function of the normal distribution is unknown, the computation of $\sigma$ is possible through approximation only, using an iteration algorithm.

In the world of the Black-Scholes model, the parameter $\mu$, which is the mean rate (in logarithmic form) of the underlying´s growth, has no influence on the option price.[4] Due to this, the spread between the riskless and the risky bond must be independent of the parameter $\mu$. Thus, the spread measures only the volatility $\sigma$ of the reserves of foreign currency, or more generally, the volatility of the debtor nation´s ability to repay the bond. The spread is not able to measure the risk of the bond in the sense of a default probability. In order to be able to compute the default probability we need to know the parameter $\mu$ as well.

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2 Derivation of equation (3) is possible in two alternative ways. Derivation analogeous to the derivation of the call option price formula of Black-Schooles or the derivation that uses the call option pricing formula of Black-Scholes and the put call parity theorem.

3 including debt repayments and interest payments

4 Due to Ito´s lemma the stochastic process in the hedge portfolio is cancelled out to create a secure portfolio. This causes the loss of information on $\mu$ because $\mu$ is cancelled out. The information on $\sigma$ remains because $\sigma$ is part of the partial derivatives of the option price.

Foreign currency reserves in the future are equal to today´s reserves plus exports (EX) minus imports (IM).[5] Thus we can compute the mean of foreign currency reserves[6] using the expected exports[7] and imports:

$$E(K^t) = K^0 + E(EX) - E(IM) \tag{4}$$

The world of the Black-Scholes model implies that the reserves are distributed log-normally. Therefor we can compute the mean as:

$$K^0 + E(EX) - E(IM) = E(K^0 \cdot e^x) \qquad \text{with } x \sim N(\mu,\sigma^2) \tag{5}$$

Using the mean of the log-normal distribution we can write equation (5) as:

$$K^0 + E(EX) - E(IM) = K^0 \cdot e^{\mu + \frac{\sigma^2}{2}} \tag{6}$$

As we already have computed σ as the implicit volatility we can solve equation (6) and compute μ as:

$$\mu = \ln\left(\frac{K^0 + E(EX) - E(IM)}{K^0}\right) - \frac{\sigma^2}{2} \tag{7}$$

Knowing the parameter μ and σ of the distribution we can compute the probability that the debtor nation will not own a sufficent amount of foreign currency to repay its debts:

$$P(K^t < S) = P(K^0 \cdot e^x < S) \qquad \text{with } x \sim N(\mu,\sigma^2) \tag{8}$$

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5 In addition, one has to account for net capital inflows. As our empirical application will focus on emerging markets, net capital imports are set equal to zero, which reflects high capital mobility in the countries under consideration.

6 The interest payments and the debt repayment are not part of the mean of the reserves as they have already been included in the strike-price S.

7 For practical use some banks take only the worst case E(EX) = 0 as does Polanski (forthcoming).

This can be simplified to

$$P\left(K^{t} < S\right) = P\left( x < \frac{\ln\left(\frac{S}{K^{0}}\right) - \mu}{\sigma} \right) \qquad \text{with } x \sim N(0,1^{2}) \tag{9}$$

Equation (9) represents the market ´s perception on the default probability of the debtor nation.

## 2.2 Standardization

The ability to compute the default probability of only one debtor nation does not deliver much new information. More interesting for an investor is to be able to compare the default probabilities of different debtor nations. In order to be able to do this, the USD bonds of the nations the investor is interested in must have the same expiration date. But this will almost never be the case: for a great number of nations, there exists just one issue of USD bonds. In order to be able to compare different debtor nations we normalize the default probabilities to a one year horizon. Now we can compare the probabilities that the debtor nations default within the next year, using the assumption that the risk premium – measured as spread of the riskless and risky bond with the same maturity[8] – is independent of the time until the bond expires. This allows to transform the bonds into artificial zero bonds with the same effective interest rate $i^{eff}$ as the original bonds, thereby having a one-year time maturity period and a repayment value of one USD. To account for the expiration date of one year, equation (1) must be rewritten as:

$$\mathrm{Put}\left(\frac{K^{0}}{S}, 1\right) = \exp\left(-\ln\left(1 + i_{secure}^{eff}\right)\right) - \exp\left(-\ln\left(1 + i_{risky}^{eff}\right)\right) \tag{10}$$

[8] The assumption of equal expiration dates of the risky and the riskless bond eliminates problems caused by the term structure of interest rates.

We set t equal to one in all equations used. Due to the standardization to one year and one USD repayment value the relevant part $\alpha$ of foreign debt ist equal to 1/S. The price to insure all outstanding debt S of same or earlier expiration dates, is equal to the price of the put option on one USD multiplied by S.

$$\mathrm{Put}\left(K^0, S\right) = S \cdot \mathrm{Put}\left(\frac{K^0}{S}, 1\right) \tag{11}$$

## 2.3 An Example

The workings of the model will now be demonstrated drawing on an example of USD bonds issued by Argentina (ID#: 004785428) and by Ecuador (ID#: 007562128). We use the rates of 01/19/1999 as a convenient date where shocks have been absent.

The data of the bonds and the basic economic data of both nations are as follows:

| | Argentina | Ecuador |
|---|---|---|
| coupon | 8.75 | 11.25 |
| market price | 90.0844 | 77.5000 |
| maturity | 12/20/2003 | 02/25/2002 |
| $i^{eff}{}_{risky}$ | 11.04 % | 21.18 % |
| $i^{eff}{}_{secure}$ | 4.58 % | 4.58 % |
| foreign debt | 104,539 Mio. USD | 15,941 Mio. USD |
| dept repayment | 6,969 Mio. USD | 638 Mio. USD |
| interest payment | 6,454 Mio. USD | 703 Mio. USD |
| total payment S | 13,416 Mio. USD | 1,341 Mio. USD |
| reserves of foreign currency $K^0$ | 25,470 Mio. USD | 1,743 Mio. USD |
| exports EX | 29,318 Mio. USD | 5,700 Mio. USD |
| imports IM | 34,899 Mio. USD | 5,510 Mio. USD |

The difference of the riskless and the risky one-year-zero bonds with a face value of one USD, has to be equal to the price of the put option on one USD foreign debt (see eq. (10)), which results in:

$$Put_{Argentina}\left(\frac{K^0}{S},1\right)=\exp(-\ln(1+4{,}58\%))-\exp(-\ln(1+11{,}04\%)) \ = 0.0556$$

$$Put_{Ecuador}\left(\frac{K^0}{S},1\right)=\exp(-\ln(1+4{,}58\%))-\exp(-\ln(1+21{,}18\%)) \ = 0.1311$$

The price of the put option on the total foreign debt that has to be repayed within the next year is (see eq. (11)):

$Put_{Argentina}(K^0, S)$ = 0.0556*13,416 Mio. USD = 746 Mio. USD
$Put_{Ecuador}(K^0, S)$ = 0.1311* 1,341 Mio. USD = 176 Mio. USD

The prices of the put options in equation (3) are equal to the observed market prices if the volatilities satisfy:

$\sigma_{Argentina}$ = **56.17 %** $\sigma_{Ecuador}$ = **61.10 %**

The parameters μ in equation (6) can be computed as:

$$\mu_{Argentina} = \ln\left(\frac{25{,}470+29{,}318-34{,}899}{25{,}470}\right)-\frac{0.5617^2}{2} \quad = \quad \underline{\mathbf{-\ 43.84\ \%}}$$

$$\mu_{Ecuador} = \ln\left(\frac{1{,}743+5{,}700-5{,}510}{1{,}743}\right)-\frac{0.6110^2}{2} \quad = \quad \underline{\mathbf{-\ \ 8.32\ \%}}$$

The probabilities that the debtor nations will not have sufficient foreign reserves to repay their foreign debts can be computed by using equation (8):

$$P(K^t < S)_{Argentina} = P\left(x < \frac{\ln\left(\frac{13{,}416}{25{,}470}\right)-0.4384}{0.5617}\right) \quad = \quad \underline{\underline{\mathbf{43.52\ \%}}}$$

$$P(K^t < S)_{Ecuador} = P\left(x < \frac{\ln\left(\frac{1{,}341}{1{,}743}\right)-0.0832}{0.6110}\right) \quad = \quad \underline{\underline{\mathbf{38.46\ \%}}}$$

## 3 Conclusion

The model clearly shows that the difference between the risky and the secure interest rate is not a relyable indicator of a bond´s default risk. In our example, the bond of Ecuador has a spread that is more than twice as high as the spread of the bond of Argentina. Nevertheless, the bond of Ecuador has a more than 10 % lower default risk.

In the option-based model, the spreads indicate only the volatility of the foreign currency reserves, but not the default risk. Ecuador is a less developed economy with few, although diversified exports (food, fruits, bananas, minerals, oil). Due to the small size of its economy Ecuador is exposed to a higher risk than Argentina. The experiences of the last decade (decrease of the oil-price, boycott of Ecuadorian bananas by the EC) seem to have forced the government of Ecuador to insure against such external risks in form of high foreign currency reserves. Ecuador owns – as percentage of GNP - about 30 % more foreign reserves than Argentina. This decreases the default risk of the foreign debt of Ecuador to a lower level than the corresponding risk for Argentina despite of higher volatility.

Our model, of course, is only valid if the Black-Scholes formula is applicable for computing the true option price. Mistakes will arise when the parameter $\mu$ is found to have an influence on the option price or when the option price bears a risk premium. The latter may arise in cases where the Delta-arbitrage process is not performed due to lack of profitability. But, both cases are thought of as highly implausible by almost all participants in the option markets.

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