OPTIMAL CATASTROPHE INSURANCE

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ABSTRACT

This paper adopts a normative approach to catastrophe insurance. It addresses the question of how innovation in the design of insurance contracts could help resolve the capacity gap in the provision of insurance against natural catastrophes. The innovation allows for more flexibility in risk retention for the consumer, thus improving consumer welfare. By decomposing the influence of catastrophes on claims amounts (severity risk) and on the probability of loss (frequency risk), we show how insurance contracts can be designed to endogenize the degree of risk sharing between groups of insureds. In particular, we examine how insurance can facilitate direct risk sharing between groups of insureds who are exposed to different catastrophes.

1. Introduction

Past decades have shown an increasing severity and frequency of losses arising from natural catastrophes: earthquakes, hurricanes, floods and large-scale fires[1]. It is still controversial whether an increasing frequency of hurricanes and floods may be attributed to climate change (global warming), but it is clear that concentration of values in catastrophe-prone coastal areas has brought about an increase in the amount of damages. From 1970 to 1990, population in the Pacific and South Atlantic coastal states of the United States increased by 51% and 45% respectively, compared to a countrywide increase of 24% over the same period. This evolution is a source of concern for the insurance industry, because the wealth elasticity of insurance demand is empirically larger than one[2]. Thus, rapid increases in exposed wealth mean that insured losses represent an increasing proportion of total losses. For example, one of the most severe catastrophes over the past years was the Kobe earthquake in 1995, which caused losses to an amount of USD 82.4 billion, but insured losses remained at a more modest 2.5 billion. This is relatively small compared to the series of losses which were inflicted upon the insurance industry since 1988. Insurers had to pay USD 12.5 billion for the Northridge earthquake (1994) and 16 billion for Hurricane Andrew (1992), more than 40 percent of the combined total cost for these two events, estimated at USD 65 billion.

Prior to Andrew, the industry had not anticipated such high damage values. Indeed, the financial press was saturated with stories of astonishment following Hurricane Hugo

[1] The insurance industry defines a catastrophe as "an event which causes in excess of $5 million in insured property damage and affects a significant number of insureds and insurers" (Cummins and Geman, 1995).

[2] See *Sigma* (No 4, 1997): The rich countries of the world are by far the most largely insured. This may seem in contradiction with the well-known theoretical result that property insurance is an inferior good (Mossin, 1968). But, as noticed by Chesney and Loubergé (1986), this result is obtained on the assumption that increases in wealth affect the non-risky portion of wealth only.

in 1989, which cost the industry over USD 5 billion. Ever since Hugo however, insured losses in excess of 1 billion have become the rule rather than the exception (see exhibit 8 in Canter *et al*. 1996).

Catastrophic losses challenge the economic role of insurance as a private wealth redistribution mechanism. Insurance makes possible the transfer of numerous risks. It is a mechanism whereby insurers collect funds from many agents exposed to similar risks, to pay for losses that will randomly affect some of these agents. The reinsurance mechanism complements direct insurance by allowing a world-wide diversification of risks. In addition, the financial capacity of the insurance industry has been able to absorb deviations of total losses from their expected value. However, financial capacity has been outpaced by potential losses in the catastrophe lines. The total capital of the US property-casualty insurance industry is estimated at USD 200 billion, of which 20 billion provided by reinsurers (Kielholz and Durrer, 1997). The coverage capacity in the catastrophe line of business (direct insurance and reinsurance) is estimated at USD 25 billion. This is less than the reference losses estimated by reinsurers at 50 billion for California earthquakes and 45 billion for East Coast storms, and this is well below the maximum losses expected from these two kinds of events: 100 billion for California earthquakes and 85 billion for East Coast storms.

The problem arises because the risk of natural catastrophes is not widely diversifiable in an insurance context where insurers supply coverage in well-defined business lines. Natural catastrophes tend to occur in selected areas of the globe: seismic regions and ocean coasts. Moreover, only a subset of these regions expresses much demand for insurance coverage[3]. Thus, reinsurers are not able to disseminate the risk

[3] Figures published in *Sigma* (No 4, 1997) show that North America, Western Europe and Japan make up 90 percent of the global non-life insurance premium income.

easily across the world, and cross-subsidization among different lines of business is not feasible in a competitive environment.

Two types of solutions to the insurance capacity gap have been proposed and put into practice. Mandatory public provision of insurance is one alternative. It relies on the financial and fiscal ability of the government to spread losses across many citizens, as well as intertemporally. This was imposed in France, where all insureds pay an additional premium on their property-liability insurance contracts in exchange for coverage against natural catastrophes, with a reinsurance guarantee provided by the State[4]. Risk securitization represents the second alternative. It relies on the huge pool of financial capacity provided by asset markets. For example, total capitalization of the US financial market amounts to approximately USD 20 trillion, with a daily standard deviation of around USD 130 billion. Thus, typical daily fluctuations in total US asset market capitalization are able to cover the maximum probable loss from a California earthquake. Risk securitization is accomplished by issuing specific conditional claims and selling them directly to financial investors. Options on natural catastrophes (*cat spreads*) started trading at the Chicago Board of Trade in 1995, and catastrophe-linked bonds (*cat bonds*) have been issued since 1997[5].

A third solution, which may be combined with risk securitization, is risk mutualization. Economic theory teaches that losses that cannot be diversified away in a portfolio of risks or an insurance-reinsurance pool should be shared by economic agents according to their respective risk-tolerance. This is the *mutuality principle* due to Karl Borch (1962). Large-scale mutualization already occurs through the financial market, via

[4] See Magnan (1995). See also Lewis and Murdock (1996) for a proposal of more government intervention in the coverage of US natural catastrophes.

[5] The developments in catastrophe risk securitization were analyzed in Niehaus and Mann (1992), Doherty (1997) and Jaffee and Russell (1997). Catastrophe options were priced and analyzed by Cummins and Geman (1995). Catastrophe-linked bonds were priced and analyzed by Loubergé et al. (1999).

widely held insurance stocks and via risk securitization. But a first layer of mutualization may also be organized within the insurance market by having the insureds share in the operational result of their insurers. As argued by Doherty and Dionne (1993), such an arrangement is particularly appropriate when the risk is partially undiversifiable: decomposition of the risk into idiosyncratic and nonidiosyncratic risk with separate allocations of the components leads to some degree of mutualization (see also Schlesinger, 1999). Under this scenario, risk mutualization occurs as a combination of three processes, instead of two: first, among insureds in an insurance company, second, among insurers using reinsurance, and third, among the wider population of economic agents, using the financial market to diversify the residual risk.

This paper concentrates on the first process. It extends the models proposed by Doherty and Schlesinger (1998), who allow for more flexible risk sharing between the insurer and the insured. We allow further flexibility by allowing risk sharing between different pools of insureds. Although catastrophes are defined as an accumulation of claims, catastrophic events are largely uncorrelated: hurricanes in Florida are uncorrelated with earthquakes in California or Japan, with tropical storms in Hawaii, and with floods in Italy or Germany. This is the *raison d'être* of catastrophe reinsurance. This observation may be used to improve the financial capacity within the insurance market by allowing the insureds to participate in the overall risk of their insurer. Thus, a Californian insured would be able to increase her utility, if instead of only purchasing an insurance contract priced according to Californian earthquake risk, she would be given the opportunity to purchase a contract based on the portfolio of her insurer's Californian earthquake and Floridian hurricane risks.

Our model is presented in the next two sections and the optimal insurance contract is derived in section 4. The model is initially based on the assumption that natural catastrophes affect the severity of losses, but not their frequency in the insured population. Frequency risk is then introduced in section 5. A more complete model, combining simultaneous frequency and severity risk is developed in section 6. The last section concludes.

2. The model

Consider two geographic regions (or two groups of insureds), A and B, facing independent risks of being struck by a catastrophe, e.g., a hurricane in Florida and an earthquake in California. Let (Ω, F, μ) be a probability space and let $L{:}\Omega \to R_+$ and $\varepsilon_j{:}\Omega \to R$ for *j* = (*A*,*B*) be well-defined random variables.

Each region is populated with a continuum of individuals. We assume that these individuals are identical in both regions: same initial wealth $W > 0$, same preferences and attitudes towards risk[6]. In the absence of catastrophe, each individual faces the prospect of losing a random amount *L*, with expected value *EL*. Losses *L* are i.i.d., and insurable at zero transaction cost. However, in each region, the catastrophic event may result in an "inflation" of claims, brought about by the simultaneity of losses (*severity risk*). In region *j* (*j* = A,B), the random loss faced by individual *i* becomes $L_{ij}(1+\varepsilon_j)$, where ε is random and is region-specific (i.e., the same for all individuals in the region). Without loss of generality, we assume $E(\varepsilon_j) = 0$. Moreover, we assume that the random catastrophic component ε is independent of *L* for all individuals, and that the catastrophes occurring in regions *A* and *B* are independent of one another: $E(L_{ij}\varepsilon_j) = 0 = E(\varepsilon_A \varepsilon_B)$, for all *i* and *j*. To avoid complications of personal bankruptcy, we assume that the distribution for the

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[6] The individuals are not necessarily expected utility maximizers. Individuals are risk averse, by which we mean they are averse to mean-preserving spreads of the wealth distribution. We will assume that they display second-order risk aversion, in the sense of Segal and Spivak (1990).

random variable $L(1+\varepsilon_j)$ has a support which is bounded between zero and W. Finally, to avoid negative loss amounts, we assume $\varepsilon_j > -1$. Thus, for example, if $\varepsilon_A = .15$, all losses in region A are 15 percent higher, whereas $\varepsilon_B = -.10$ would indicate that all losses in region B are 10 percent lower.

We need to caution the reader that $\varepsilon = 0$ does not characterize the no-catastrophe case. Indeed, although we consider a mean value of zero for ε, this includes catastrophe years. Thus, we would expect a modal value for ε that is negative.

Competitive insurers provide damage insurance in the two regions. Insurance is assumed to be proportional, with coverage $\alpha \in [0,1]$ chosen by the insured. Three types of contracts are available, and the insured is free to combine the three types, with the same α :

1. Fixed premium contract. In this contract, the insurer retains the catastrophic component of the risk by charging the insured a fixed premium based on the expected value of losses. This risk is then either assumed, or reinsured, or hedged, or securitized (or a combination of these actions is chosen) against the payment of a risk premium λ, the same for each type of catastrophe. Thus, although the catastrophe risk is statistically uncorrelated with market risk, we assume that a risk premium is required to compensate shareholders and/or financial investors, due to imperfections in the market, such as agency costs or asymmetric taxes[7]. This risk premium is passed through to the policyholders as a loading. The fixed premium in region j is thus:

$$P_j^f = \alpha_j (1+\lambda) E\left[(1+\varepsilon_j) L\right] = \alpha_j (1+\lambda) EL.$$

[7] See Garven & Loubergé (1996) or Eeckhoudt, Gollier & Schlesinger (1997). A risk premium may also be justified in the presence of parameter uncertainty (see Hogarth & Kunreuther, 1992).

2. Variable premium contract. In this contract, the insurer does not assume the catastrophic risk of its policyholders. The insured pays an initial premium equal to the expected value of losses. An *ex post* adjustment in the premium then occurs to take into account the actual severity of losses within her group (A or B), so that the premium is initially random. Assuming zero interest rates, the variable premium in region *j* is thus:

$$P_j^v = \alpha_j (1 + \varepsilon_j) EL.$$

3. Participating premium contract. With this contract, the insurer acts as a mutual insurer. The insured shares in the more or less favorable loss experience of her insurer. She pays initially the expected value of her losses. An *ex post* adjustment in the premium occurs eventually, depending on the overall result of the insurer. Again, the premium is initially random and is defined as $P_j^p = \alpha_j (1+m) EL,$ where *m* is a random element which is a weighted average of ε_A and ε_B, and is defined in the next section.

We assume that the risk premium, λ, is positive due to the size of the catastrophic risk. Although the insurer can "pass off" this risk, it cannot do so without a risk loading charge, since the catastrophic risk cannot be fully diversified. The variable-premium and participating-premium contracts impose no systematic risk on the insurer, and we (perhaps boldly) assume the risk-loading charge is zero. As long as their risk-loading charges are less than λ, we can modify the model to obtain similar qualitative results.

The variable and participating premiums allow the insured to share in the catastrophe risk, either locally, or more widely. In principle, such risk sharing could also occur using the financial markets by having the insured purchase shares of the insurer's equity[8]. However, equity prices are more comprehensive and more forward-looking. This risk sharing also could occur using the derivatives markets for one's own personal

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[8] This is called "homemade mutualization" by Doherty and Dionne (1993).

account, for instance by writing *CBOT* cat call spreads on the Florida index, or the California index, or both. The insured would then pay a fixed insurance premium, with a loading λ, and would get back part of this loading by writing options. Alternatively, the catastrophic risk could be excluded from coverage, and the individual would be forced to obtain the desired coverage against this risk by hedging on the derivatives market. However, this would involve retention of the idiosyncratic part of the risk by the individual. The individual has essentially a random level of the ε -risk and thus cannot easily hedge the overall risk level of $L(1+\varepsilon)$. By offering a menu of contracts, the insurer acts as a financial intermediary, taking advantage of lower (or zero) information and transaction costs, as well as "pooling" the levels of ε -risk.

The combined insurance premium is *P*, defined as $P = \beta^0 P^f + \beta^1 P^v + \beta^2 P^p$, with $(\beta^0, \beta^2, \beta^3) \in S^2$, the unit simplex on $\Re^3$. The insured has the choice of α and of every β^i.

3. Definition of *m*.

The factor *m* affecting the participating premium is endogenous. It depends on the share of the catastrophe risk remaining with the insurer once all insureds have decided how to combine the three kinds of contracts.

Consider the situation in region *A*.

- A fraction $\alpha_A \beta_A^0$ of risk ε_A is assumed by the insurer and does not trigger any recovery or allowance. It is compensated by the risk premium λ.

- A fraction $\alpha_A \beta_A^1$ of risk ε_A is assumed by the insureds in region *A*. This fraction leads to a recovery of $\alpha_A \beta_A^1 \varepsilon_A EL$.

- Assuming the same number n of individuals in both regions, a fraction $[\alpha_A\beta_A^2 / (\alpha_A\beta_A^2 + \alpha_B\beta_B^2)]$ of the remaining (ε_A + ε_B) risk is assumed by the insureds in region A. This fraction leads to a recovery of $\alpha_A\beta_A^2 mEL$.

The remaining (ε_A + ε_B) risk is $\alpha_A\varepsilon_A(1-\beta_A^0-\beta_A^1)+\alpha_B\varepsilon_B(1-\beta_B^0-\beta_B^1)$. Thus,

$$m = \frac{\alpha_A\varepsilon_A(1-\beta_A^0-\beta_A^1)+\alpha_B\varepsilon_B(1-\beta_B^0-\beta_B^1)}{\alpha_A\beta_A^2+\alpha_B\beta_B^2}$$
$$= \frac{\alpha_A\beta_A^2\varepsilon_A+\alpha_B\beta_B^2\varepsilon_B}{\alpha_A\beta_A^2+\alpha_B\beta_B^2}$$

As it turns out, m represents the unit of overall risk remaining to be shared. It may also be shown that this value of m leads to a fair amount of technical loss/profit for the insurer, given the values of β^0 in each region. Assuming the same number of policyholders in both regions, the total loss/gain after recoveries or allowances is $nEL\{\alpha_A\varepsilon_A(1-\beta_A^1)+\alpha_B\varepsilon_B(1-\beta_B^1)-m(\alpha_A\beta_A^2+\alpha_B\beta_B^2)\}$ Plugging the above value of m in this expression yields the fair loss/gain incurred by the insurer:
$nEL\{\alpha_A\varepsilon_A\beta_A^0+\alpha_B\varepsilon_B\beta_B^0\}$

4. Optimal insurance

4.1. Optimal coinsurance

Using the canonical model of optimal insurance purchasing under a proportional contract, the final wealth of the representative individual in region A is written as:

(1) $\quad Y_A = W - P_A - (1-\alpha_A)(1+\varepsilon_A)L$

with:

$$P_A = \alpha_A\{\beta_A^0(1+\lambda)EL+\beta_A^1(1+\varepsilon_A)EL+\beta_A^2(1+m)EL\}$$
$$= \alpha_A EL\{1+\varepsilon_A+\beta_A^0(\lambda-\varepsilon_A)+\beta_A^2(m-\varepsilon_A)\}$$

Rearranging yields:

(2) $$Y_A = (1+\varepsilon_A)\{W - \alpha_A EL - (1-\alpha_A)L\} - \{\varepsilon_A W + \gamma_A^0 EL(\lambda - \varepsilon_A) + \gamma_A^2 EL(m - \varepsilon_A)\}$$

where $\gamma_A^0 = \alpha_A \beta_A^0$ and $\gamma_A^2 = \alpha_A \beta_A^2$. Using these definitions, we may rewrite m as

$$m = \frac{\gamma_A^2 \varepsilon_A + \gamma_B^2 \varepsilon_B}{\gamma_A^2 + \gamma_B^2} = m(\gamma_A^2, \gamma_B^2, \varepsilon_A, \varepsilon_B).$$

Noting that γ can be made independent of α by an appropriate choice of β, the second term in brackets in equation (2) may be treated as an independent background risk with respect to $[W - \alpha_A EL - (1-\alpha_A)L]$. For this latter term, the optimal α equals one for any realized value of $(1+\varepsilon_A)$ and for any risk-averse decision-maker. This follows since the first term has a constant mean value for any α, plus the assumption that ε and L are independent (see Schlesinger 1997, 1999).

4.2. Optimal contracting

Assuming symmetry of regions *A* and *B*, $\alpha_B^* = \alpha_A^* = 1$, where an asterisk denotes an optimal value. Moreover, assuming rational expectations among the identical individuals yields $\beta_A^{0*} = \beta_B^{0*}$ and $\beta_A^{2*} = \beta_B^{2*}$. The final wealth of the representative individual in region *A* may thus be rewritten as:

(3) $$\begin{aligned} Y_A &= (1+\varepsilon_A)\{W - EL\} - \{\varepsilon_A W + \beta_A^0 EL(\lambda - \varepsilon_A) + \beta_A^2 EL(m - \varepsilon_A)\} \\ &= (W - EL) - EL\{\beta_A^0 \lambda + \beta_A^1 \varepsilon_A + \beta_A^2 m\} \end{aligned}$$

with $m = \dfrac{\varepsilon_A + \varepsilon_B}{2}$.

Using this value of m in (3), and defining $t = \dfrac{\beta_A^1}{\beta_A^1 + \beta_A^2}$, we obtain:

(4) $$\begin{aligned} Y_A &= (W - EL) - EL\left\{\beta_A^0 \lambda + (\beta_A^1 + \beta_A^2)\left[t\varepsilon_A + (1-t)\frac{\varepsilon_A + \varepsilon_B}{2}\right]\right\} \\ &= (W - EL) - EL\left\{\beta_A^0 \lambda + (1 - \beta_A^0)\left[\frac{1}{2}(\varepsilon_A + \varepsilon_B) + \frac{1}{2}t(\varepsilon_A - \varepsilon_B)\right]\right\} \end{aligned}$$

Now, define $z = \varepsilon_A + \varepsilon_B$ and $e = \varepsilon_A - \varepsilon_B$. The term in square brackets in (4) becomes $\left[\frac{1}{2}z + \frac{1}{2}te\right]$. Given that E(*z*) = 0, E(*e*) = 0, and E(*e*|*z*) = 0, *z* + *te* is a mean-preserving spread of *z* as defined by Rothschild and Stiglitz (1970). Thus *z* dominates *z* + *te* by second-order stochastic dominance, i.e. for every risk averter in our enlarged definition. As a result, $t^* = 0$, which is the same as $\beta_A^1 = 0$. As long as $\beta_A^2 \neq 0$, this leads to:

(5) $$Y_A = (W - EL) - EL\left\{\beta_A^0 \lambda + (1 - \beta_A^0)\frac{\varepsilon_A + \varepsilon_B}{2}\right\}.$$

Given that the random term $[(\varepsilon_A + \varepsilon_B)/2]$ has an expectation lower than the fixed term λ, it follows that $\beta_A^{0*} < 1$ for every risk averter, which in turn implies $\beta_A^{2*} > 0$.[9]

To sum up, the individual insures fully ($\alpha^* = 1$) to eliminate the diversifiable risk. She then optimally shares the global catastrophe risk with the insurer, to avoid paying the full risk premium λ. In contrast with what might be expected *a priori*, it is suboptimal for her to be pooled only with risks of the same class. This result is in accordance with Borch's (1962) mutuality principle.

5. The case of frequency risk

To introduce this case, the occurrence of a catastrophe is assumed to have no impact on the severity of individual losses. We relax this assumption in the following section of the paper. To simplify the severity component of the model, assume an all-or-nothing loss: either $L = 0$ with probability 1-*p*, or $L = M < W$ with probability *p*. The latter

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[9] If the individual displays first-order risk aversion, as in Segal and Spivak (1990), it may be the case that $\beta_A^{0*} = 1$. In this case t is not well defined in the above analysis. However, the results still follow via convergence arguments. In other words, allowing for first-order risk aversion allows the possibility of the solution $\beta_A^{0*} = 1, \beta_A^{1*} = \beta_A^{2*} = 0$.

probability is an *ex ante* measure based on long-term statistical records of claims, including claims due to catastrophes. We assume that p is the same in both regions, A and B.

However, catastrophes are rare events, so that the current period frequency of losses among a population of insured individuals in region j is random. For $j = (A,B)$, let $f_j = p(1+\delta_j)$ represent the random frequency, with $\delta_j : \Omega \to R$ and with $E(\delta_A) = E(\delta_B) = E(\delta_A \delta_B) = 0$. The support of δ is contained in [-1, (1-p)/p]. We can think of $p(1+\delta_j)$, for a realized value of δ_j, as the *ex post* probability of a randomly chosen insured having a loss during the given period. In other words, $p(1+\delta_j)$ represents the actual relative loss frequency for the current period whereas p represents the long-run average loss frequency per period. Again, we caution the reader that the case where $\delta = 0$ does not represent the no-catastrophe case.

If total losses are the same, the insurer is indifferent as to whether catastrophes provoke a general increase in the severity of losses, for a given frequency, or whether they yield an increase in the frequency of losses for a given severity[10]. If $EL = pM$, and the random components ε and δ are the same, the total claims from region j are the same, using either model: $n\{(1+\varepsilon_j)EL\} = n\{(1+\delta_j)pM\}$.

Using the same contract design as in section 2, we define:

$P_j^f = \alpha_j(1+\lambda)pM$ (fixed premium contract)

$P_j^v = \alpha_j(1+\delta_j)pM$ (variable premium contract)

$P_j^p = \alpha_j(1+m)pM$, with $m = \dfrac{\alpha_A\beta_A^2\delta_A + \alpha_B\beta_B^2\delta_B}{\alpha_A\beta_A^2 + \alpha_B\beta_B^2}$. (participating contract)

The final wealth of the representative individual from region A is thus:

$$Y_A = \{W - \alpha_A pM - (1-\alpha_A)L\} - \alpha_A pM\{\beta_A^0\lambda + \beta_A^1\delta_A + \beta_A^2 m\}. \quad (6)$$

[10] Of course, this statement relies on the absence of transactions costs.

This may be rewritten as:

(6') $Y_A = \{W - \alpha_A (1+\lambda) pM - (1-\alpha_A) L\} - pM\{\gamma_A^1(\delta_A - \lambda) + \gamma_A^2(m-\lambda)\}$, using the same definition of γ as in the preceding section.

Again, the second term in the preceding equation may be treated as an independent background risk. However, because the first term now incorporates the loading, the optimal insurance coverage is less than one if the individual has second order risk aversion[11]. It may be one if she has first order risk aversion.

Using again the symmetry of regions *A* and *B*, and assuming rational expectations, we obtain $\alpha_A^* = \alpha_B^*$ and $m = (\delta_A + \delta_B)/2$. Then, from (6), and using the definitions of *t*, *z* and *e*, leads to:

(6'') $Y_A = \{W - \alpha_A pM - (1-\alpha_A) L\} - \alpha_A pM\left\{\beta_A^0 \lambda + (1-\beta_A^0)\frac{1}{2}(z+te)\right\}$

As in the preceding section, risk aversion leads to

$t^* = \beta_A^{1*} = 0,\ 0 < \beta_A^{0*} < 1$, and $\beta_A^{2*} = 1 - \beta_A^{0*}$.

The representative individual insures partially, and chooses to share in the global catastrophe risk, to save on the loading, as previously. To get an intuition of why insurance is partial in this model, note that the α_A factor in the second term of (6) cannot be "undone" by appropriate choices of β_A^i, since the β_A^i must sum to one. Thus, increasing the insurance level will always increase the level of this undesirable second term, and hence an optimal insurance level is less than one.

[11] Assuming standard risk aversion, Eeckhoudt and Kimball (1992) have shown that the optimal insurance coverage under a loaded premium and an independent background risk is less than it would be under no background risk. However, they consider an actuarially fair background risk, whereas the background risk defined by (6') has a positive expected value.

6. Simultaneous frequency risk and severity risk

In this section, we combine the basic models presented in sections 2 and 5 above. Catastrophes affect both the frequency of losses in the population and the severity of these losses. As before, the random frequency of losses is given by $f_j = p(1+\delta_j)$, with $E(f_j) = p$, and j = (A,B). For simplicity, we assume the severity of a loss would be the constant M, adjusted only by the perfectly correlated catastrophe factor ε_j. If a loss occurs in region j, its magnitude is thus $M(1+\varepsilon_j)$. Dropping subscript j, the *ex ante* random loss is now defined as:

(7) $L = \eta(\delta)M(1+\varepsilon)$,

with $\eta(\delta)$ a Bernoulli variable taking values 1 or 0 with (random) probabilities $p(1+\delta)$ and $1-p(1+\delta)$. As before, we assume that the ε as well as the δ are identical within each region, with $E(\varepsilon)=E(\delta)=0$.

Taking into account the fact that the random variables ε and δ may be positively correlated, we find $E(L) = pM[1+Cov(\delta,\varepsilon)]$.Then, using the same contracts design as previously, we define[12]:

$P_j^f = \alpha_j(1+\lambda)E(L_j) = \alpha_j(1+\lambda)[1+Cov(\delta_j,\varepsilon_j)]pM$ (fixed premium contract)

$P_j^v = \alpha_j(1+\delta_j)(1+\varepsilon_j)pM$ (variable premium contract)

$P_j^p = \alpha_j(1+m)pM$ (participating contract)

with $m = \dfrac{\alpha_A\beta_A^2(\delta_A+\varepsilon_A+\delta_A\varepsilon_A)+\alpha_B\beta_B^2(\delta_B+\varepsilon_B+\delta_B\varepsilon_B)}{\alpha_A\beta_A^2+\alpha_B\beta_B^2}$.

Using these premiums and dropping the subscript for ease of exposition, we derive the final wealth for an individual:

$$Y = W-(1-\alpha)L-(1-\beta^1-\beta^2)(1+\lambda)\alpha E(L)-\beta^1\alpha pM(1+\delta)(1+\varepsilon)-\beta^2\alpha pM(1+m)$$

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[12] Note that the premium loading λ will not necessarily be the same as previously. In particular, a positive correlation between ε and δ is likely to lead to a higher market premium for the catastrophic risk.

This expression may be rearranged to yield:

(8) $$Y = \{W - (1+\lambda)E(L) - (1-\alpha)L\} - pM\{\gamma^1[(1+\delta)(1+\varepsilon) - (1+\lambda)(1+C)] + \gamma^2[(1+m) - (1+\lambda)(1+C)]\ \}$$

where $\gamma = \alpha\beta$ as before and $C = Cov(\delta, \varepsilon)$.

Again, the second term in this expression may be treated as an independent background risk. Looking at the first term in brackets, we obtain that the optimal insurance coverage is less than one if the individual has second order risk aversion, and that it is less than or equal to one if she has first order risk aversion. Without surprise, this is the same as under frequency risk.

Further, assuming symmetry of regions *A* and *B*, we obtain:

(9) $$Y = W - \alpha pM - (1-\alpha)L - \alpha pM\left\{\beta^0(\lambda + C + \lambda C) + (1-\beta^0)(\frac{z+te}{2})\right\},$$

where $t = \dfrac{\beta^1}{\beta^1 + \beta^2}$ as before. However, in this case we have:

$$z = \delta_A + \varepsilon_A + \delta_A\varepsilon_A + \delta_B + \varepsilon_B + \delta_B\varepsilon_B$$
$$E(z) = Cov(\delta_A, \varepsilon_A) + Cov(\delta_B, \varepsilon_B)$$
$$e = (\delta_A + \varepsilon_A + \delta_A\varepsilon_A) - (\delta_B + \varepsilon_B + \delta_B\varepsilon_B)$$
$$E(e) = Cov(\delta_A, \varepsilon_A) - Cov(\delta_B, \varepsilon_B) = 0$$

with the symmetry assumption being used in obtaining $E(e) = 0$. Given that $E(e|z) = 0$, $z+te$ is a mean-preserving spread of z. Thus, the optimal t is again zero for every risk averter.

These results do not differ qualitatively from what we obtain in the frequency risk model:

$$\alpha^* \leq 1, \quad t^* = \beta_A^{1*} = 0, \ \ 0 < \beta_A^{0*} < 1, \ \text{and}\ \beta_A^{2*} = 1 - \beta_A^{0*}.$$

However, any comparative-static analysis will yield ambiguous results without specifying parameters of the model for a particular case.

7. Conclusion

This paper extends the previous literature on catastrophe insurance by considering the case where insureds are able to take advantage of other catastrophe risk handled by their own insurer. Insureds are offered a menu of contracts and they are able to mix these contracts. With two classes of catastrophes, three basic contract types are proposed: a fixed premium contract providing partial or full coverage, a variable premium contract offering the opportunity to the insured to share in the local catastrophe risk (as she would do, for example, when investing on her own account in cat options on a local index), and a participating premium contract offering the opportunity to participate in the insurer's portfolio of catastrophe-linked risks.

We find that the optimal mix for a risk-averse insured combines the fixed premium contract and the participating premium contract. It is not optimal for the individual to include a contract share in the local catastrophe risk. In addition, the result is shown to be robust to a generalization of the model, to take into account the simultaneous occurrence of severity and frequency risk. Although our analysis considers only the simplest case of uncorrelated catastrophes, the framework also could be used when correlations are not zero. Of course, our qualitative results may no longer hold, but the greater degree of flexibility can only benefit the insurance consumer.

In accordance with Borch's (1962) mutuality principle, our results imply that endogenizing the participation levels of insureds would be a welfare-improving innovation in the property-liability insurance market. They provide a further instrument for efficient risk sharing between insurers and insureds. The results further imply that larger insurers, who more likely can write policies covering several catastrophic exposures, might have an additional competitive advantage in the global marketplace.

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