Stable Models In Credit Risk

I. Khindanova[1], M. Knoch[2], S. T. Rachev[2], E. Schwartz[3]

[1] University of California at Santa Barbara
[2] Universität Karlsruhe
[3] University of California at Los Angeles

Abstract: We discuss the main approaches to quantify the risk of losses arising from a defaulting counterparty to a financial transaction that have been developed over the last 25 years. Every existing method faces major problems in assessing the numerous and partly non-observable factors influencing credit risk. One shortcoming common to all methods is the classical normal assumption for interest rate changes and asset returns. We suggest the introduction of log-stable (non-Gaussian) processes as more realistic model for bond returns and credit spreads.

Keywords: risk management, credit risk, corporate bonds, stable distributions, log-stable processes

1 Introduction to Credit Risk

Every financial institution has an exposure to four sorts of risk: market-, credit-, liquidity- and operational risk. The central issue of financial risk management has always been market risk, the risk of loss as a result of changes in market prices (i.e. foreign exchange- or interest rate movements). These changes occur daily and a firm's business is constantly affected by them. Therefore a large number of methods to evaluate and reduce market risk have been established.

Fewer attention has been given to credit risk, which refers to the loss arising from the default of a counterparty.

Twenty years ago financial institutions relied exclusively on the largely subjective judgement of an expert whether or not to grant credit to a counterparty. A fundamental progress in quantitative analysis of credit risk was the work of Fischer Black and Myron Scholes .In 1973 they noted that their option pricing formula could also be applied to valuing corporate debt[1] and one year later Robert Merton introduced the corresponding valuation formula[2]. Since then two schools of thought have been developed for modelling credit risk:

- **Structural models**: in this approach the default is a foreseeable event and determined by the value of the firm's assets (based on Arbitrage Pricing Theory),

[1]Black and Scholes (1973)
[2]Merton (1974)

- **Reduced form models**: in this approach the default is unpredictable and is modelled by the theoretical probability of bankruptcy. This method includes Markov and Poisson models.

Both approaches face the same difficulties that are specific for credit risk. Default is a very rare event. For a typical firm the probability to default in any year is around 2%. High rated firms (AAA or Aaa) even exhibit average default rates not exceeding 0.02%. Hence, data for estimating model parameters or empirical validation is rather scarce.

In addition to that the causes for default and its technicalities are very diverse and hard to grasp. They depend not only on quantitative but also on qualitative variables such as legal provisions, bankruptcy laws and other country specific circumstances.

Nevertheless in recent years credit risk has become increasingly threatening for the financial institutions. Reasons that led to a growing interest in reliable credit risk models are (i) a worldwide structural increase in the number of bankruptcies, (ii) globalization of markets with new chances and more complex risks (shortage of information about the credit quality of counterparties in foreign or emerging markets), (iii) a trend towards disintermediation by the highest quality and largest borrowers, (iv) more competitive margins on loans, (v) a declining value of assets in many markets and (vi) a dramatic growth of off-balance sheet instruments with inherent default risk exposure, including credit derivatives.[3]

In the following we only consider zero-bonds for simplicity. While the structural approach requires an extended theory to value any kind of coupon bonds[4], in reduced form models these bonds can simply be viewed as a portfolio of independent zero-bonds.

1.1 Quantitative Credit Risk Models

Basically, any credit risk model – structural or reduced form – uses the same relationship between the price of a riskless (treasury) bond and a risky (corporate) bond with of the same maturity. The price of a risky bond is simply the expectation of its discounted returns. The return at maturity is either the full nominal value B (this amount is received with survival probability $1-p$), or a certain fraction of the nominal value, the so-called recovery rate $R \in [0,1)$ (received with default probability p). Let $B^*(t,T)$ denote the discounted value of a riskless bond at time t paying B dollars at T. Then the price of the corresponding risky bond is

$$B(t,T) = ((1-p) + pR)B^*(t,T). \tag{1}$$

We note that the price of a bond with inherent default risk depends on three variables:

[3]Altman (1998)

[4]Geske (1979)

- the default probability
- the recovery rate
- the value of the corresponding riskless bond (the riskless interest rate, respectively).

The derivation of default probabilities is the main difference between the structural and reduced form approach. Default probabilities can either be given exogenously in the form of historical averages (reduced form), or calculated endogenously (structural). Recent models of both types use historical averages also to estimate recovery rates. For an estimate of the riskless interest rate any credit risk model refers to existing term structure models (e.g. Vasicek, Cox-Ingersoll-Ross, etc.). We will discuss the limitation that is involved by the normal assumption underlying these term structures in the last two sections.

2 Structural Models

The structural approach to credit risk was introduced by Robert Merton in 1974[5]. He applied the Black-Scholes option pricing formula to valuing corporate debt (known as Contingent Claims Analysis) as it was suggested by Black and Scholes themselves in their seminal paper[6].

The basic idea is that default occurs when the firm's assets are exhausted. 'Exhausted' means that the value of all the assets falls below the value the of firm's outstanding debt.

Therefore the probability to default is determined by the dynamics of the assets. In structural models a stochastic process driving the dynamics of the assets is assumed. The firm is in financial distress when the stochastic process hits a certain lower boundary at (respectively before) maturity of the obligations.

In this case the lenders are repaid only with the residual value of the assets (due to limited liability of the shareholders). Otherwise – if the assets exceed the firm's debt – the shareholders pay back the debt in full.

In other words, the shareholders have a call option on the firm's assets with a strike price equal to the face value of the outstanding debt.

Using the Black-Scholes formula the price of a risky zero-bond is then

$$B(t,T) = V(t)(1 - \Phi(d_1)) + Be^{-r(T-t)}\Phi(d_2) \qquad (2)$$

with

$$d_1 = (\ln \tfrac{V(t)}{B} + (r + \tfrac{1}{2}\sigma^2)(T-t))/(\sigma\sqrt{T-t}), \qquad (3)$$

$$d_2 = (\ln \tfrac{V(t)}{B} + (r - \tfrac{1}{2}\sigma^2)(T-t))/(\sigma\sqrt{T-t}), \qquad (4)$$

[5] Merton (1974)

[6] Black-Sholes (1973)

where $V(t)$ is the value of the firm's assets at time t, and r is the (constant) riskless interest rate.

Applying the option pricing formula to corporate liabilities involves some additional shortcomings beyond the standard assumptions of the Black-Scholes world. While stock prices can easily be observed when valuing an option, the values of a firm's entire assets and liabilities are almost impossible to observe. Secondly, the recovery in case of default depends in this setting exclusively on the firm's remaining assets (safety covenants or other types of outstanding debt are neglected). Furthermore, Merton's model has limited predictive power due to the unrealistic assumption that default can only occur at maturity of the bond.

Since Merton's model had only few success in empirical tests it was modified by Black and Cox (1976) who introduced a constant lower boundary at which a firm defaults at an arbitrary time before maturity [7], and most notably by Longstaff and Schwartz (1995) who assumed the recovery in case of default to be constant, too.

With these modifications the bond pricing formula in the Longstaff-Schwartz model eventually has the same form as equation (1) with R a constant and p the probability that the first passage time of $V(t)$ to a lower boundary K is less than T

$$p = \Pr[\min\{\tau \in (t,T) : V(t)/K = 1\} < T]. \tag{5}$$

The price of the riskless bond $B^*(t,T)$ is calculated using the term structure model by Vasicek (1977), but basically any stochastic term structure can be included in the model.

Even with its most recent modifications the structural approach to credit risk reveals some potential shortcomings:

- The correlation between the two stochastic processes driving the dynamics of interest rates and asset values is assumed to be zero.
- It is hardly possible to observe the value of a firm's entire assets.
- Both, changes in asset values and changes in interest rates are assumed to be normally distributed. This distributional assumption might be too restrictive to fit empirical data (see section 4).

At least the first two of the problems mentioned above are avoided by a class of credit risk models that is based on corporate ratings, the so-called reduced form models.

[7] This extension is actually a simplification of Merton's model that makes it more applicable in practice.

3 Reduced Form Models

The reduced form approach to credit risk was first introduced by Jerome Fons in 1994. Fons built up a simple model that offered a practical alternative to those based on option pricing theory (OPT). While the OPT models were not that successful in practice due to their complex model structure Fons relies more on rather easily observable input data. The components needed to determine the credit spread of a risky bond are corporate ratings, statistics of historical default and recovery rates and the riskless interest rate. The main advantage of Fons' approach is that the default process is not anymore directly linked to the value of the assets. The value of the assets influences the model only to the extent to which it enters into the rating. So one might consider the structural approach as a special case of the reduced form where the ratings are measured continuously (rating scale and time) and depend exclusively on the value of the assets.

Reduced form models are increasingly implemented in software modules for risk management. Popular examples are *CreditRisk+* (Credit Suisse Financial Products)[8] and *CreditMetrics* (J.P.Morgan) [9].

Every reduced form model requires a discrete set of rating categories $I = \{1, 2, \ldots, K\}$ with ascending credit quality from 1 to $K-1$ and K the state that designates default. Then the rating history of a firm is modelled as a Markov chain [10] where default is an absorbing state [11].

Consider a discrete time-homogeneous finite Markov chain $X = (X_t)_t \in \{0, \ldots, T\}$ defined on the state space I with transition matrix

$$P = (p_{ij})_{i,j \in I} = \begin{pmatrix} p_{11} & \cdots & p_{1(K-1)} & p_{1K} \\ \vdots & \ddots & \vdots & \vdots \\ p_{(K-1)1} & \cdots & p_{(K-1)(K-1)} & p_{(K-1)K} \\ 0 & \cdots & 0 & 1 \end{pmatrix}, \tag{6}$$

where p_{ij} is the probability for a rating change from category i to j in one step.

Once this matrix is estimated (using historically tabulated rating changes) we can obtain the probabilities for rating changes over any arbitrary time horizon $(t, t+n)$ by calculating [12]

$$P(t, t+n) = P^n. \tag{7}$$

[8]For details on *CreditRisk+* see the technical document available under http://www.csfp.co.uk.

[9]The technical document of CreditMetrics (Gupton, Finger and Bhatia (1997)) is available under http://www.creditmetrics.com.

[10]An introduction to basic properties of Markov chains can be found in Norris (1997).

[11]A state is called absorbing when the probability for a transition from this state to any other equals zero.

[12]The validity of this equation requires the absence of autocorrelations.

Hence, the probability that an i-rated firm defaults before T is

$$p^{(i)} = p_{iK}(t, T-t). \tag{8}$$

Substituting this probability into equation (1) yields an adequate bond price for every rating category at any risk horizon

$$B^{(i)}(t,T) = ((1 - p_{iK}(t, T-t)) + p_{iK}(t, T-t)R)B^*(t,T). \tag{9}$$

Again the recovery rate is assumed to be constant at R and $B^*(t,T)$ is calculated using one of the existing interest rate models. Thus, in the case of a reduced form model the price of a corporate bond depends on the validity of the normal assumption for riskless interest rates, too.

In the following section we discuss this assumption and suggest a new class of processes to model the dynamics of financial returns.

4 Modifying Credit Risk Models By Stable Paretian Distributions

In order to obtain the price of the riskless bond existing credit risk models utilize the results from the vast theory of (stochastic) term structure models [13]. Following this theory we can rewrite the price of the riskless bond as

$$B^*(t,T) = B\exp\{-\int_t^T r(u)du\}, \tag{10}$$

where $r(t)$ is the short rate. The dynamics of r are given by the stochastic differential equation

$$dr(t) = \mu(t,r)dt + \sigma(t,r)dW(t) \tag{11}$$

with continuous functions μ and σ, and $W(t)$ a Wiener process. The assumption that the short rate process has normally distributed increments is the standard paradigm in modern finance.

However, empirical marginal distributions of financial returns, especially over short time horizons, seem to deviate from the normal distribution. They exhibit more mass near the mean (higher peaks) and a higher probability for extreme outcomes (heavier tails) [14].

Therefore we suggest the use of stable Paretian distributions [15] for modelling financial returns. The class of stable distributions is characterized by the following definition:

[13] An overview on term structure models can be found in Duffie (1996).

[14] see Mittnik and Rachev (1999)

[15] For an introduction to stable Paretian distributions and stochastic processes based on these distributions see Samorodnitsky and Taqqu (1994).

A random variable X is said to be stable if for any $a > 0$ and $b > 0$ there exist constants $c > 0$ and $d \in \mathbf{R}$ such that

$$aX_1 + bX_2 =^d cX + d, \tag{12}$$

where X_1 and X_2 are independent copies of X and $=^d$ denotes the equality in distribution.

Depending on four parameters μ (drift), σ (scale parameter), β (skewness parameter) and α (index of stability), the shape of a stable distribution is more flexible to fit empirical data. Small values of α produce the desired higher peaks and heavier tails (so-called *leptocurtic* shapes). Since for $\alpha = 2$ the distribution reduces to the Gaussian distribution we can view the use of stable distributions as a true generalization of the normal assumption.

A stochastic process, starting at 0, and having stationary, independent and stable (non-Gaussian) distributed increments is a so-called *Lévy Motion*. The term structure equation (11) can be modified by the concept of subordination so that it replaces the Brownian motion driving market uncertainty by a Lévy Motion [16]. Note that under the stable hypothesis there is no explicit representation of the bond price in the existing term structure models, so one approach is to rely on various simulation techniques for stable processes.

5 Stable Approach to the Term Structure of Credit Risk

Beside the modification of existing structural and reduced form models by stable processes (as we have done in the last section) we suggest a new class of credit risk models which is completely based on stable Paretian distributions. In a VAR-like approach we estimate a distribution for the relative changes of bond prices in each rating category over a given time horizon, and then compute expected and unexpected losses in the form of percentiles. Since we try to fit the empirical distribution function with a stable Paretian distribution we are able to easily apply this approach to portfolios of risky assets.

Table I shows the estimated parameter values for the seven rating categories together with the Kolmogorov distance between the estimated and the empirical cumulative distribution function (cdf). The four parameters of each distribution are estimated in two ways (maximum likelihood estimation and $\mathcal{L}^1$-distance). The sample data consists of prices for the 5% 5-year senior unsecured bond in different rating categories.

[16] Hurst, Platen and Rachev (1996)

Table I
Computed Kolmogorov distances (ks) between the estimated and empirical cdf

Bond	Method	α	β	σ	μ	ks
AAA	l1	1.5	-1	.0541	104.64	.2992
AAA	mle	1.2	-0.9915	.0409	104.58	.6605
AA	l1	1.5	-1	.1365	104.22	.2687
AA	mle	1.4	-0.9929	.1214	104.21	.3130
A	l1	1.5	-1	.3598	103.22	.3503
A	mle	1.4445	-0.8651	.2566	103.35	.2510
BBB	l1	1.5	-1	1.034	99.97	.2274
BBB	mle	1.8	-1	1.260	100.25	.0974
BB	l1	1.5	-1	1.687	94.44	.3642
BB	mle	1.187	-0.3443	1.117	94.61	.2867
B	l1	1.5	-1	3.892	84.12	.2519
B	mle	1.428	-0.8713	3.843	83.98	.2660
CCC	l1	1.5	-1	16.65	64.23	.1821
CCC	mle	2.0	-0.0726	11.60	64.15	.0235

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