Revised December, 1999

CHANGES IN RISK AND ASSET PRICES*

Christian Gollier
GREMAQ and IDEI
University of Toulouse
F-31042 Toulouse Cedex
(FRANCE)
E-mail: gollier@cict.fr

Harris Schlesinger
Department of Finance
University of Alabama
Tuscaloosa, AL 35487-0224
(USA)
E-mail: hschlesi@cba.ua.edu

Abstract

We examine asset prices in a representative-agent model of general equilibrium. Assuming only that individuals are risk averse, we determine conditions on the changes in asset risk that are both necessary and sufficient for the asset price to fall. We show that these conditions neither imply, nor are implied by the conditions for second-degree stochastic dominance. For example, if the payoff on an asset becomes riskier in the sense of second-degree stochastic dominance, the equilibrium price of the asset need not necessarily fall. We further demonstrate how our results can be imbedded into a market that is incomplete in the sense of containing an uninsurable background risk, such as a risk on labor income. We extend our model to show how a miscalibration of the asset risk can lead to a partial explanation of high equity premia (i.e., the "equity premium puzzle").

\asset\asset3

*This paper was written while Schlesinger was a visiting professor at IDEI, University of Toulouse. Financial support from the Fédération Française des Sociétés d'Assurance is gratefully acknowledged. The authors thank workshop participants at the University of Alabama, Hebrew University of Jerusalem, University of Konstanz, University of Munich and the University of Pennsylvania, as well as Günter Franke, Fatma Lajeri and an anonymous referee, for helpful comments on an earlier version of this paper.

1. Introduction

Understanding the mechanism of asset pricing has long been a goal of financial economists. One approach is the so-called representative agent model, as put forth by Lucas (1978), which has since established itself as a major part of both the macroeconomic theory and the microeconomic theory of security returns. For instance, the model has been used extensively in examining such vexing empirical anomalies as the equity-premium puzzle (Mehra and Prescott (1985)).[1] A key result of the representative-agent model is that the price of an asset equals the expected value of that asset under a transformed probability distribution, the risk-neutral probability distribution.[2] This basic paradigm of "risk-neutral pricing" has imbedded itself throughout the financial literature and is at the foundation of many complex theories.

Our goal in this paper is to examine the effect of a change in the actual distribution of an asset's payoffs on its price, i.e. on its risk-neutral expectation. For example, if the payoff on an asset becomes riskier in the sense of second-degree stochastic dominance, will the price of the asset necessarily fall? The surprising answer (at least to some) is "no." Although the individual agents will all have a lower certainty equivalent for replacing the riskier asset, assuming they are risk averse, risk-neutral pricing looks only at the marginal valuation of the asset. At equilibrium prices, the individual is indifferent to buying or selling more of the asset, not indifferent to replacing his or her asset position with certainty. The problem, then, is more closely related to the standard portfolio problem under an increase in risk.[3] Indeed, our model essentially takes

---

[1]See Kocherlakota (1996) for excellent discussion of the background of the representative-agent approach vis a vis its alternatives, such as the CAPM. Kocherlakota also presents a broad survey of the empirical successes and failures of this approach in addressing both the equity-premium puzzle and the riskfree-rate puzzle (Weil (1989)).
[2]See Rubinstein (1976) for the development of the risk-neutral approach.
[3]See Rothschild and Stiglitz (1971) for an original statement of the problem. See also a nice restatement and extension by Diamond and Stiglitz (1974) and by Hadar and Seo (1990).

the portfolio problem -- how much to invest in a risky asset at a given price -- and imbeds it into a general equilibrium model of prices in an exchange economy.

Unlike Rothschild and Stiglitz (1971), Diamond and Stiglitz (1974), Labadie (1986), Hadar and Seo (1990) and others, we do not focus on restrictions on preferences which will cause mean-preserving spreads to reduce demand for a risky asset. Rather, we focus on changes in risk that will induce all risk averters to demand less of the asset. Although one cannot assume, in general, that asset demand is downward sloping, it turns out always to be downward sloping at the market-clearing price, which therefore is unique. Assuming only that individuals are risk averse, we determine conditions on the changes in asset risk that are both necessary and sufficient for the asset price to fall. We show that these conditions neither imply, nor are implied by the conditions for second-degree stochastic dominance. We provide a set of extent conditions for changes in asset risk that are sufficient to lower the equilibrium asset price.

We next demonstrate how our results can be imbedded into a market that is incomplete in the sense of containing an uninsurable background risk, such as a risk on labor income. Weil (1992) uses such a model in a two-period setting to address the equity-premium puzzle. We illustrate Weil's main result in our simple static model and we extend the model to show how a miscalibration of the asset risk also can lead to a partial explanation for high equity premia, if the miscalibration error takes the form of white noise.

Before proceeding, we wish to make two caveats. First, our model considers a single risky asset, which might best be viewed as the "market portfolio" for the given economy. Thus, we essentially consider two economies, identical in every way except for the riskiness of the market portfolio in each. This is not the same as comparing the prices of two assets within a single economy. In this context, our paper considers whether or not there is a monotonic relationship between the riskiness of the market portfolio and its expected return.

As a second caveat, we forewarn the reader that ours is a static model. Certainly there are dynamic features of exchange economies that need to be taken into account if one expects to develop a full positive theory of asset pricing. However, an understanding of something as basic as how risk affects asset prices would appear to be of fundamental importance. All of the effects we examine here become even more complex within a more general intertemporal setting.[4] In this regard, ours might best be viewed as a negative result: there need not be a risk-versus-return tradeoff in equilibrium. Of course, if this tradeoff cannot be guaranteed in our simple static model, it obviously must not hold when our model is embedded into a more realistic setting. And although one can correctly argue that the sufficient condition presented in our Proposition below might not remain sufficient in a more general setting, our necessary condition for a change in asset risk to guarantee a reduction in the asset's price will remain necessary.

2. Equilibrium Prices

We consider a static Lucas (1978) "tree economy." The economy consists of risk-averse individuals, all of whom may be portrayed by a "representative agent."[5] The economy is competitive in that individuals maximize expected utility with prices taken as given.

Initial wealth consists of one unit of the risky asset plus an allocation of a risk-free asset. Because our model is static, the risk-free rate in our model can be thought of in essence as zero. We let $w > 0$ denote the value of wealth that is initially invested in

---

[4]However, if the riskfree rate is zero, consumers behaving myopically is intertemporally efficient when utility belongs to the HARA class (Mossin (1968)); and with a positive risk free rate, a restriction to the subclass of CRRA utility will allow for myopia (Gollier, Lindsey and Zeckhauser (1997)).

[5]We can assume that all agents are identical, although this assumption is not always necessary as is pointed out by Constantinides (1982).

the risk-free asset and define the random variable $\tilde{x}$ as the final value of the risky asset, including all incremental cash flows. The distribution function for $\tilde{x}$, F, is assumed to be chosen from those with support in the interval $[a,b] \subset \Re$ such that $E\tilde{x} > 0$, where E denotes the expectation operator. Let $D^{+}[a,b]$ denote the set of all such distribution functions. The assumption $E\tilde{x} > 0$ ensures a positive equilibrium price. Agents' preferences are assumed to be smooth in the sense of being representable by a twice-differentiable von Neumann-Morgenstern utility function $u(\cdot)$.[6] The agent can adjust her portfolio via buying and selling the two assets. Letting P represent the price of the risky asset and β denote the demand for additional units of $\tilde{x}$, the agent solves the following optimization program:

(1) $$\max_{\beta} Eu(\tilde{y}) \,, \quad \text{where } \tilde{y} \equiv w + \tilde{x} + \beta(\tilde{x} - P) \,.$$

The first-order condition for maximizing the agent's expected utility is [7]

(2) $$E(\tilde{x} - P)u'(\tilde{y}) = 0.$$

Since our focus is on equilibrium, we assume that (2) is satisfied with an excess demand of zero, i.e. $\beta^* = 0$. Rearranging equation (2), this implies an equilibrium asset price of

(3) $$P = \frac{E\tilde{x}u'(w + \tilde{x})}{Eu'(w + \tilde{x})}.$$

It is useful to note that (1) is simply the standard portfolio problem and the solution(s) to (3) show all values of P for which there is no excess demand or supply of the risky asset.

---

[6] This assures second-order risk aversion in the sense of Segal and Spivak (1990).
[7] The second-order condition follows trivially from the assumption of risk aversion.

To keep the discussion relatively unencumbered, we focus solely on the price (3) of the asset. However, interested readers can easily adjust the analysis to consider asset returns,

$$(4) \qquad E\tilde{R} \equiv \frac{(E\tilde{x})(Eu'(w+\tilde{x}))}{E\tilde{x}u'(w+\tilde{x})}.$$

As is fairly common in the finance literature, (3) can be written as

$$(5) \qquad P = \int_a^b x d\eta(x) \equiv \hat{E}\tilde{x},$$

where $\eta(x) \equiv (Eu'(w+\tilde{x}))^{-1}\int_a^x u'(w+t)dF(t)$ is the so-called risk-neutral probability distribution corresponding to $w+\tilde{x}$ for the utility u, and where $\hat{E}$ denotes the expectation operator under this distribution. In other words, the actual equilibrium (market-clearing) price of the risky asset, P, is simply the risk-neutralized expectation of $\tilde{x}$.

3. Tatônnement Adjustment

Although our focus is not on out-of-equilibrium adjustment, some discussion of it is necessary for us to be able to compare equilibrium prices following a change in asset risk. To this end, consider the first-order condition as given in (2), without the restriction that individual optima represent an equilibrium, i.e. without assuming that $\beta^* = 0$. We find the slope of the demand curve at any price by examining how the optimal excess demand at that price, which we label $\beta(P)$, would change with a perturbation of P. In particular, consider

$$(6) \qquad \frac{d^2 Eu(\tilde{y})}{\partial\beta\partial P} = E\left\{\beta[-u''(\tilde{y})(\tilde{x}-P)] - u'(\tilde{y})\right\}.$$

Concavity of $Eu(\tilde{y})$ in β implies that excess demand for $\tilde{x}$ will rise or fall as the sign of (6) is positive or negative respectively. Unfortunately the sign of (6) is indeterminate *a priori*. As an example of possibly upward sloping demand, consider a case where the agent's preferences exhibit decreasing absolute risk aversion (DARA).

DARA can be completely characterized as "$-u'(\cdot)$ is a more concave utility function than $u(\cdot)$." As is well known,[8] an individual who is more risk averse will invest less in a risky asset, ceteris paribus. Thus, an individual with utility $-u'(\cdot)$ will invest less in the risky asset than someone with utility $u(\cdot)$. This implies that $dE(-u'(\tilde{y}))/d\beta = E[-u''(\tilde{y})(\tilde{x}-P)] < 0$ when evaluated at $\beta(P)$, the optimal β for *u*. This result shows that the derivative in (6) will be negative for positive values β, but might be positive if β is negative. That is, the risky asset might be a Giffen good at some price levels for which $\beta < 0$. The intuition here is quite usual for trading an endowed asset. When selling $\tilde{x}$ (i.e. $\beta < 0$), an increase in P makes it attractive to sell more (the substitution effect); but this effect is mitigated by an income effect which, under DARA, induces a reduction in risk aversion that makes owning more $\tilde{x}$ attractive. Of course, if preferences do not satisfy DARA, other comparative statics are possible.

Fortunately for us, we are not concerned with the shape of the entire demand curve. We need only concern ourselves with the slope of demand at the fixed (aggregate-endowment) level of supply; that is at $\beta = 0$. At the equilibrium price, (6) is unambiguously negative. As a consequence, the excess-demand curve crosses the horizontal axis, $\beta = 0$, only once, from above. This implies that the equilibrium price is unique and stable. Two examples of $\beta(P)$ are shown in Figure 1.

---

[8]See for example Arrow (1971) or Pratt (1964).

4. Changes in Risk and Lower Asset Prices

Consider a change in the distribution of payoffs on the risky asset from F to G, represented by a change in random variables from $\tilde{x}_1$ to $\tilde{x}_2$, where $F, G \in D^+[a,b]$. We let P_1 and P_2 denote the corresponding equilibrium prices and examine restrictions on the distributional changes which are both necessary and sufficient for $P_1 \geq P_2$, independent of the particular increasing and concave utility function of the representative agent. In other words, consider the set $U(P_1)$ of all risk-averse utility functions yielding P_1 as an equilibrium price. We look at conditions on the distributional changes that guarantee that the new asset price is smaller, independent of the particular utility function that generated P_1 as the equilibrium price.

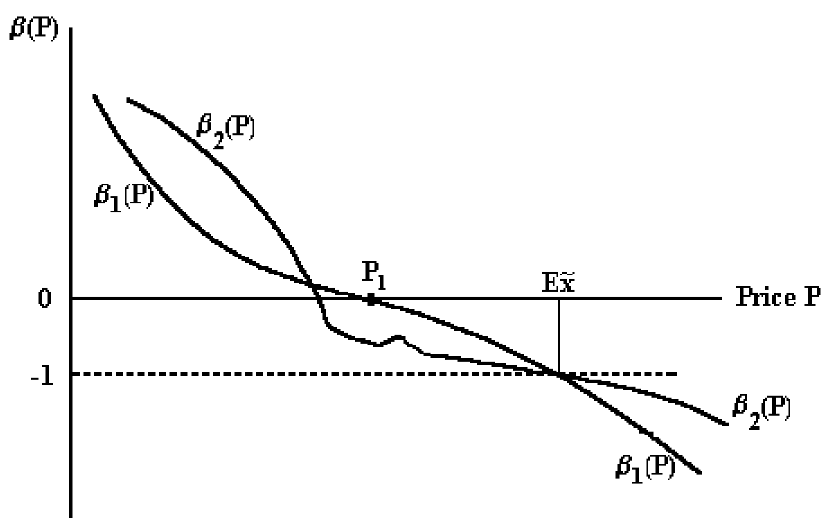

Figure 1

Since the excess demand function $\beta_1(P)$ crosses the horizontal axis exactly once and from above, we only need to consider conditions for which $\beta_2(P_1) \leq 0$. This would automatically imply $P_2 \le P_1$. An example is drawn in Figure 1 for which $\beta_2(P) > \beta_1(P)$ for some out-of-equilibrium prices.

It is useful to note that $\beta_1(E\tilde{x}_1) = -1 = \beta_2(E\tilde{x}_2)$, since a "fair price," $P = E\tilde{x}$, induces every risk averter to fully insure, which in our model entails $\beta = -1$. Thus, if $E\tilde{x}_1 = E\tilde{x}_2$, as is the case we illustrate in Figure 1, the demand curves will intersect at this price. Prices above $E\tilde{x}$ will cause the individual to desire a short position in $\tilde{x}$, i.e. $\beta < -1$. However, in our model this out-of-equilibrium condition is of no consequence, unless $\beta_2(P_1) \le -1$. But in such a case, whether or not we restrict short sales does not matter. All that matters to us is that $\beta_2(P_1) < 0$.

We are interested in determining the conditions on any distributional change in the payoffs that will guarantee that $P_2 \le P_1$. This is given in the following proposition.

Proposition: *Suppose that the equilibrium price of asset $\tilde{x}_1$, with distribution $F \in D^+[a,b]$, is P_1. Then the equilibrium price of asset $\tilde{x}_2$, with distribution $G \in D^+[a,b]$, will be no larger than P_1, independent of the particular concave utility function that generated P_1 [i.e. $\forall\, u \in U(P_1)$], if and only if $\exists \gamma \in \Re$ such that*

$$\text{(7)} \qquad \int_a^x (t - P_1)\,dG \le \gamma \int_a^x (t - P_1)\,dF \quad \forall x \in [a,b].$$

[Remark: When condition (7) holds for some γ, $\tilde{x}_2$ is said to be “Centrally Riskier” than $\tilde{x}_1$ around P_1 (see Gollier (1995))].

Proof:

Since we have established the single-crossing property of the excess demand function, we only need to determine conditions for which

$$\text{(8)} \qquad E(\tilde{x}_1 - P_1)u'(w + \tilde{x}_1) = 0 \qquad \Rightarrow \qquad E(\tilde{x}_2 - P_1)u'(w + \tilde{x}_2) \le 0$$

for all increasing and concave functions u.[9] The first condition in (8) above expresses the fact that u is in $U(P_1)$, whereas the second condition is equivalent to $\beta_2(P_1) < 0$. These conditions are the same as those in the standard portfolio problem under an exogenous change in risk, which has been examined in recent papers by Gollier (1995) and by Gollier and Kimball (1996). In particular, that (7) is both necessary and sufficient for (8) to hold follows directly from applying Proposition 1 in Gollier (1995, p. 525) to the standard portfolio problem. ■

Although condition (7) in the Proposition above is the same as that for the standard portfolio problem, the set-up of the two problems is somewhat different. In the standard portfolio problem, individuals start off with a fixed wealth and zero risky asset, and choose α to maximize $Eu(\hat{w} + \alpha(\tilde{x} - P))$. If we normalize $\tilde{x}$ such that $\alpha^* = 1$, the model is similar to ours with one major difference. A change from $\tilde{x}_1$ to $\tilde{x}_2$ in our model also changes the starting point, since we allocate one unit of risky asset to the individual. Viewed differently, if we define $\hat{w} = w + P$, then the equilibrium wealth prospects are identical, but a change of $\tilde{x}$ in the standard problem would need to be accompanied by a change in $\hat{w}$ (to account for equilibrium price changes) for the comparative statics to be identical. However, any difference in the two models is due only to a wealth effect. Since our Proposition is valid for all risk-averse preferences (e.g. it is valid in cases of both increasing and decreasing absolute risk aversion), any quantitative differences in the two models are of no consequence here. That is, condition (7) turns out to be necessary and sufficient for either $\beta_2(P_1) \le 0$ or $\alpha_2(P_1) \le 0$. Of course, if (7) does not hold, it is possible to find examples for which $\beta_2(P_1)$ and $\alpha_2(P_1)$ differ in sign.

---

[9] It is interesting to note that, if $\tilde{x}_2$ is a mean-preserving increase in risk over $\tilde{x}_1$, then (8) will hold whenever $(x - P_1)U'(W + X)$ is concave in x. Several authors have examined this sufficient condition on preferences. Condition (7), which is a restriction on distributions, is both necessary and sufficient for the implication in (8) to hold.

5. Stochastic Dominance and Asset Prices

Rothschild and Stiglitz (1971) showed that second-order stochastic dominance of $\tilde{x}_1$ over $\tilde{x}_2$ is not sufficient for $\beta_2(P_1) \le \beta_1(P_1)$. Thus, if $\tilde{x}_2$ is riskier than $\tilde{x}_1$ in the sense of Rothschild and Stiglitz (1970), it does not necessarily follow that $\tilde{x}_1$ is Centrally Riskier than $\tilde{x}_2$. This emphasizes the fact that the price of a risky asset (or equivalently, its risk-neutralized expected value) should not be seen as always adjusting in tandem with its risk premium. Second-order stochastic dominance also is not necessary for $\beta_2(P_1) \le \beta_1(P_1)$, as is illustrated by our numerical example below. In our example, $\tilde{x}_2$ is not second-order stochastically dominated by $\tilde{x}_1$, yet any risk-averse economy with $P_1 > 1$ incurs a reduction in the equilibrium price due to this change in distribution. Thus, second-order stochastic dominance is neither necessary nor sufficient for the equilibrium price to be affected in an unambiguous way. The same is easily shown to be true for first-order stochastic dominance as well.

<u>Example:</u> Let $w = 0$ and let $\tilde{x}_1$ be a discrete random variable distributed as $(0, \frac{1}{2}; 3, \frac{1}{2})$. Let $\tilde{x}_2$ be distributed as $(0, \frac{1}{3}; 1, \frac{1}{3}; 3, \frac{1}{3})$. The change is risk is made by transferring some probability mass from the tails to an atom at $x = 1$. In some sense the risk is reduced, but the mean is also reduced. Thus, there is no second-order stochastic dominance. Suppose that we observed an equilibrium price $P_1 = 1.1$. Can one guarantee that the change in distribution of the payoffs from $\tilde{x}_1$ to $\tilde{x}_2$ will reduce the equilibrium asset price in this economy, under the initial observation that $P_1 = 1.1$? This is indeed the case, since condition (7) is satisfied with $P_1 = 1.1$ and $\gamma = 2/3$. Thus, $\tilde{x}_2$ is Centrally Riskier than $\tilde{x}_1$ around $P_1 = 1.1$.

Unfortunately, defining $\tilde{x}_1$ and $\tilde{x}_2$ as in the example above does not always lead to $P_1 < P_2$ when $P_1 \ne 1.1$. Indeed, $\tilde{x}_2$ is not Centrally Riskier than $\tilde{x}_1$ around P_1, when $P_1 < 1$. For example, consider utility yielding the following marginal utility function:

$$u'(z) = \begin{cases} 1 & if \quad z < 0.1 \\ 0.2 & if \quad z \geq 0.1 \end{cases}$$

It is easily checked that $P_1 = 0.5$ in this economy, but $P_2 = 4/7 > P_1$. Thus, the change in risk from $\tilde{x}_1$ to $\tilde{x}_2$ always reduces the asset price when $P_1 = 1.1$, but it may increase the equilibrium price when $P_1 \neq 1.1$.

We should be careful to point out that our Proposition is dependent upon the price P_1 as the above example illustrates. Indeed, if we search for conditions for which the change in risk from $\tilde{x}_1$ to $\tilde{x}_2$ always reduces the asset price, regardless of the initial price P_1, then second-order stochastic dominance is necessary.[10]

6. Sufficient Increases in Risk for Lower Asset Prices

Although section 4 portrays conditions that are both necessary and sufficient for changes in the distribution of asset payoffs to yield a lower equilibrium price, condition (7) may not always be easily verifiable. In this section, we examine whether several restrictions on changes in risk, each of which has appeared in previous literature, might be sufficient in reaching the conclusion that $P_2 \leq P_1$ in our Proposition. Verifying (7) for a fixed value of γ, for example, is a much easier task. For example, the case where $\gamma = 1$, as originally examined by Rothschild and Stiglitz (1971), may be easily verifiable.

Meyer and Ormiston (1985) introduced the notion of a *strong increase in risk*, where some probability mass is taken from the initial support of the distribution of

[10] A proof can be found in Gollier & Schlesinger (1998). In particular, the left-hand side of (8) is assumed to hold in our Proposition, so that P_1 is the equilibrium price.

payoffs and is transferred outside of the convex hull of the initial support, while preserving the mean.[11] They show that their condition is sufficient to reduce the demand for risky assets by any risk-averse investor. It can be easily verified that a strong increase in risk is a particular case of centrally riskier shifts in distribution, in which condition (7) is satisfied with $\gamma = 1$.

The restriction of transferring the probability mass outside of the initial support, though trivial to verify, is quite a strong restriction, and one that might not apply in many real world situations. Both Black and Bulkley (1989) and Dionne, Eeckhoudt and Gollier (1993) relax this notion by obtaining weaker sufficient conditions called respectively *relatively strong increases in risk* and *relatively weak increases in risk* around P_1. These definitions allow for more generality in that they allow for some spreading of probability mass within the interior of the initial support. However, this added generality is gained at a cost of more-complex conditions to verify, which involve likelihood ratios.

Dionne and Gollier (1992) obtain a very appealing sufficient condition called a *simple increase in risk* around P_1 in which the two cumulative distribution functions must cross only once *at* P_1. This condition is easy to verify and supports many nice real-world scenarios. It is essentially equivalent to requiring an increase in risk such that, at the original price P_1,

(i) the expected profit remains the same

(ii) the odds of getting any final positive profit level or higher is increased, and

(iii) the odds of getting any final negative profit level or lower is increased.

In such a case, the equilibrium price will adjust downwards. Again, one can easily show that a simple increase in risk around P_1 is a particular case of a Centrally Riskier shift in the distribution around P_1, with $\gamma = 1$.

---

[11] Actually, the Meyer and Ormiston result can be considered as an extension of the note by Eeckhoudt and Hansen (1980), who consider increases in risk that shift mass from the initial distribution to the two extremum of the probability support.

For the set of distribution shifts satisfying first-order stochastic dominance, Landsberger and Meilijson (1990) obtain an unambiguous comparative static result for the demand of a risky asset, if the shift in distribution satisfies the well-known monotone likelihood ratio property. While this property is examined often in the literature, it is rather restrictive. Many first-order stochastic dominance shifts that lead to a lower asset price do not satisfy this property. Moreover, this condition is sometimes difficult to verify. A subset of distribution shifts satisfying the monotone likelihood ratio property was examined recently by Eeckhoudt and Gollier (1996). They define distributional shifts sich that the ratio G(t)/F(t) is nondecreasing in t as satisfying the *monotone probability ratio* criteria. This easily verifiable condition is also sufficient for a lower equilibrium asset price.

Both the monotone likelihood ratio property and the monotone probability ratio property have the advantage of being independent of P_1. In other words, they guarantee a lower equilibrium and asset price under G than under F, regardless of the price under asset-distribution F.

7. Background Risk and the Equity Premium Puzzle

The analysis thus far has been based within the context of a market with no background risk. But, as mentioned by Mehra and Prescott (1985), some empirical puzzles, such as the equity premium puzzle, might be answerable if we assume that markets are incomplete.[12] Weil (1992), for example, assumes an uninsurable idiosyncratic risk for each individual, due to uncertain second-period labor income, and shows how this type of background risk might lead to a higher equity premium. In this

[12] Some compelling new evidence by Goetzmann and Jorion (1997), however, shows that the equity premium might not be quite as high as is typically supposed. Thus, it may be a smaller "puzzle" that needs to be rationalized by the theory.

section, we present a simple extension of our own model that captures the essence of Weil's model. We extend this setting in the next section to consider how a miscalibration of the background risk might offer some additional insight into the equity premium puzzle.

In our model, we can consider the equity premium as essentially $E\tilde{x} - P$. Using empirical observations, this difference is often regarded as too high to be supported by realistic preferences within a representative agent model.[13] In our static model, we achieve essentially the same effect as Weil by replacing the fixed initial wealth, w, with $w + \tilde{\varepsilon}$, where we assume $\tilde{\varepsilon}$ is independent of asset payoffs and $E\tilde{\varepsilon} = 0$.[14] In this case, we proceed by defining the derived utility function (see Kihlstrom et al. (1991)) as

$$v(y) \equiv Eu(y + \tilde{\varepsilon}) \; \forall y,$$

where expectations are taken over the distribution of $\tilde{\varepsilon}$. The function $v(\cdot)$ represents a well-defined von Neuman-Morgenstern utility of wealth and v inherits both monotonicity and risk aversion from u. Consequently, our Proposition will hold for v as well for u; or put differently, conditions on changes from $\tilde{x}_1$ to $\tilde{x}_2$ that ensure $P_2 \leq P_1$, with a fixed background wealth, w, will also apply when background wealth is risky, $w + \tilde{\varepsilon}$.

In applying the Proposition, however, we must caution that the equilibrium price itself will most surely be different in the presence of background risk. Indeed, if P_1 denotes the equilibrium price of $\tilde{x}$ in a particular market economy without background risk, then the introduction of independent background risk $\tilde{\varepsilon}$ can cause the equilibrium price to either rise or fall to some new equilibrium price, say $\hat{P}_1$. Therefore, applying the Proposition to P_1 will lead to conditions for which $\hat{P}_2 \leq P_1$. In other words, we will only

---

[13] For example, using the fairly common assumption of constant relative risk aversion, the level of relative risk aversion implied by the data typically falls at unrealistically high levels, such as 15 or 20. With risk aversion equaling 20, for example, a person with $100 of initial wealth would have a certainty equivalent of $3.42 for a 50-50 gamble between $100 and $0.

[14] See Doherty and Schlesinger (1983) and Eeckhoudt and Kimball (1992) for discussion of the case where $\tilde{\varepsilon}$ and $\tilde{x}$ are not independent.

determine whether we can guarantee that $\hat{\beta}_2(P_1) \le \hat{\beta}_1(P_1)$, where $\hat{\beta}_i$ denotes excess demand for asset $\tilde{x}_i$ in the incomplete-market economy. This in turn will not be sufficient to infer whether $\hat{\beta}_2(\hat{P}_1) \le \hat{\beta}_1(\hat{P}_1) = 0$, and so we do not know whether $\hat{P}_2 \le \hat{P}_1$.

In addressing the equity-premium puzzle, we do not need to apply the Proposition at all. Rather, we need only consider the effect of $\tilde{\varepsilon}$ on the demand for the risky asset. Although the effect of adding $\tilde{\varepsilon}$ is ambiguous, a priori, Weil (1992) assumes that preferences are not only risk averse, but also exhibit Kimball's (1993) standard risk aversion. In such a case, it is simple to show that the derived utility function is more risk averse than u.[15] Therefore, under the same risky payoff $\tilde{x}_1$, the equilibrium price in the market with background risk, $\hat{P}_1$, will be less than the price in a market without background risk, P_1. Since the expected payoff, $E\tilde{x}_1$ remains unchanged, Weil argues that a market analyst who ignores the idiosyncratic risk $\tilde{\varepsilon}$ (or who uses macro data to replace $\tilde{\varepsilon}$ with $E\tilde{\varepsilon} = 0$) and calculates price P_1 will overstate the equilibrium price, or equivalently understate the true (empirical) equity premium.[16]

8. Miscalibrated Risk and the Equity Premium

The background-risk model of the previous section leads to an interesting extension in the context of changes in risk. Suppose again that $\tilde{x}_1$ and $\tilde{\varepsilon}$ are independent with $E\tilde{\varepsilon} = 0$. Now define $\tilde{x}_2 =_d \tilde{x}_1 + \tilde{\varepsilon}$, where "$=_d$" denotes "equal in distribution." In

[15]See Eeckhoudt and Kimball (1992). Standard risk aversion is fully characterized by preferences exhibiting both DARA and decreasing absolute prudence, where absolute prudence is defined as $-u'''(y)/u''(y)$. Although Weil assumes the sufficient condition of standard risk aversion, we know from Gollier and Pratt (1996) that the weaker condition of risk vulnerability is sufficient, and also necessary, for v to be more risk averse than u. A sufficient condition for risk vulnerability is absolute risk aversion being decreasing and convex.

[16]This manifests itself in the expected return (4) in our model. Weil's model is a bit more complex, since background risk in his model also induces a change in the riskfree rate of return. To be sure, Kocherlakota's (1996) criticism of Weil's lack of sufficient dynamic structure applies even more strongly to our model. However, Kocherlakota is not correct in claiming that individuals have a natural temporal hedge of the $\tilde{\varepsilon}$ risk. The invalidity of this argument was originally made by Samuelson (1963), and examined more recently by Gollier, Lindsey and Zeckhauser (1997). Moreover, dynamic hedging strategies are likely to be themselves imperfect. See, for example, Constantinides and Duffie (1996).

other words, rather than attach the "noise" term $\tilde{\varepsilon}$ to initial wealth, we attach it here to the original asset distribution, represented via $\tilde{x}_1$. Clearly $\tilde{x}_2$ is riskier than $\tilde{x}_1$ in the second-degree stochastic dominance sense of Rothschild and Stiglitz (1970). Consider final wealth, $\tilde{y}_2 = w + \tilde{x}_2 + \beta_2(\tilde{x}_2 - P_1)$. Considering whether there is excess demand for the asset under price P_1, we consider

$$(12) \qquad \left.\frac{dEu(\tilde{y}_2)}{d\beta_2}\right|_{\beta_2=0} = Eu'(\tilde{y}_2)\cdot(\tilde{x}_1 + \tilde{\varepsilon} - P_1)$$

$$= E[u'(w+\tilde{x}_1+\tilde{\varepsilon})\cdot(\tilde{x}_1 - P_1)] + E[u'(w+\tilde{x}_1+\tilde{\varepsilon})\cdot\tilde{\varepsilon}]$$

$$= Ev'(w+\tilde{x}_1)\cdot(\tilde{x}_1 - P) + \text{cov}(u'(\tilde{y}_2),\tilde{\varepsilon}).$$

We see in the above model that a change in risk from $\tilde{x}_1$ to $\tilde{x}_1 + \tilde{\varepsilon}$ yields a partial derivative in (12) consisting of two terms. The first term is identical to the valuation of $dEu/d\beta$ evaluated at $\beta = 0$ and at P_1 under the addition of an uninsurable background risk added to initial wealth (which in turn is equivalent to replacing *u* with the derived utility function *v*); exactly the case we studied in the previous section (Weil's model). The second term is a covariance term, which is negative due to risk aversion. Thus, conditions that are sufficient to render the first term on the right-hand side of (12) nonpositive, will also be sufficient to render all of (12) negative. Standard risk aversion is one such a condition.[17]

In Weil's (1992) background risk model, the $\tilde{\varepsilon}$ risk was private information and therefore uninsurable due to observability asymmetries. However, it appears to us that even if $\tilde{\varepsilon}$ is identical for all individuals, i.e. perfectly correlated, there is also a good case to be made for uninsurability.

---

[17]See Kimball (1993) for a justification of standardness. We should point out that all constant relative risk aversion utility functions, for example, are standard. Moreover, utility satisfying a weak version of standardness, namely CARA utility, is sufficient for (12) to be negative. Observe that the weaker condition of risk vulnerability as introduced by Gollier and Pratt (1996) is also sufficient to sign (12) as negative.

We thus have an extension of Weil's arguments for the equity-premium puzzle under miscalibrated risk: Suppose that the market analyst examines empirical data and calculates a distribution function for $\tilde{x}_1$ as F. Obviously such a calculation is based on historical data, which is by necessity only a sampling distribution of the true distribution. If consumers all possess the same distributional information as the analyst, but consumers include a spurious noise term $\tilde{\varepsilon}$ in their projected distribution, we once again have a model in which the analysts' projected P_1 is higher than the empirical equilibrium price, P_2. This, of course, leads to a higher empirical equity premium than the analyst's prediction. Hence, we have another potential explanation for the equity-premium puzzle. Moreover, if the same level of background risk $\tilde{\varepsilon}$ embeds itself by attaching to $\tilde{x}_1$, rather than to w, the effect on the equity premium is greater than in Weil's model, the difference being accounted for by the extra (covariance) term on the right-hand side of equation (12).

Of course one might think that this conclusion is obvious: any underestimation of the level of "risk" will always lead to a higher equilibrium price, thus yielding a higher than expected equity premium. However, as our analysis of section 5 demonstrates, if risk increases are in the form of a second-order stochastic dominance deterioration, this conclusion is false. Indeed, even under a change from $\tilde{x}_1$ to $\tilde{x}_2 =_d \tilde{x}_1 + \tilde{\varepsilon}$, as given in this section, risk aversion alone is not sufficient to unambiguously conclude that $P_2 \leq P_1$, and we need stronger assumptions, such as standard risk aversion, to obtain definitive results.

9. Conclusion

Our main objective in this paper has been to derive necessary and sufficient conditions on changes in the distribution of payoffs for a risky asset that would always reduce the asset's equilibrium price, assuming only risk aversion on the part of consumers. For example, although it is well known that second-order stochastic dominance is not sufficient to achieve definitive comparative statics in the standard portfolio problem, we wondered whether some attribute of economic equilibrium might lead to its sufficiency in determining prices. This turned out not to be the case. The relevant restriction on the change in distribution is called *Central Riskiness* and it is specific to the initial equilibrium price that has been observed. Second order stochastic dominance is neither necessary nor sufficient for a Centrally Riskier shift in the distribution. Since the conditions we obtained are not always easy to check, we presented several simple sufficient conditions for central riskiness. These are conditions that currently exist in the literature and that are sufficient for signing the effect of a change in asset risk on the asset's equilibrium price.

Additionally, we examined the equity premium puzzle. We first examined a simplified version Weil's model by adding a background risk to initial wealth. We then attached the background risk to the initial asset-payoff distribution, rather than to initial wealth. We showed how a miscalibration of risk, together with an assumption that preferences are standard offers a new potential explanation for empirically high equity premia.

REFERENCES

ARROW, K., (1971), *Essays in the Theory of Risk Bearing*, Markham Publishing Co, Chicago.

BLACK, J.M. and G. BULKLEY (1989), A Ratio Criterion for Signing the Effect of an Increase in Uncertainty, *International Economic Review*, 30, 119-130.

CONSTANDINIDES, G., (1982), Intertemporal Asset Pricing with Heterogeneous Consumers and Without Demand Aggregation, *Journal of Business*, 55, 253-267.

CONSTANDINIDES, G. and D. DUFFIE, (1996), Asset Pricing with Heterogeneous Consumers, Working Paper, University of Chicago.

DIONNE, G., L. EECKHOUDT and C. GOLLIER, (1993), Increases in Risk and Linear Payoffs, *International Economic Review*, 34, 309-319.

DIONNE, G and C. GOLLIER, (1992), Comparative Statics Under Multiple Sources of Risk with Applications to Insurance Demand, *The Geneva Papers on Risk and Insurance Theory*, 17, 21-33.

DOHERTY, N. and H. SCHLESINGER, (1983), Optimal Insurance in Incomplete Markets, *Journal of Political Economy*, 91, 1045-1054.

EECKHOUDT, L. and C. GOLLIER, (1995), Demand for Risky Assets and the Monotone Probability Ratio Order, *Journal of Risk and Uncertainty*, 11, 113-122.

EECKHOUDT, L. and P. HANSEN (1980), Minimum and Maximum Prices, Uncertainty and the Theory of the Firm, *American Economic Review*, 70, 1064-1068.

EECKHOUDT, L. and M.S. KIMBALL, (1992), Background Risk, Prudence and the Demand for Insurance, *in Contributions to Insurance Economics*, G. Dionne Ed., Kluwer, 239-254.

GOETZMANN W.N. and P. JORION, (1997), A Century of Global Stock Markets, NBER Working paper No. 5901.

GOLLIER, C., (1995), The Comparative Statics of Changes in Risk Revisited, *Journal of Economic Theory*, 66, 522-535.

GOLLIER, C., (1996), A Note on Portfolio Dominance, forthcoming in the *Review of Economic Studies*.

GOLLIER, C. and M.S. KIMBALL, (1996), New Methods in the Classical Economics of Uncertainty: Comparing Risks, Discussion Paper, University of Toulouse.

GOLLIER, C. and J.W. PRATT, (1996), Risk Vulnerability and the Tempering Effect of Background Risk, *Econometrica*, 64, 1109-1123.

GOLLIER, C. and H. SCHLESINGER, (1998), Changes in Risk and asset Prices, Discussion Paper, University of Toulouse.

GOLLIER, C., J. LINDSEY and R.J. ZECKHAUSER, (1997), Investment Flexibility and the acceptance of Risk, *Journal of Economic Theory*, 76, 219-241.

HADAR, J. and SEO, (1990), The Effects of Shifts in a Return Distribution on Optimal Portfolios, *International Economic Review*, 31, 721-736.

KIHLSTROM, R., D. ROMER and S. WILLIAMS, (1981), Risk Aversion with Random Initial Wealth, *Econometrica*, 49, 911-920.

KIMBALL, M.S., (1993), Standard Risk Aversion, *Econometrica*, 61, 589-611.

KOCHERLAKOTA, N.R., (1996), The Equity Premium: It's Still a Puzzle, *Journal of Economic Literature*, 34, 42-71.

LABADIE, P., (1986), Comparative Dynamics and Risk Premia in an Overlapping Generations Model, *Review of Economic Studies*, 53, 139-152.

LANDSBERGER, M. and I. MEILIJSON, (1990), A Tale of Two Tails : An Alternative Characterization of Comparative Risk, *Journal of Risk and Uncertainty*, 3, pp. 65-82.

LANDSBERGER, M. and I. MEILIJSON, (1993), Mean-Preserving Portfolio Dominance, *Review of Economic Studies*, 60, 479-485.

LUCAS, R.E., (1978), Asset Prices in an Exchange Economy, *Econometrica*, 46, 1429-1446.

MEHRA, R. and E. PRESCOTT, (1985), The Equity Premium: A Puzzle, *Journal of Monetary Economics*, 10, 335-339.

MEYER, J. and M.B. ORMISTON, (1985), Strong Increases in Risk and their Comparative Statics, *International Economic Review*, 26, 425-437.

MOSSIN, J., (1968), Optimal Multiperiod Portfolio Policies, *Journal of Business*, 41, 215-229.

PRATT, J.W., [1964], Risk Aversion in the Small and in the Large, *Econometrica*, 32, 122-136.

ROTHSCHILD, M. and J. STIGLITZ, (1970), Increasing Risk : I. A Definition, *Journal of Economic Theory*, 2, 225-243.

ROTHSCHILD, M. and J. STIGLITZ, (1971), Increasing Risk: II. Its Economic Consequences, *Journal of Economic Theory*, 3, 66-84.

RUBINSTEIN, M., (1976), The Valuation of Uncertain Income Streams and the Pricing of Options, *Bell Journal of Economics*, 7, 407-425.

SAMUELSON, P.A., (1963), The Fallacy of the Law of Large Numbers, *Sciencia*, 98, 108-113.

SEGAL, U. and A. SPIVAK, (1990), First 0rder Versus Second Order Risk Aversion, *Journal of Economic Theory*, 51, 111-125.

WEIL, P., (1989), The Equity Premium Puzzle and the Risk-Free Rate Puzzle, *Journal of Monetary Economics*, 24, 401-421.

WEIL, P., (1992), Equilibrium Asset Prices with Undiversifiable Labor Income Risk, *Journal of Economic Dynamics and Control*, 16, 769-790.