Capital structure and the prediction of bankruptcy

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Preliminary version, all comments welcome. This version: July 2002

ABSTRACT:
This paper addresses the theoretical foundations of bankruptcy prediction, using the neo-classical theory of capital structure as a starting point. The paper intends to demonstrate the feasibility of such an approach in a simple setting, i.e. by using a simple theoretical model and a limited empirical analysis. A model of optimal capital structure is constructed and rewritten as a model of default probability. Its empirical implications are derived and tested on a sample of Norwegian data. It is concluded that this approach clearly has its limitations, but also that it may be a valuable contribution compared to the multitude of theory-less empirical studies and a useful alternative to the default theory based on option pricing.

Keywords: Default Probabilities, Capital structure, Logistic regression

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1. Introduction

The recent bankruptcies of Enron and Worldcom have underlined the importance of default prediction both in academia and in industry. It now seems more necessary than ever to develop early warning systems that can help prevent or avert corporate default and that facilitate the selection of firms to collaborate with or invest in. Research on default prediction has been conducted for many decades and a very large number of empirical studies has been published since the pioneering work of Beaver (1966, 1968) and Altman (1968). The initial approach to predicting corporate failure has been to apply a statistical classification technique (usually discriminant analysis) to a sample containing both failed and non-failed firms. Examples of such studies are Deakin (1972) and Altman et al. (1977). After that, emphasis shifted toward probit or logit analysis. Martin (1977) and Ohlson (1980) were among the first to apply these techniques, followed by e.g. Wiginton (1980), Zmijewski (1984), Zavgren (1985), Aziz and Lawson (1989), Lennox (1999) and Westgaard & Van der Wijst (2001)). Other statistical techniques have also been introduced, such as recursive partitioning (Frydman et al. (1985)), catastrophe theory (Gregory et al. (1991)), multidimensional scaling (Mar Molinero and Ezzamel (1991)), neural networks (Tam and Kiang (1992)), multinominal logit models (Johnsen and Melicher (1994)), multicriteria decision aid methodology (Zopounidis and Doumpos, 1999) and rough sets (Dimitras et al., 1999). Reviews studies can be found in Jones (1987), Karels and Prakash (1987) and Dimitras et al. (1996).

The general conclusions from this extensive research effort seem to be that each study by itself provides a reasonable discrimination between failed and non-failed firms, but also, and perhaps more significantly, that the various studies hardly show any agreement on what factors are important for failure prediction. Indeed, it can be said that more than 30 years of empirical research on bankruptcy prediction failed to produce agreement on which variables are good predictors and why. This discord of conclusions can, of course, partly be attributed to the fact that the studies refer to different periods, countries and industries. Another factor may be that virtually all of these studies lack a theoretical framework to guide the empirical research effort. In the absence of a theory that provides testable hypotheses, each empirical result has to be evaluated on its own merits and one can only hope that patterns emerge from the multitude of results. This is obviously not the case in the default prediction.

This paper addresses the theoretical underpinnings of bankruptcy prediction, taking the well-known neo-classical theory of capital structure as a starting point. Thus, it follows an alternative approach compared to the well-known Merton model (Merton, 1974), that is based on option pricing theory and that is elaborated into the KMV model. The origins of a capital structure based default theory lie, on the one hand, in models that relate the risk of ruin to the valuation of corporate claims (see e.g. Gordon, (1971), Scott (1977), and Vinso (1979)). A later elaboration can be found in Scott (1981). On the other hand, they lie in the models of optimal capital structure that were developed in the wake of the famous Modigliani-Miller irrelevance theorem (Modigliani and Miller (1958, 1963), Baxter (1967), Kraus and Litzenberger (1973), Scott (1976), and Kim (1978)). Practically all models of optimal capital structure use a default (or bankruptcy) condition in the derivation of optimal capital structure. This condition captures the essence of the default decision: it obtains when the value of the various cash flows available to firm are insufficient to cover the debt obligations. With this, the theory derives properties of the optimal capital structure in its comparative statics, which are the basis for empirical analyses. Surprisingly, these models are seldom, if ever, rewritten to explicitly state the probability of bankruptcy and its characteristics, i.e. how it is influenced by the determinants of optimal capital structure. Since the early eighties, this line of theoretical research appears to be completely outstripped by the option based default theories. This paper aims to contribute to a capital structure based default theory by demonstrating its feasibility in a simple setting, i.e. by using a simple theoretical models and limited empirical analysis.

The organization of this paper is as follows. Section 2 firstly restates a simple model of optimal capital structure and subsequently rewrites the model to elicit the determinants of the probability of default. Testable hypotheses are formulated on the basis of the comparative statics of the model. These hypotheses are tested empirically on Norwegian firm data from the period 1995-2001. Section 3 describes the data and methodology, section 4 presents and discusses the results. Conclusions are formulated in section 5.

2 Default probabilities in a capital structure framework

2.1 A simple model of optimal capital structure

The model used here is a simple, single period model of optimal capital structure. The model allows only two market imperfections: taxes and bankruptcy costs. This suffices to capture the essence of the so-called trade-off theory, in which optimal capital structures are set as a trade-off between tax advantages and expected bankruptcy costs. The model is adopted from Van der Wijst (1989), in which a family of related models is elaborated. The reader is referred to this publication for further details. Some details of the calculations are included in appendix B.

The major assumptions of the model are as follows. Capital markets are assumed to be costless and competitive. Corporate profits are taxed at a fixed rate and according to a wealth tax system that allows the deduction of all payments to the debt holders, including principal repayments, from the firm's taxable income. However, there are no other tax deductible items or tax shields and there are no personal taxes. All market participants are assumed to be insatiable and to act rationally. The firm‘s set of income generating assets is assumed to be fixed; i.e. all investment decisions are already made but the financing decision not. Firms only issue equity and debt. Debt claims are only subject to the risk of default. Finally, investors are assumed to be risk neutral and to have limited liability.

In this setting, a firm's cash flow is the only source that can be used to cover the obligations to the debt holders. Consequently, if these obligations exceed the firm's cash flow, the firm defaults and is declared bankrupt. Thus, the bankruptcy condition, *b*, is:

$$b = \tilde{x} < R \tag{2.1}$$

where $\tilde{x}$ is a random variable representing the firm's cash flow before interest and taxes and R is the payment to the debtholders. $\tilde{x}$ is assumed to be normally distributed with mean μ_x and standard deviation σ_x . If, at the end of the period, b obtains, stockholders are protected by limited liability and receive nothing. Otherwise they receive the cash flow after taxes and interest. The end of period value of equity, Y_e, is:

$$\begin{aligned} &Y_e = 0 && \text{if } \tilde{x} < b \\ &Y_e = (1-\tau)(\tilde{x} - R) && \text{if } \tilde{x} \geq b \end{aligned} \tag{2.2}$$

where τ is the corporate tax rate. For risk neutral investors, the equilibrium value of equity, V_e.,is the present value, discounted at the risk free interest rate, of the expectation of Y_e, :

$$V_e = \frac{E(Y_e)}{(1+r)} = \frac{(1-\tau)\int_b^{\infty}(\tilde{x} - R)f(\tilde{x})d\tilde{x}}{(1+r)} \tag{2.3}$$

where *r* is the risk free interest rate. The debt holders' value at the end of the period, Y_d, can be derived in a similar way. If the bankruptcy condition obtains, the firm is transferred to the debtholders, which means that they receive the cash flow minus the bankruptcy costs. Limited liability prevents them from having to accept a negative cash flow. So the end of period value of debt is:

$$\begin{aligned} &Y_d = 0 && \text{if } \tilde{x} \leq 0 \\ &Y_d = \tilde{x} - B(\tilde{x}) && \text{if } 0 < \tilde{x} < b \\ &Y_d = R && \text{if } \tilde{x} \geq b \;\; \text{(b=R)} \end{aligned} \tag{2.4}$$

were $B(\tilde{x})$ is the amount of bankruptcy costs as a function of the cash flow $\tilde{x}$ [1]. The equilibrium value of the debt is the present value of the expectation of Y_d:

[1] $B(\tilde{x})$ is not greater than $\tilde{x}$, always positive in the bankruptcy zone and zero otherwise. $B(\tilde{x})$ is also assumed to be twice differentiable with a positive first derivative and a second derivative equal to or larger than zero.

$$V_d = \frac{\int_0^b (\tilde{x} - B(\tilde{x})) f(\tilde{x}) d\tilde{x} + R(1-F)}{(1+r)} \tag{2.5}$$

where F is defined as the probability to default: $F = \int_{-\infty}^{b} f(\tilde{x}) d\tilde{x}$. Since it is defined as a cumulative density function, the probability of default will always be between 0 and 1.

The total value of the firm is found by adding V_e and V_d , which, after rearranging terms[2], is:

$$V = \frac{\int_0^\infty \tilde{x} f(\tilde{x}) d\tilde{x} - \tau \int_b^\infty \tilde{x} f(\tilde{x}) d\tilde{x} - \int_0^b B(\tilde{x}) f(\tilde{x}) d\tilde{x} + \tau R(1-F)}{(1+r)} \tag{2.6}$$

Optimal capital structure and debt capacity are found by differentiating V and V_d with respect to R:

$$\frac{\partial V}{\partial R} = \frac{\tau(1-F) - B(R) f(R)}{(1+r)} \tag{2.7}$$

$$\frac{\partial V_d}{\partial R} = \frac{(1-F) - B(R) f(R)}{(1+r)} \tag{2.8}$$

Where B(R) and f(R) are the bankruptcy cost function and the pdf of the cash flow respectively, both evaluated at the point of optimal capital structure.

Setting equation (2.8) equal to zero gives the maximum amount of debt the debt holders are willing to supply, which is the firm's debt capacity. Equation (2.7) set equal to zero gives the amount of debt that maximizes the value of the firm, i.e. the optimal capital structure. It can be shown that for normally distributed cash flows the second order conditions for (2.7) and (2.8) are satisfied. Because the corporate tax rate, τ, has a value between zero and one, the amount of debt in the optimal capital structure is less than the amount debt holders are willing to supply. This means that equation (2.8) does not limit the amount of debt the company can obtain, i.e. optimal capital structure is reached before debt capacity. Rewriting equation (2.7) gives:

$$\frac{\tau(1-F)}{(1+r)} = \frac{B(R) f(R)}{(1+r)} \tag{2.9}$$

The left hand side represents the present value of the marginal expected tax saving, while the right hand side represents the present value of marginal bankruptcy costs. Thus, the optimal capital

[2] See appendix B for some intermediate calculations

structure is reached as the marginal benefits of debt financing equal the marginal costs. A more extensive discussion of the model and further computational details including comparative statics can be found in Van der Wijst (1989).

2.2 The bankruptcy probability model

In this section, the capital structure model is reformulated as a bankruptcy probability model and further analysed. Equation (2.9) represents the optimal choice of capital structure as a function of the tax rate, bankruptcy costs and the distributional properties of the cash flow including the probability of default. Rearranging the terms of equation (2.9) gives an expression for the probability of default:

$$F = 1 - \frac{B(R) f(R)}{\tau} \qquad (2.10)$$

where all variables are as defined before.

Note that (2.10) reflects the consequences for the probability of default of the decision to maximize the value of the firm using capital structure as an instrument. The default probability itself is neither a goal variable (to be minimized or optimised) nor a direct instrument. The default probability is, of course, manipulated indirectly by choosing levels of R. In equation (2.10) the default probability is dependent on the tax rate, bankruptcy costs and the distributional properties of the cash flow.

To further analyse the model, its comparative statics[3] are calculated. These show the effect on the probability of default, F, of changes in the variables in the model. The comparative statics of the model are given below, some more detailed calculations are appended.

- The default probability F depends on the debt level in the following way:

$$\frac{\partial F}{\partial R} = -\frac{f(R)}{\tau}\left[\frac{B(R)(\mu_x - R)}{\sigma_x^2} + B'(R)\right] < 0 \quad \text{if } \mu_x \geq R \qquad (2.11)$$

Since f(R), the corporate tax rate, the bankruptcy costs, the variance of the cash flow and the first derivative of the bankruptcy costs are all positive, (2.11) is only strictly negative if $\mu_x \geq R$. Otherwise the sign depends on the relative sizes of the other variables and cannot be determined unambiguously. This means that the effect of leverage on the probability on default cannot be determined unambiguously, and in the range where it can be determined unambiguously its influence is contrary to what conventional wisdom predicts.

[3] See e.g. Silberberg (1981) for a discussion of the methodology of comparative statics.

- The change in F due to a change in the tax rate is:

$$\frac{\partial F}{\partial \tau} = \frac{B(R) f(R)}{\tau^2} > 0 \tag{2.12}$$

Both the bankruptcy costs, f(R) and τ, the corporate tax rate, are positive. This means that an increase in the corporate tax rate will increase the probability of default. It makes debt financing more attractive on the margin, leading to more debt in the optimal capital structure and higher default probabilities.

- Derivation of F with respect to the bankruptcy costs gives:

$$\frac{\partial F}{\partial B(R)} = -\frac{f(R)}{\tau} < 0 \tag{2.13}$$

Since f(R) and τ are both positive, (2.13) will be negative. An increase in the bankruptcy costs makes debt financing less attractive on the margin, leading to less debt in the optimal capital structure which reduces the probability of default.

- The change in the standard deviation of the cash flow will affect the probability of default as follows:

$$\frac{\partial F}{\partial \sigma_x} = -\frac{B(R)\frac{\partial f}{\partial \sigma_x}}{\tau} = \frac{f(R)B(R)}{\tau}\left[\frac{1}{\sigma_x} - \frac{(R-\mu_x)^2}{\sigma_x^3}\right] \tag{2.14}$$

Although (2.14) looks a bit complicated at first sight, f(R), B(R), τ and σ_x are all positive, so the equation within the square brackets will determine the sign of (2.14). This can be narrowed down to:

$$\begin{aligned} \sigma_x^2 - (R-\mu_x)^2 &< 0 \quad \text{if } R-\mu_x > \sigma_x \\ &= 0 \quad \text{if } R-\mu_x = \sigma_x \\ &> 0 \quad \text{if } R-\mu_x < \sigma_x \end{aligned} \tag{2.15}$$

Thus, the comparative static of the standard deviation of the cash flow depends on whether the difference between the expected earnings and the debt obligations is larger or smaller than the standard deviation in earnings.

- Finally, a change in the expectation of the future cash flows μ_x on F is:

$$\frac{\partial F}{\partial \mu_x} = -\frac{B(R)\dfrac{\partial f}{\partial \mu_x}}{\tau} = \frac{f(R)B(R)}{\tau\sigma_x^2}(\mu_x - R) > 0 \quad \text{if} \quad \mu_x > R \qquad (2.16)$$

$$= 0 \quad \text{if} \quad \mu_x = R$$

$$< 0 \quad \text{if} \quad \mu_x < R$$

Since f(R), the bankruptcy costs B(R), the tax rate and the variance of the cash flow are all positive, the sign of (2.16) depends on the relation between the expectation of the cash flow and the level of debt.

The comparative statics of the default probability model are summarised in Table 1 below.

COMPARATIVE STATICS: $\frac{\partial F}{\partial ..}$	EXPECTED EFFECT ON F
R, debt	**Positive or Indetermined**
τ , tax rate	**Positive**
B(R), bankruptcy costs	**Negative**
σ , std.dev. of cash flow	**Positive or Negative**
μ , expected cash flow	**Positive or Negative**

Table 1: The influence of the variables in the model on the probability of default

The most striking aspect of table 1 is, of course, that neither capital structure nor the distributional properties of the cash flow (expectation and variance) have a straightforward effect on the probability of default. A comparable conclusion was reached in the comparative statics analysis of the optimal capital structure model (i.e. the probability of default has an ambiguous influence on optimal capital structure, see Van der Wijst, 1989, p.72-73). This challenges the conventional wisdom that the probability of default increases, other things equal, with leverage and cash flow variance and decreases with cash flow expectation. Note that is not possible to make assumptions regarding R, μ_x and σ_x that bring all comparative statics in line with conventional wisdom. If leverage and cash flow expectation are to have the 'conventional wisdom' effect, it must be assumed that μ_x <R, but this would give cash flow variance a negative effect, contrary to conventional wisdom[4]. More research is necessary to determine whether these ambiguous results spring from the extreme ends of the distribution or the central area. At present, we can only formulate hypotheses for the tax rate and the bankruptcy costs, which are hypothesized to have a positive resp, negative effect on default

[4] Note that R - μ_x > σ_x implies that μ_x < R

probability. These hypotheses are tested in section 4 using the data described in the next section.

3. Data and Methodology

3.1 Dataset

The data used in this study come from a dataset that comprises all Norwegian limited liability companies (AS Companies) for the period 1995-2000. Companies with total assets or total sales less than 100 000 Norwegian Kroner (approximately $12.500) are excluded from our analysis. This is done to exclude non-operative companies established for e.g. tax advantages only. Second, all companies in our analysis must have delivered their financial statements for every year between 1995 and 2000. Companies that are about to go bankrupt or have gone bankrupt will in many cases not hand in their financial statements in time or even not hand them in at all. Using these selection criteria we are left with roughly 70,000 companies and among these are 149 companies that went bankrupt in the year 2000. 149 companies is a small number compared to the total number of 4,661 companies that went bankrupt in 2000. The reason is the strong restriction we put on the data, demanding that all companies have accounting data available for all years between 1995 and 2000. A random sample of 1394 non-bankrupt companies was selected for the analysis in addition to the 149 bankrupt companies. The construction of the dataset for this study is rather crude and the procedure is chosen for convenience only.

3.2 Empirical model and proxy variables

Many variables in the theoretical models refer to future expected values that cannot be measured directly. Instead, empirical proxy variables have to be used, taken from the available accounting data. The proxy variables used in the analysis are:

- Debt: the level of debt divided by total assets: DEBT/TA
- Taxes: observed tax rate, the amount of tax paid over earnings: TAX/EBIT
- Cash flow expectation (μ_x): accounting cash flow $CF = \dfrac{\text{net profit} + \text{depreciation}}{\text{total assets}}$.
- Standard deviation of cash flow (σ_x): standard deviation of CF calculated over 1995-2000
- Bankruptcy costs B(x): approximated by the size of the company (ln(sales))

Since these variables are straightforward (transformations of) accounting numbers they do not need much discussion. With the exception of cash flow standard deviation, they refer to the year 2001. The leverage and cash flow variables are included in the analysis without any explicit hypothesis regarding their influence. The tax rate (TAX/EBIT) is hypothesized to be positively related to the probability of default. Bankruptcy costs are generally assumed to be inversely related to size, i.e. bankruptcy costs as a fraction of firm value decreases with size. In our model, bankruptcy costs have a negative effect on the probability of default, so this leads to the hypothesis that size is positively related to default probability[5]. In addition to these 5 financial variables, 8 dummy variables for industrial classification (first digit NACE code, explained in appendix A) are included to capture the effect of omitted variables, if any, that would otherwise disturb the analysis.

3.3 Estimation procedure

The analytical technique used for this study should allow for a binary dependent variable and the testing of the hypotheses formulated in the previous subsection. The latter requirement prohibits the use of classification techniques as discriminant analysis, as the discriminant function coefficients are not unique, only their ratios are (see e.g. Eisenbeis (1977)). The binary dependent variable essentially rules out usual regression analysis, including the linear probability model. Linear functions are inherently unbounded, while probabilities are bounded by 0 and 1. This makes logit and probit analysis the most obvious candidates for the 'regression' analysis of dichotomous variables. Both models always return values between 0 and 1. The logit model solves the problem of the bounded dependent variable by transforming the probabilities in such a way that they are no longer bounded. The upper bound is removed by transforming the probability p to the odds ratio p/(1-p). The lower bound is removed by taking the logarithm of the odds ratio: ln(p/(1-p)). The logit model is linear in the log-odds and this makes the coefficients somewhat easier to interpret than those of the probit model. However, both models are very close and rarely lead to different qualitative conclusions, so that it is difficult to distinguish between them statistically. As a general proposition, the question of the choice between them is unresolved (Greene, 1993). Without decisive arguments pro or contra, logit analysis is used to estimate the influence of the above mentioned variables on the probability of default

The logit model formulated here for the Norwegian data contains a two state dependent variable (state 1 = declared bankrupt in 2001, state 2 = not bankrupt in 2001). The independent variables are the 5 proxy variables described in section 3.2 plus the 8 industry dummies. The logistic regression analysis is performed with the statistical software SPSS. (see SPSS (1999) and Kinnear and Gray (2001)).

4. Logistic regression results

The results of the analysis are presented in table 2, where the estimated coefficients and their significance are given. The significance level is calculated with the Wald statistic, that has a chi-square distribution. For variables with a single degree of freedom, as is the case here, the Wald statistic is the square of the ratio of the coefficient to its standard error. Using a 5% significance level,

[5]Note that that size is usually assumed to be negatively related to default probability because larger cash flows are assumed to have less variability and, thus, to be safer. The analyses in this paper show that when the cash flow variability effect is accounted for, the bankruptcy costs effect remains, with a positive effect.

the coefficient of the tax rate is seen to be positive, but not significantly so. The hypothesis regarding the tax rate is not supported in the data, but not rejected either. The analysis provides support for the hypothesized effect of size (measured as ln(sales)) as a proxy for bankruptcy costs: the coefficient is significantly positive.

For the other variables, no unequivocal hypotheses could be derived from the theory, but the results are seen to be in line with what could be expected according to conventional wisdom. Leverage and cash flow standard deviation have a significantly positive effect on default probability, cash flow itself has a significantly negative effect. Only one industry dummy is significantly different from zero, indicating that the other variables to a large extend capture the inter-industry differences in default probability.

As a measure for 'goodness of fit' the Nagelkerke R^2 is used. This statistic measures the proportion of explained "variation" in the logistic model. It is similar in intent to the R^2 in a linear regression model, although the variation in a logistic regression model must be defined differently (see Nagelkerke (1991)). According to this measure, the logistic regression model explains 17.4% of the "variation" in the outcome variable. So it must be concluded that the variables used here give an only partial explanation of the probability of default.

Variable	Parameter
Constant	-3.349 (0.000)
Debt / TA	0.485 (0.020)
Tax / earnings	0.000 (0.354)
Cash flow	-1.459 (0.000)
Standard dev. of cash flow	0.926 (0.005)
ln (sales) (bankr. costs)	0.116 (0.046)
Dummy for sector 2	0.259 (0.625)
Dummy for sector 3	-0.089 (0.896)
Dummy for sector 4	0.026 (0.959)
Dummy for sector 5	-0.420 (0.372)
Dummy for sector 6	-1.064 (0.085)
Dummy for sector 7	-1.329 (0.015)
Dummy for sector 8	0.225 (0.739)
Dummy for sector 9	-1.252 (0.167)
R^2 (Nagelkerke)	0.174

Table 2: Estimated parameters of the model (significance level in parentheses).

5. Conclusions

The purpose of this paper is to contribute to a theory of default that is based on capital structure theory, by demonstrating its feasibility in a simple setting. As a preliminary conclusion it can be said that the approach is indeed feasible, but that it clearly has its limitations. One limitation is that some undesirable aspects of the capital structure model are transferred into default probability model. More specifically, the model shows no straightforward relationship between default probability on the one hand and leverage and cash flow characteristics (expectation and variance) on the other. Conventional wisdom generally assumes that such a straightforward relationship exists. In addition, the models appeared to have a limited explanatory power, even if the variables for which no hypothesis could be formulated are included in the empirical analysis. On the other hand, the advantages of a theory based methodology have become apparent along the way: it provides a clear frame of reference in which the results can be evaluated and directions for future research can be specified. This by itself can be a valuable contribution compared to the multitude of theory-less empirical studies and a useful addition to default theory based on option pricing.

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Appendix A. Explanation of the NACE industry codes

NACE CODE	DESCRIPTION
1	Mining industry
2	Miscellaneous industry
3	Production of electrical and optical products, production of transportation vehicles and other production
4	Power and water supply, building and construction operations
5	Trading of goods, repairing of vehicles and working with domestic appliances, hotels and restaurants
6	Transportation, communication, financial service companies
7	Management of properties, business services and rental business, public administration
8	Education and healthcare
9	Miscellaneous services, paid housework, international organs and organizations

Appendix B. Some details of the calculations in section 2

In section 2.1 the total value of the company V, is calculated as the sum of V_d and V_e. The calculations leading to (2.6) are given below:

$$V = V_e + V_d \tag{A.1}$$

given that

(2.3) $$V_e = E(Y_e) = \frac{(1-\tau)\int_b^\infty (\tilde{x} - R) f(\tilde{x}) d\tilde{x}}{(1+r)} \tag{A.2}$$

(2.5) $$V_d = \frac{\int_0^b (\tilde{x} - B(\tilde{x})) f(\tilde{x}) d\tilde{x} + R(1-F)}{(1+r)} \tag{A.3}$$

V becomes:

$$V = \frac{(1-\tau)\int_b^\infty (\tilde{x} - R) f(\tilde{x}) d\tilde{x}}{(1+r)} + \frac{\int_0^b (\tilde{x} - B(\tilde{x})) f(\tilde{x}) d\tilde{x} + R(1-F)}{(1+r)} \tag{A.4}$$

$$= \frac{(1-\tau)\int_b^\infty (\tilde{x} - R) f(\tilde{x}) d\tilde{x} + \int_0^b (\tilde{x} - B(\tilde{x})) f(\tilde{x}) d\tilde{x} + R(1-F)}{(1+r)} \tag{A.5}$$

$$= \frac{\int_b^\infty (\tilde{x} - R) f(\tilde{x}) d\tilde{x} - \tau \int_b^\infty (\tilde{x} - R) f(\tilde{x}) d\tilde{x} + \int_0^b (\tilde{x} - B(\tilde{x})) f(\tilde{x}) d\tilde{x} + R(1-F)}{(1+r)} \quad \text{(A.6)}$$

$$= \frac{\int_b^\infty \tilde{x} f(\tilde{x}) d\tilde{x} - R\int_b^\infty f(\tilde{x}) d\tilde{x} - \tau \int_b^\infty \tilde{x} f(\tilde{x}) d\tilde{x} + \tau R \int_b^\infty f(\tilde{x}) d\tilde{x} + \int_0^b \tilde{x} f(\tilde{x}) d\tilde{x} + \int_0^b B(\tilde{x}) f(\tilde{x}) d\tilde{x} + R(1-F)}{(1+r)}$$

(A.7)

$$= \frac{\int_b^\infty \tilde{x} f(\tilde{x}) d\tilde{x} - R(1-F) - \tau \int_b^\infty \tilde{x} f(\tilde{x}) d\tilde{x} + \tau R(1-F) + \int_0^b \tilde{x} f(\tilde{x}) d\tilde{x} + \int_0^b B(\tilde{x}) f(\tilde{x}) d\tilde{x} + R(1-F)}{(1+r)} \quad \text{(A.8)}$$

since we defined $F = \int_{-\infty}^b f(\tilde{x}) d\tilde{x}$ (A.9)

$$= \frac{\int_0^b \tilde{x} f(\tilde{x}) d\tilde{x} + \int_b^\infty \tilde{x} f(\tilde{x}) d\tilde{x} - \tau \int_b^\infty \tilde{x} f(\tilde{x}) d\tilde{x} + \int_0^b B(\tilde{x}) f(\tilde{x}) d\tilde{x} + +\tau R(1-F) + R(1-F) - R(1-F)}{(1+r)} \quad \text{(A.10)}$$

$$= \frac{\int_0^\infty \tilde{x} f(\tilde{x}) d\tilde{x} - \tau \int_b^\infty \tilde{x} f(\tilde{x}) d\tilde{x} - \int_0^b B(\tilde{x}) f(\tilde{x}) d\tilde{x} + \tau R(1-F)}{(1+r)} \quad \text{(A.11)}$$

which is like (2.6).

In the calculations of the comparative statics below, the derivation of the normal distribution is used often.

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{1}{2\sigma_x^2}(x-\mu_x)^2} \quad \text{(A.12)}$$

The derivative of f with respect to x :

$$\frac{\partial f(x)}{\partial x} = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} * (-\frac{1}{2\sigma_x^2} 2(x-\mu_x)) = \frac{f(x)}{\sigma_x^2}(\mu_x - x) \quad \text{(A.13)}$$

The derivative of f with respect to x, when x = R is

$$\frac{\partial f}{\partial R} = \frac{f(R)}{\sigma_x^2}(\mu_x - R) \quad \text{(A.14)}$$

The derivative of f with respect to μ_x

$$\frac{\partial f(x)}{\partial \mu_x} = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} * (-\frac{1}{2\sigma_x^2} * (-1) * 2(x-\mu_x)) = -\frac{f(x)}{\sigma_x^2}(\mu_x - x) \quad (A.15)$$

Then the derivative of f with respect to μ_x, when x = R is

$$\frac{\partial f(R)}{\partial \mu_x} = -\frac{f(R)}{\sigma_x^2}(\mu_x - R) \quad (A.16)$$

The derivative of f with respect to σ_x is:

$$\begin{aligned}\frac{\partial f(x)}{\partial \sigma_x} &= -\frac{1}{\sqrt{2\pi}\sigma_x^2} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} + \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} * \frac{(-1)(x-\mu_x)^2}{2} * \frac{(-2)}{\sigma_x^3} \\ &= f(x)\left[\frac{(x-\mu_x)^2}{\sigma_x^3} - \frac{1}{\sigma_x}\right]\end{aligned} \quad (A.17)$$

Then the derivative of f with respect to σ_x, when x = R is

$$\frac{\partial f(R)}{\partial \sigma_x} = f(R)\left[\frac{(R-\mu_x)^2}{\sigma_x^3} - \frac{1}{\sigma_x}\right] \quad (A.18)$$

The function for the probability of default is:

$$(2.10):\ F = 1 - \frac{B(R)f(R)}{\tau} \quad (A.19)$$

The derivative of F with respect to R using (A.3) is:

$$\begin{aligned}(2.11):\ \frac{\partial F}{\partial R} &= -\frac{1}{\tau}(B(R)\frac{\partial f(R)}{\partial R} + \frac{\partial B(R)}{\partial R} f(R)) \\ &= -\frac{1}{\tau}(B(R)\frac{f(R)(\mu_x - R)}{\sigma^2} + B'(R)f(R))\end{aligned} \quad (A.20)$$

The derivative of F with respect to τ :

$$(2.12):\ \frac{\partial F}{\partial \tau} = 0 - \frac{B(R)f(R)*-1}{\tau^2} = \frac{B(R)f(R)}{\tau^2} \quad (A.21)$$

The derivative of F with respect to B:

$$(2.13):\ \frac{\partial F}{\partial B(R)} = -\frac{f(R)}{\tau} \quad (A.22)$$

The derivative of F with respect to σ_x using the result from (A.18):

$$(2.14):\ \frac{\partial F}{\partial \sigma_x} = -\frac{B(R)\dfrac{\partial f}{\partial \sigma_x}}{\tau} = \frac{f(R)B(R)}{\tau}\left[\frac{1}{\sigma_x} - \frac{(R-\mu_x)^2}{\sigma_x^3}\right] \tag{A.23}$$

The derivative of F with respect to μ_x using the result from (A.16):

$$(2.16):\ \frac{\partial F}{\partial \mu_x} = -\frac{B(R)\dfrac{\partial f}{\partial \mu_x}}{\tau} = \frac{f(R)B(R)}{\tau\sigma_x^2}(\mu_x - R) \tag{A.24}$$