Fair Valuation of Insurance Contracts under Lévy Process Specifications
Preliminary Version

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Abstract

The valuation of insurance contracts using concepts from financial mathematics (in particular, from option pricing theory), typically referred to as *Fair Valuation*, has recently attracted considerable interest in academia as well as among practitioners. We will investigate the valuation of so-call participating (with-profits) contracts, which are characterised by embedded rate guarantees and bonus distribution rules. We will study two model specific situations, one of which includes a bonus account. While our analysis reveals information on fair parameter settings of the contract, the main focus of the study will be on the impact of different Lévy process specifications on the fair values obtained. Our findings imply that regardless of the models current German regulatory requirements are not compatible with the fair valuation principle. We also find that a change in the underlying asset model will imply a significant change in prices for the guarantees, indicating a substantial model risk.

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1 Introduction

With-profit life insurance policies which contain an interest rate guarantee have recently generated considerable practical concern in many countries. In particular in Germany there has been an extensive discussion between life-insurance companies and their regulator on level of guarantees and participation rates. While some life-insurance companies argued that participation rates need to be different to properly account for the value of different guaranteed return levels the regulator opposed this view on the basis of equal treatment of policy holders.

Based on the so-called fair value principle, i.e. valuation of the life-insurance contracts by means of option pricing theory, one can support the point of view of the insurance companies, see Buchwald and Müller (2004) and Kling and Ruß (2004). Similar results, in a different setting specific to the Danish market have been obtained in Hansen and Miltersen (2001). A general model has been proposed in Miltersen and Persson (2001) where also a bonus distribution mechanism was included.

In all the cited papers the model of the underlying costumer asset account, to which the guaranteed return rule has to be applied, was based on a Brownian motion driven Black-Scholes setting. In our study we will generalize the dynamics of the asset account by using different Lévy process specifications. This is motivated by the substantially changed investment policies of insurance companies in recent years which contributed to higher fluctuations of the insurers investment portfolios underlying the contracts. This generalisation of the model setting also allows to investigate the impact of a variation of investment strategies on fair rates. Recall the a Gaussian-based model is complete specified by the first two moments of a distribution. In contrast, Lévy-type models allow for much more flexible strategies which can be distinguished according to higher moments of the asset returns, e.g. with respect to kurtosis and thus to the tails of the return distribution. Additionally, we will investigate the impact of different types of Lévy process models on fair contract parameters. In this respect our study generalizes Ballotta (2004), who focused on a Merton-style jump-diffusion model applied to a simple contract model.

The paper is organized as follows. In section 2 we specify a simple contract, introduce the fair value principle and present results in the standard Black-Scholes setting. Section 3 is devoted to the analysis of the simple contract under Lévy process specifications. In section 4 we study the impact of the introduction of a bonus account under different Lévy process specifications. The proposed contract specifications in the simple and the more complex case are as in Miltersen and Persson (2001).

2 The simple model

In this section we specify a simple contract specification which does not include a bonus account.

2.1 The simple contract

At $t = 0$ policy holders deposit X into account C which is invested by insurer for T years. The insurer promises an annual rate of return of C in year i of

$$g_i + \delta(\xi_i - g_i)^+,$$

where g_i is the minimum rate of return, ξ_i is random rate of return of insurers (investment) performance and δ is the participation rate. The insurers liabilities

$$\begin{aligned} C_t &= C_{t-1} \exp\left\{g_t + \delta[\xi_t - g_t]^+\right\} \\ &= X \exp\left\{\sum_{i=1}^{t} g_i + \delta[\xi_i - g_i]^+\right\} \end{aligned}$$

with initial liabilities $C_0 = X$. The insurers assets are given by

$$A_t = X \exp\left\{\sum_{i=1}^{t} \xi_i\right\}.$$

2.2 Fair valuation principle

We suppose the we can use a pricing measure (equivalent martingale measure, EMM) $\mathbb{Q}$ so that any (random) cash flow Z_T at time T can be valued using the pricing formula

$$V_t(Z_T) = \mathbb{E}\left(e^{-\int_t^T r_u du} Z_T \middle| \mathcal{F}_t\right) \tag{1}$$

where $\mathbb{F} = (\mathcal{F}_t)$ is the relevant financial market filtration and (r_t) short rate process. Thus

$$V_0(A_T - C_T) = 0$$

Since $V_0(A_T) = X$ we have the *fair value condition*

$$V_0\left(\frac{C_T}{X}\right) = V_0\left(e^{\sum_{i=1}^{T} g_i + \delta[\xi_i - g_i]^+}\right) = 1 \tag{2}$$

We now assume a constant interest rate, then

$$
\begin{aligned}
V_0\left(\frac{C_T}{X}\right) &= I\!\!E\left(e^{-rT}e^{\sum_{i=1}^T g_i+\delta[\xi_i-g_i]^+}\right) = I\!\!E\left(e^{-rT}e^{\sum_{i=1}^T (g_i \bigvee (\delta\xi_i+(1-\delta)g_i))}\right) \\
&= I\!\!E\left(e^{-rT}e^{\sum_{i=1}^T ((\delta g_i \bigvee \delta\xi_i)+(1-\delta)g_i)}\right) \\
&= e^{\sum_{i=1}^T (1-\delta)g_i} I\!\!E\left(e^{-rT}e^{\sum_{i=1}^T (\delta g_i \bigvee \delta\xi_i)}\right) \\
&= e^{\sum_{i=1}^T (1-\delta)g_i} \prod_{i=1}^T I\!\!E\left(e^{-r}e^{\delta g_i} \bigvee e^{\delta\xi_i}\right)
\end{aligned}
$$

The expectation can be written

$$
\begin{aligned}
I\!\!E\left(e^{-r}e^{\delta g_i} \bigvee e^{\delta\xi_i}\right) &= e^{-r} I\!\!E\left(\left[e^{\delta\xi_i} - e^{\delta g_i}\right]^+ + e^{\delta g_i}\right) \qquad (3) \\
&= e^{-r} I\!\!E\left(\left[e^{\delta\xi_i} - e^{\delta g_i}\right]^+\right) + e^{\delta g_i - r}
\end{aligned}
$$

The first term corresponds to a European call option on a modified security with strike $e^{\delta g_i}$ and payoff $e^{\delta\xi_i}$ at maturity.

2.3 Black-Scholes asset specification

For the standard Black Scholes model we have

$$dA_t = A_t(rdt + \sigma dW_t).$$

This implies

$$\xi_t = \left(r - \frac{1}{2}\sigma^2\right) + \sigma(W_t - W_{t-1}).$$

Thus

$$I\!\!E\left(e^{\delta\xi_t}\right) = e^{\delta\left(r-\frac{1}{2}\sigma^2\right)+\frac{1}{2}\delta^2\sigma^2}$$

and the volatility of the modified asset is $\delta\sigma$. So we obtain for the expectation in (3)

$$\begin{aligned}
& e^{-r}\mathbb{E}\left(\left[e^{\delta\xi_i} - e^{\delta g_i}\right]^+\right) + e^{\delta g_i - r} \\
= \quad & e^{-r}e^{\delta\left(r+\frac{1}{2}(\delta-1)\sigma^2\right)}\Phi\left(\frac{r+\left(\delta-\frac{1}{2}\right)\sigma^2 - g_i}{\sigma}\right) \\
- \quad & e^{\delta g_i - r}\Phi\left(\frac{r - g_i - \frac{1}{2}\sigma^2}{\sigma}\right) + e^{\delta g_i - r} \\
= \quad & e^{-r}e^{\delta\left(r+\frac{1}{2}(\delta-1)\sigma^2\right)}\Phi\left(\frac{r+\left(\delta-\frac{1}{2}\right)\sigma^2 - g_i}{\sigma}\right) \\
+ \quad & e^{\delta g_i - r}\Phi\left(\frac{-r + g_i + \frac{1}{2}\sigma^2}{\sigma}\right)
\end{aligned}$$

The fair valuation equation (2) implies

$$\begin{aligned}
V_0\left(\frac{C_T}{X}\right) = \quad & e^{\sum_{i=1}^{T}(1-\delta)g_i} \times \\
& \times \prod_{i=1}^{T} \left(e^{(\delta-1)\left(r+\frac{1}{2}\delta\sigma^2\right)}\Phi\left(\frac{r+\left(\delta-\frac{1}{2}\right)\sigma^2 - g_i}{\sigma}\right)\right. \\
& \left. + e^{\delta g_i - r}\Phi\left(\frac{-r + g_i + \frac{1}{2}\sigma^2}{\sigma}\right)\right) \\
= \quad & 1
\end{aligned}$$

If we assume (as typical) constant guarantee g, then we obtain

$$\begin{aligned}
V_0\left(\frac{C_T}{X}\right) = \quad & e^{(1-\delta)gT} \\
& \times \left(e^{(\delta-1)\left(r+\frac{1}{2}\delta\sigma^2\right)}\Phi\left(\frac{r+\left(\delta-\frac{1}{2}\right)\sigma^2 - g}{\sigma}\right)\right. \\
& \left. + e^{\delta g - r}\Phi\left(\frac{-r + g + \frac{1}{2}\sigma^2}{\sigma}\right)\right)^T \\
= \quad & 1.
\end{aligned}$$

This is equivalent to the condition

$$1 = e^{(1-\delta)\left(g-r-\frac{1}{2}\delta\sigma^2\right)}\Phi\left(\frac{r+\left(\delta-\frac{1}{2}\right)\sigma^2-g}{\sigma}\right)e^{g-r}\Phi\left(\frac{g-r+\frac{1}{2}\sigma^2}{\sigma}\right). \quad (4)$$

Thus the fairness condition is independent of the time horizon.

Figure 1 shows the fair sets of participation rates against guarantees for a Black-Scholes model for different volatilities σ. Participation rates are in percentage points. The yellow curve corresponds to $\sigma = 0.1$, the green curve corresponds to $\sigma = 0.2$, the blue curve corresponds to $\sigma = 0.3$ and the purple curve corresponds to $\sigma = 0.4$. The riskless interest rate, here and in what follows, is $r = 0.1$. The figure shows clearly that lower volatility of the underlying asset process leads to higher participation rates. However, with none of the above volatility specifications the German regulatory environment of $g = 2.75$ % and $\delta \geq 90$ % is matched, i.e. such a contracts does not satisfy the fair value requirement! (Observe that we also assumed a very high riskfree rate, i.e. the situation is even worse for the current interest rate regime.)

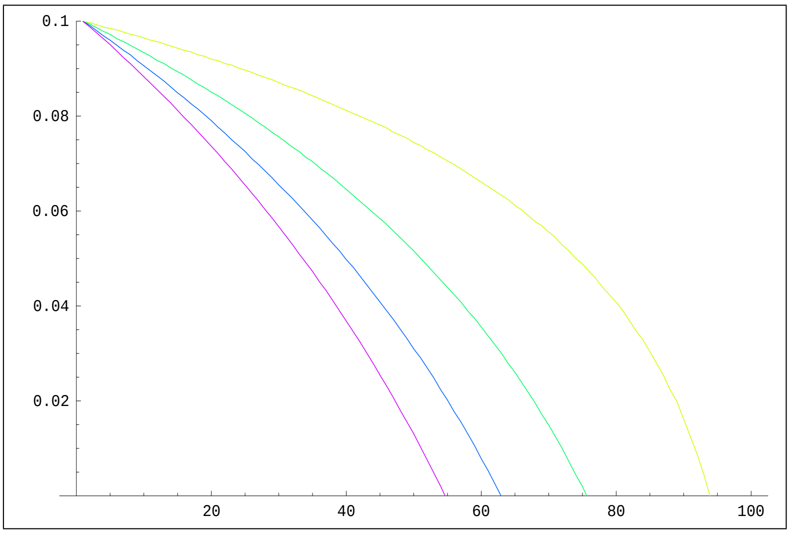

Figure 1: Isoquants for the Black-Scholes-model under different volatilities

3 Fair valuation for Lévy-type models

3.1 Basic results on Lévy-type models

Recall that in the standard Black-Scholes model the asset price dynamics are defined via the stochastic differential equation (SDE)

$$dS_t = S_t(\mu dt + \sigma dW_t),$$

with constant coefficients and a standard Brownian motion W. The solution of the SDE is

$$S_t = S_0 \exp\left\{\mu t - \frac{\sigma^2}{2}t + \sigma W_t\right\}.$$

Hence *log* returns are normally distributed. However, empirical densities of log returns exhibit stylized facts which are not consistent with normal distributions, i.e. they show more mass near the origin, less mass in the flanks and considerably more mass in the tails. This has motivated to consider Lévy-type models, i.e. asset price models, which generate more realistic log return distributions. We consider here general exponential Lévy process model for asset prices

$$S_t = S_0 \exp(L_t),$$

with a Lévy process L that satisfies some integrability condition. Examples of such models are, e.g. the generalized hyperbolic model, see Eberlein (2001), with the normal inverse Gaussian model, see Barndorff-Nielsen (1998), as a special case. The Meixner model, see Schoutens (2003) and the Variance-Gamma model, see Madan and Seneta (1990) and Carr, Chang, and Madan (1998). For a more complete overview see Schoutens (2003) or Cont and Tankov (2004).

3.1.1 Generalized hyperbolic model

This is based on the generalized hyperbolic distributions for log returns. For these distributions the densities are given by:

$$\begin{aligned} d_{GH}(x;\lambda,\alpha,\beta,\delta,\mu) &= a(\lambda,\alpha,\beta,\delta,\mu) \\ &\quad\times(\delta^2+(x-\mu)^2)^{(\lambda-\frac{1}{2})/2} \\ &\quad\times K_{\lambda-\frac{1}{2}}(\alpha\sqrt{\delta^2+(x-\mu)^2}) \\ &\quad\times \exp\{\beta(x-\mu)\} \end{aligned} \tag{5}$$

where

$$a(\lambda,\alpha,\beta,\delta,\mu) = \frac{(\alpha^2-\beta^2)^{\lambda/2}}{\sqrt{2\pi}\alpha^{\lambda-\frac{1}{2}}\delta^\lambda K_\lambda(\delta\sqrt{\alpha^2-\beta^2})}$$

and K_ν denotes the modified Bessel function of the third kind

$$K_\nu(z) = \frac{1}{2}\int_0^\infty y^{\nu-1}\exp\left\{-\frac{1}{2}z(y+y^{-1})\right\}dy$$

We will consider the Normal Inverse Gaussian Distribution (NIG), where the parameter $\lambda = -1/2$. So the density is

$$\begin{aligned} d_{NIG}(x) &= \frac{\alpha}{\pi}\exp\left\{\delta\sqrt{\alpha^2-\beta^2}+\beta(x-\mu)\right\} \\ &\quad \times \frac{K_1\left(\alpha\delta\sqrt{1+\left(\frac{x-\mu}{\delta}\right)^2}\right)}{\sqrt{1+\left(\frac{x-\mu}{\delta}\right)^2}}. \end{aligned}$$

3.1.2 The Meixner model

Here the Lévy-type asset model is based on Meixner(α,β,δ,m)-Process. Recall that the density of Meixner(α,β,δ) is

$$f_M(x;\alpha,\beta,\delta) = \frac{(2\cos(\beta/2))^{2\delta}}{2\alpha\pi\Gamma(2\delta)}\exp(\beta x/\alpha)\left|\Gamma\left(\delta+\frac{ix}{\alpha}\right)\right|^2 \tag{6}$$

where $\alpha > 0, -\pi < \beta < \pi, m\delta > 0$ with semi-heavy tails. One can define a Lévy process X with distribution of X_t being Meixner$(\alpha,\beta,\delta t)$. It is possible to add a drift term to obtain

$$Y_t = X_t + mt$$

which we denote as Meixner$(\alpha,\beta,\delta t, mt)$

3.1.3 Option pricing

As Lévy type models typically exhibit incomplete market models we need to choose an equivalent martingale measure for option pricing. Standard methods to choose such a measure lead to the so-called Esscher and the mean-correcting measure. The methodology is explained in detail in Schoutens (2003). We review briefly the Esscher case. We need the following assumptions

- L_1 possesses a moment-generating function $M(z,1) = I\!E(\exp(zL_1))$ on some open interval (a,b) with $b-a > 1$.

- There exists a real number $\theta^* \in (a, b-1)$ such that

$$M(\theta^*, 1) = M(\theta^* + 1, 1).$$

We call a change of measure $I\!\!P$ to a locally equivalent measure $\mathbb{Q}$ an Esscher transform if

$$\left.\frac{d\mathbb{Q}}{dI\!\!P}\right|_{\mathcal{F}_t} = Z_t^\theta = \frac{\exp(\theta L_t)}{M(\theta, t)}.$$

Then there exists an equivalent martingale measure $\mathbb{Q}$ such that the discounted asset price process $e^{-rt}A_t$ is a $\mathbb{Q}$-martingale. The density process leading to such a martingale measure is given by $Z_t^{\theta^*}$.

3.2 Lèvy asset models

We use the Meixner model as our main example. So we assume

$$A_t = A_0 \exp(X_t)$$

Then

$$\xi_t = X_t - X_{t-1}$$

are Meixner$(\alpha, \beta, \delta, m)$. Under the Esscher measure ξ_t are Meixner$(\alpha, \beta + \alpha\theta^*, \delta, m)$ with

$$\theta^* = -\frac{1}{\alpha} \times \left(\beta + 2\arctan\left(\frac{-\cos(\frac{\alpha}{2}) + e^{((m-r)/(2\delta))}}{\sin(\frac{\alpha}{2})}\right)\right)$$

Now the first term in the expectation equation (see (3)) is

$$e^{-r} I\!\!E\left(\left[e^{\bar{\delta}\xi_i} - e^{\bar{\delta}g_i}\right]^+\right)$$

This corresponds to a European call option on a modified security with strike $e^{\bar{\delta}g_i}$ and payoff $e^{\bar{\delta}\xi_i}$ at maturity where ξ_t is Meixner$(\alpha, \beta + \alpha\theta^*, \delta, m)$ under the pricing measure. Thus

$$I\!\!E\left(e^{\delta\xi_t}\right) = \left[\frac{\cos((\alpha + \beta\theta^*)/2)}{\cosh(-i(\alpha\bar{\delta} + \alpha + \beta\theta^*)/2)}\right]^{2\delta}$$

where $\bar{\delta}$ is the participation rate, and the volatility of the modified asset is $\delta\sigma$

3.3 Fair valuation analysis in Lévy models

We start by looking at the sensitivities of the fair value approach towards the choice of the pricing measure. Figure 2 shows the fair sets of participation rates against guarantees for a Meixner model for the parameter set $\alpha = 0.2 \cdot \sqrt{2}$, $\beta = 0$ and $\delta = 1$ (notation as in Schoutens (2003)). The green curve was calculated under the Esscher measure, the blue curve under the mean-correcting martingale measure. Since there are hardly any visible differences we will concentrate for the subsequent analysis on valuation based on the Esscher measure.

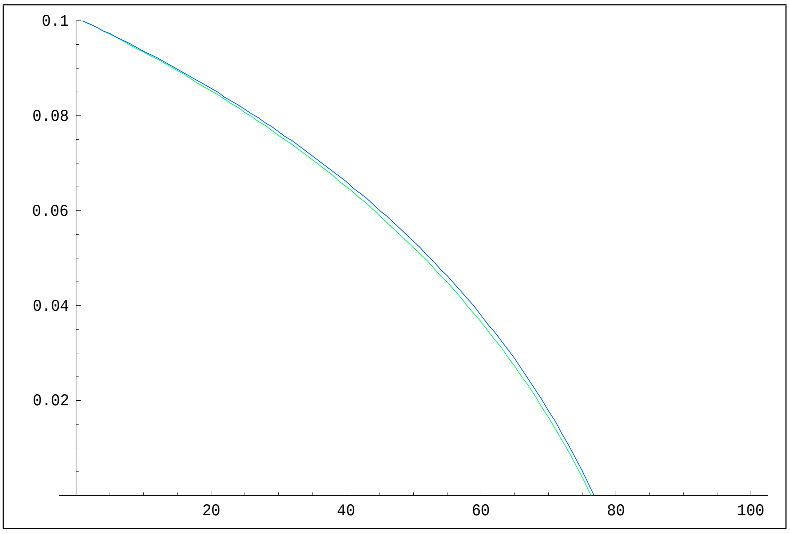

Figure 2: Isoquants for the Meixner-model under the Esscher- and the mean-correcting martingale measure

We now investigate the impact of changing the kurtosis on the fair valuation parameter sets. We start with a Meixner model that replicates the Brownian case. In figure 3 the fair sets of participation rates against guarantees (denoted in percentage points) for a Meixner model under the Esscher measure for different values of α are shown. For all curves, we chose $\beta = 0$ and $\delta = 1$. The yellow curve corresponds to $\alpha = 0.1 \cdot \sqrt{2}$, the green curve corresponds to $\alpha = 0.2 \cdot \sqrt{2}$, the blue curve corresponds to $\alpha = 0.3 \cdot \sqrt{2}$, the purple curve corresponds to $\alpha = 0.4 \cdot \sqrt{2}$. The above choice of parameters results in the standard deviations (volatilities) being 0.1, 0.2, 0.3 and 0.4, respectively. Skewness is always zero, kurtosis always 4.

Similar to figure 1 we see that higher risk taking of the insurance com-

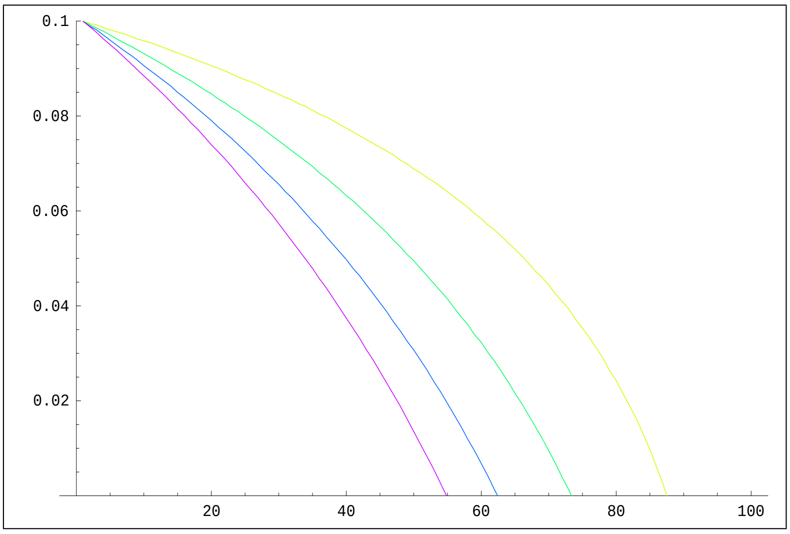

Figure 3: Isoquants for the Meixner-model under the Esscher-measure for different volatilities and low kurtosis

pany in terms of asset returns, i.e. higher volatility of the asset returns, requires a reduction of the participation rate. More importantly, we also observe that by matching the first four moments of the underlying distribution we observe a similar behaviour of the isoquants, although the Meixner model requires for all four choices of volatility a slightly higher compensation for the guarantees, i.e. the participation rate needs to be reduced. Again none of the above volatility specifications implies that settings of the German regulatory environment of $g = 2.75$ % and $\delta \geq 90$ % correspond to the fair value requirement!

In contrast to figure 3, we choose in figure 4 parameter sets that result in a much higher kurtosis. Figure 4 shows the fair sets of participation rates against guarantees for a Meixner model under the Esscher measure for different values of α. For all curves, we chose $\beta = 0$ and $\delta = 0.1$. The yellow curve corresponds to $\alpha = 0.1 \cdot \sqrt{20}$, the green curve corresponds to $\alpha = 0.2 \cdot \sqrt{20}$, the blue curve corresponds to $\alpha = 0.3 \cdot \sqrt{20}$, the purple curve corresponds to $\alpha = 0.4 \cdot \sqrt{20}$. Again, the above choice of parameters results in the standard deviations (volatilities) being 0.1, 0.2, 0.3 and 0.4, respectively. Skewness is always zero, however kurtosis always 13. A comparison with figure 3 shows that *kurtosis does matter.* In particular, we see that higher kurtosis allows the insurance company to increase participation rates.

As an alternative to the Meixner model we use the NIG-model as Lévy

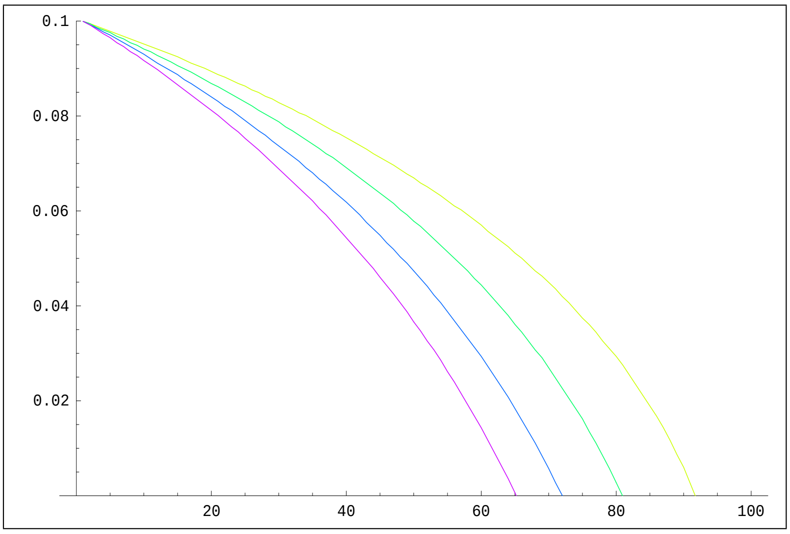

Figure 4: Isoquants for the Meixner-model under the Esscher-measure for different volatilities and high kurtosis

model in figure 5. Again this figure shows the fair sets of participation rates against guarantees for an NIG-model under the Esscher measure for different values of the parameters. For all curves, we chose $\alpha = 0$ and $\beta = 0$. The yellow curve corresponds to $\delta = 0.01$, the green curve corresponds to $\delta = 0.04$, the blue curve corresponds to $\delta = 0.09$, the purple curve corresponds to $\delta = 0.16$.[1] The above choice of parameters results in the standard deviations (volatilities) being 0.1, 0.2, 0.3 and 0.4, respectively. Skewness is always zero, however the values of kurtosis are 303, 78, 36.33 and 21.75, respectively. Compared to the isoquants with identical volatility we can again observe an increase in participation rates with increasing kurtosis.

Figure 6 is based on the following idea. We fixed a Meixner-distribution with parameters $\alpha = 0.5$, $\beta = -1$ and $\delta = 0.1$. On the basis of this distribution, we priced a European call option with time to maturity 1 and moneyness 1 under the Esscher-measure. As always, $r = 0.1$. Then we calculated the implied volatility of above option, which was $\sigma = 0.124672$. Then we contrasted the isoquant based on the Black-Scholes-model with $\sigma = 0.124672$ with the isoquant based on the Meixner model with above parameters. Figure 6 displays the result. Our Meixner-distribution has standard deviation 0.127399, skewness -2.14406 and kurtosis 17.597. The

[1] Parameterization as in Schoutens (2003).

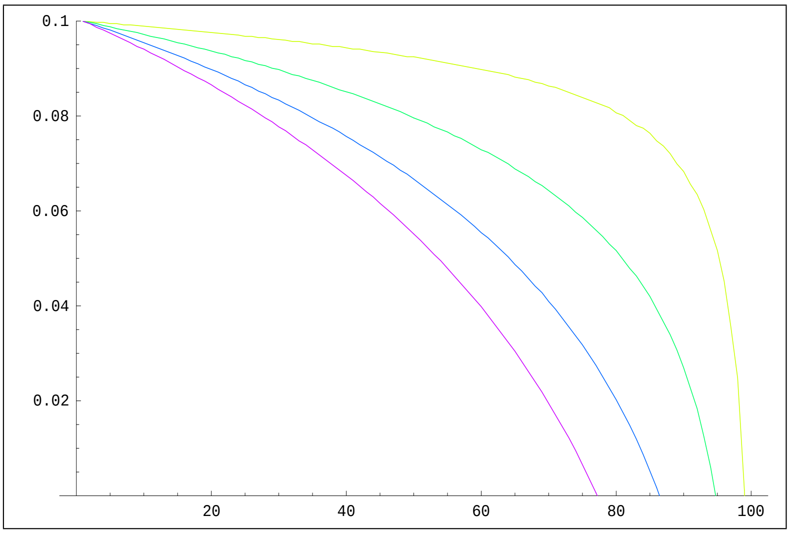

Figure 5: Isoquants for the NIG-model under the Esscher-measure for different volatilities and high kurtosis

two isoquants, though calibrated to the same option price, show a different behaviour and even intersect, indicating a change in model implied riskiness.

The effect of the underlying model is even more visible in figure 7 which is based on the same idea as figure 6. The parameters for the Meixner-distributions are given in table 1. Calibration is done as outlined above. The corresponding Black-Scholes implied volatilities σ are displayed in table 2. The curves, though calibrated to match the same option prices, differ significantly.

For our analysis below we will use the following set of parameters

	dashed red line	dashed green line	dashed blue line	dashed purple line
α	0.4	1	1.6	2.2
β	-1	-1	-1	-1
δ	0.1	0.1	0.1	0.1
std deviation	0.101919	0.254799	0.407678	0.560557
skewness	-2.14406	-2.14406	-2.14406	-2.14406
kurtosis	17.597	17.597	17.597	17.597

Table 1: Parameters and moments for Meixner-distributions

The above investigation show clearly, that the distributional assumptions underlying the fair-value calculations do matter! Using solely Black-Scholes

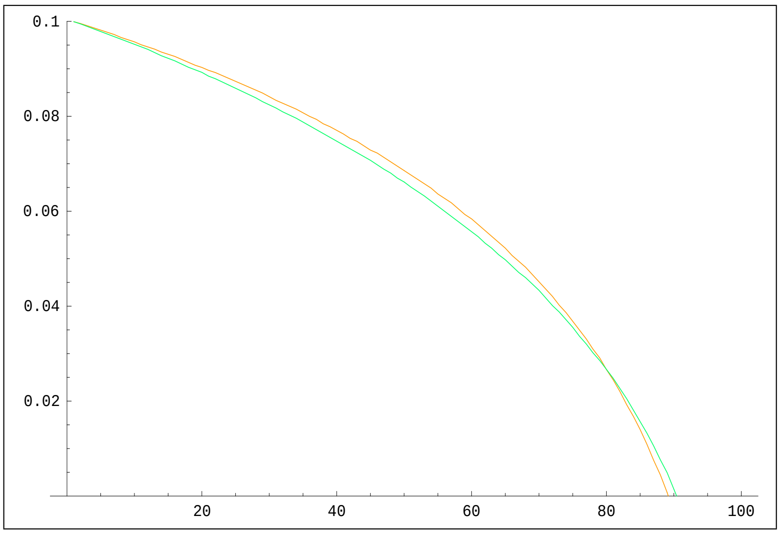

Figure 6: Meixner-isoquant under Esscher-measure versus BS-isoquant for calibrated implied volatility

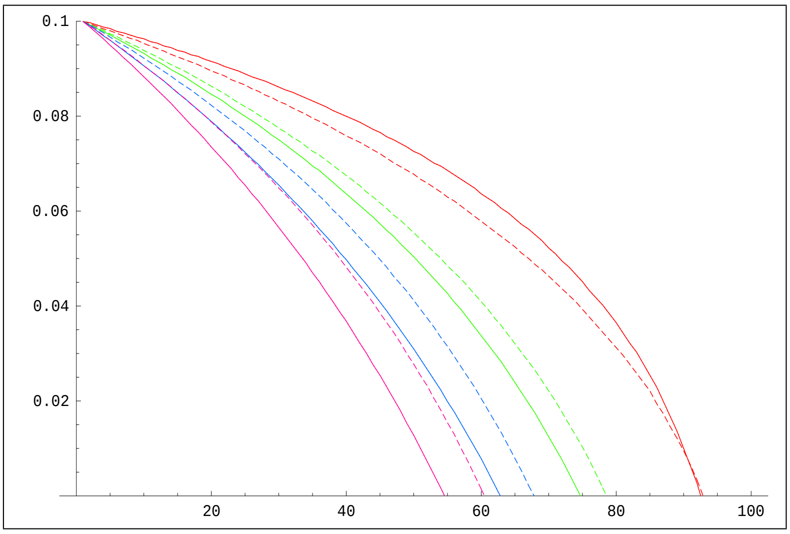

Figure 7: Meixner-isoquant under Esscher-measure versus BS-isoquant for calibrated implied volatility

	solid red line	solid green line	solid blue line	solid solid solid purple line
σ	0.107511	0.205834	0.300866	0.401245

Table 2: Black-Scholes Implied volatilities

may pose a potential threat to insurance companies, since it may lead to a mispricing of options embedded in insurance contracts!

4 The extended model

We consider now the inclusion of a bonus account B. We start with a model introduced in Miltersen and Persson (2001). Here we first define the insurer's account C as

$$C_t = C_{t-1} + A_{t-1}\left(e^{\delta(\xi_i - g_i)^+} - 1\right) \tag{7}$$

Since the initial balance of the account C is zero, we can write

$$C_t = \sum_{i=1}^{t} A_{i-1}\left(e^{\delta(\xi_i - g_i)^+} - 1\right).$$

The bonus account is the determined as the residual amount

$$B_t = Xe^{\sum_{i=1}^t \xi_i} - A_t - C_t. \tag{8}$$

In this model the insurer has to cover any deficit on the bonus account at time T. The fair value principle now requires

$$V_0(C_T) - V_0(B_T^-) = 0,$$

implying the condition

$$X = V_0(A_T) - V_0(B_T^+). \tag{9}$$

A variant of the above approach is used to generate a more realistic model (resembling the German practice). We assume that the insurer sets a maximal rate $z > g$ internally. Then the rate paid in period t is

$$\tilde{r}_t = g + \min\left\{\max(\delta\xi_t - g, 0), z - g\right\}.$$

Also only a fraction y of the true market return of the assets is used to calculate the rate paid. However, any differences are paid back at maturity via the bonus account.

In the final version we will evaluate the above two contract specifications for different types of Lévy process specification (Meixner, NIG) and discuss their parameter sensitivities.

5 Conclusions

All models imply that the current German regulatory environment forces the insurance companies to misprice contracts with respect to the fair valuation principle.

Changing the underlying asset model will imply a significant change in prices for the guarantees.

However, different guarantees require different participation rates independent of the underlying modelling assumptions.

The bonus account allows to smooth return rates and reduces the costs for guarantees.

References

Ballotta, L., 2004, Alternative framework for the fait valuation of participating life insuarnace contracts, Preprint, Cass Business School, London.

Barndorff-Nielsen, O.E., 1998, Processes of normal inverse Gaussian type, *Finance and Stochastics* 2, 41–68.

Buchwald, B., and M. Müller, 2004, Einheitliche Verzinsung in der Lebensversicherung, *Versicherungswirtschaft* 12.

Carr, P., E. Chang, and D.B. Madan, 1998, The Variance-Gamma process and option pricing, *European Finance Review* 2, 79–105.

Cont, R., and P. Tankov, 2004, *Financial Modelling with Jump Processes.* (Chapman & Hull).

Eberlein, E., 2001, Applications of generalized hyperbolic Lévy motions to finance, in O.E. Barndorff-Nielsen, T. Mikosch, and S. Resnick, eds.: *Lévy processes: Theory and Applications* (Birkhäuser Verlag, Boston,).

Hansen, M., and K.R. Miltersen, 2001, Minimum rate of return guarantees: The Danish case, Working Paper.

Kling, A., and J. Ruß, 2004, Differenzierung der Überschüsse, *Versicherungswirtschaft* 4, 254–256.

Madan, D.B., and E. Seneta, 1990, The Variance-Gamma (VG) model for share market returns, *Journal of Business* pp. 511–524.

Miltersen, K.R., and S.-A. Persson, 2001, Guaranteed investment contracts: Distributed and undistributed excess returns, Working Paper.

Schoutens, W., 2003, *Lévy Processes in Finance*. (John Wiley & Sons, Chichester).