Do Good or Bad Borrowers Pledge More Collateral?

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Abstract

In this paper, we analyze the optimal use of collateral in order to reduce interest rate payments and the present value of bankruptcy costs. For this purpose, we consider a framework similar to Merton (1974) but with the additional feature that the borrower can bring in collateral. Bankruptcy costs arise in the case of a default. Although pledging collateral induces some further costs, collateral acts as a powerful device to reduce the interest rate payments and the present value of bankruptcy costs and can therefore considerably increase the wealth of borrowers. In general, we find that a bad borrower, who is characterized by higher bankruptcy costs, riskier projects, and contributes less to the project, pledges more collateral than a good borrower. These relations, however, require the existence of perfect information between borrowers and lenders. Under asymmetric information in terms of the project's riskiness or the contribution of the borrower to the project, these relations invert and good borrowers tend to pledge more collateral.

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1 Introduction

Collateral is a widespread feature which can be observed in many loan contracts. Nevertheless, there is still a big debate in the literature for why borrowers pledge collateral. One of the main arguments for the use of collateral is the presence of asymmetric information between borrowers and lenders because collateral can serve as a self-selection mechanism. In general, lenders can offer incentive compatible contracts to borrowers that reveal their true type. Stiglitz/Weiss (1981), who are the pioneers in this field, demonstrate the ability to screen the wealth of risk-averse borrowers by using differently collateralized loan contracts. In the standard case considered by Bester (1985, 1987), Chan/Kanatas (1985) and Besanko/Thakor (1987a, 1987b), information asymmetry is related to the riskiness of the project of the borrower. All these models have in common that collateral is associated with costs that differ for good and bad borrowers. Typically, the present value of the dissipative cost of collateral in the case of default of a given loan contract is lower for good rather than bad borrowers as the probability of default is lower. Therefore, a good borrower can reveal his or her type by accepting costly collateral while a bad borrower does not pledge any collateral. The advantage for the good borrower is that he is identified as a good borrower by the lender and can take a loan at more favorable terms than a bad borrower.

This outcome that good rather than bad borrowers tend to pledge more collateral is in conflict with several empirical studies such as Berger/Udell (1990), Jimenez/Saurina (2003), Jimenez/Salas/Saurina (2004), Gonas/Highfield/Mullineax (2004), and Dey/Dunn (2004) who find evidence for the opposite relationship that primarily riskier borrowers pledge collateral.

To some extent, this observed relation can be explained by models regarding incentive problems of borrowers. These models dealing with incentive problems build the second major strand in the literature for the use of collateral. Chan/Thakor (1987), Boot/Thakor/Udell (1991), and Pozzolo (2002) regard borrowers who can increase the probability of success of their project by increasing effort. In this context collateral is an important device to force borrowers to extract more effort. The reason for this incentive comes from the loss of collateral caused by a default. Therefore, a higher level of collateral is a higher incentive for the borrower to prevent a default by extracting a higher level of effort. Pozzolo (2002) presents conditions for the functional form of the probability of success of the project under which riskier borrowers pledge more collateral. This relation holds under symmetric information between borrowers and lenders where collateral is used as a mechanism for the borrower to credibly commit to a not too low level of effort. Boot/Thakor/Udell (1991) also consider asymmetric information within this model. The result is that bad borrowers still obtain their optimal contract but good borrowers pledge more collateral than under perfect information. Therefore, it is not generally clear whether under asymmetric information bad borrowers still pledge more collateral or not.

In addition, there are some other motives that speak for the use of collateral.*

*A broad survey about the use of collateral is provided by Coco (2000).

Bester (1994) points out that under costly state verification collateral can mitigate the incentive for the borrower to pretend a default in order to extract additional surplus. In line with empirical observations, this model exhibits the relation that riskier borrowers pledge more collateral. However, Neus (2004) shows that under a typically-assumed mean-preserving spread of the project, this relation does no longer hold but it is inverted. Furthermore, Stulz/Johnson (1985) describe the important role of collateral to mitigate the underinvestment problem presented by Myers (1977). If a firm needs an outside financing to run a profitable project and the firm has already debt outstanding, it might not be worthwhile for the firm to raise additional funds in form of an unsecured loan to run the project. However, using a secured loan with collateral rather than an unsecured loan, the optimal decision might change and the execution of the project becomes worthwhile. Moreover, Rajan/Winton (1995) mention the ability of collateral as an incentive for the lender to increase monitoring. Monitoring is an important devise even for the borrower because it produces externalities for other claimants such as employees and additional bond holders.

As a result, the literature describes several settings in which collateral is a useful device. However, all these approaches, which impressively exhibit the ability of collateral to mitigate incentive or information problems, do not convincingly speak for a more pronounced use of collateral by bad borrowers as observed empirically. In particular, the reason to use collateral to extract more effort or to reduce the probability for an untruthful default might be relevant for small firms. However, in a relatively large firm the possibility to extract surplus by defaulting either contradicts to valid insolvency laws or is almost non-executable especially under an active supervisory board. Additionally, if a firm has numerous decision makers, the aggregate effort of them can hardly be affected by collateral. This is especially true if collateral comes from the firm rather from the individual persons.

There are three recent approaches that are in line with the observed behavior of borrowers to pledge collateral. Coco (1999) considers a model with asymmetric information in which borrowers have different attitudes toward risk and can choose between different types of projects. In this framework, an equilibrium exists in which risk-avers borrowers decide for less risky projects and pledge less collateral relative to risk-neutral borrowers. Hence, bad borrowers with risky projects use more collateral in this setting.

De Meza/Southey (1996) present a model in which some borrowers are overoptimistic. As a consequence, borrowers that are worse than they feel like take loans to run projects and secure them with collateral to get these loans from rational lenders. Since primarily overoptimistic borrowers take loans and secure them with collateral, one can see why collateralized loans are so risky.

Aivazian/Gu/Qiu (2004) propose a dynamic model in which borrowers can postpone an investment in the hope of better projects. In this model, there exists a cutoff interest rate such that bad borrowers pledge more collateral than good borrowers if the interest rate from the loan is below the cutoff rate.

However, it might not be necessary to give up the assumption of rational borrowers or to introduce a sophisticated model framework to explain that bad borrowers prefer collateral. A natural motive for the use of collateral, that has been disregarded so far, is the reduction of bankruptcy costs that arise in the case of a default. Strictly speaking, bankruptcy costs not only contain explicit costs that occur due to a liquidation but all other kind of costs if a firm comes into default without being liquidated such as the loss of reputation or of important employees etc. If a borrower pledges collateral, the default barrier reduces and therefore capital costs reduce and a costly default is less likely. Since on the other hand pledging collateral is also associated with costs, the optimal use of collateral to minimize the arising costs is a complex tradeoff between different effects. If collateral is in use, there are two types of costs involved. Chan/Kanatas (1985) and Bester (1985) introduce costs for pledging collateral which are proportional to the size of collateral. These costs can be understood as cost to maintain the collateral on an agreed level etc. Additionally, a dissipative cost of collateral arises in the case of default because the valuation of collateral is usually higher for the borrower than for the lender. Alternatively, if the collateralized assets are liquidated after a default possible bankruptcy costs can reduce its value. Hence, even without information asymmetries, incentive problems, non-rational borrowers, collateral is a reasonable device to reduce the present value of bankruptcy costs and cost from pledging collateral. Thus, a reasonable first step to understand the use of collateral is to regard the possibility to reduce the elementary costs, i.e. bankruptcy costs and cost for pledging collateral, by bringing in collateral. Therefore, before regarding more sophisticated models with e.g. information or incentive problems it is helpful to understand this fundamental case.

The goal of this paper is to characterize the optimal size of collateral to reduce the present value of bankruptcy costs and cost for pledging collateral. Even though the reduction of suchlike costs can be successfully achieved by the use of collateral, other models do not focus on the isolated consideration of this issue but rather introduce more complex models where bankruptcy costs and cost for pledging collateral are overlapped by other issues. The analysis of this straightforward motive for the use of collateral raises two questions. The first question is how good can collateral reduce costs and therefore increase the present value of the borrower. The second question is whether this motive for the use of collateral can explain the observed behavior that bad rather than good borrowers pledge more collateral. For the analysis, we consider a simple firm value model similar to Merton (1974) but with bankruptcy costs in the case of default. Now the borrower can bring in collateral in addition to the assets of the firm to secure the loan. The consequence of collateral is that in the case of default the lender receives not only the liquidation value from the assets of the firm but also the liquidation proceeds from the collateral. Additionally, the potential loss of collateral increases the incentive for the borrower to pay back the loan which reduces the default probability. Hence, a higher collateral volume is associated with lower interest rate payments of the loan. However, pledging collateral has two disadvantages for a borrower. Firstly, pledging collateral induces some further costs. Secondly, in the case of default not only the assets of the firm but also the collateral is lost. Therefore, the optimal size of collateral is a complex tradeoff of these different effects.

In the first step, we characterize the optimal use of collateral under symmetric information with perfect competition among lenders. The basic result is that collateral is a device to remarkably reduce bankruptcy costs which can considerably increase the net present value of the borrower of the project. More than that we find conform with empirical observations that bad borrowers tend to pledge more collateral than good borrowers. Bad borrowers are not necessarily characterized by riskier projects but alternatively by higher bankruptcy costs of their assets or a lower contribution of them to the initial firm value.

In the second step, we analyze the changes that occur through the introduction of asymmetric information between borrowers and lenders. Incentive compatible contracts exist that screen borrowers if information asymmetry refers to the riskiness of the project or the contribution of the borrower to the project. However, separation fails if bad borrowers are characterized by high bankruptcy costs. Under the socially-optimal equilibrium with information asymmetries, the bad borrower always obtains his or her first best solution as with symmetric information. Surprisingly, good borrowers tend to pledge more collateral than bad borrowers in the presence of asymmetric information concerning the riskiness of the project or the contribution of the borrower to the firm value. Therefore, the relationship between the type of a borrower and the size of collateral inverts through the introduction of asymmetric information. A further remarkable finding is that good borrowers obtain a higher present value from the project than bad borrowers under perfect information. However, under asymmetric information concerning the riskiness of the project it is the other way round because a bad borrower obtains a higher present value from the project than a good borrower. Hence, we can conclude that the motive to reduce bankruptcy costs through the use of costly collateral can explain why bad borrowers pledge more collateral. However, this relationship disappears if information asymmetries are present.

The paper is organized as follows. Section 2 presents the model framework. Optimal strategies for the use of collateral under symmetric information between borrowers and lenders are described in Section 3. Section 4 analyzes the use of collateral and the ability to screen under asymmetric information. Section 5 concludes.

2 The Model

In this model, we consider a borrower who seeks a financing volume equal to I to run a profitable project. The financing volume is provided by a lender in the form of a loan. The loan is a standard zero-coupon contract with face value F and maturity date T. The borrower is a legally independent subsidiary of a holding company. The holding company holds assets which can be utilized to collateralize the loan of its subsidiary. We assume that the holding company's assets are sufficiently large relative to the financing volume I such that we can think of an unlimited size of feasible collateral for the subsidiary. Hence, we take the view of outside collateral in this paper as assets are available that can be used as collateral but need not have to be.

Alternatively, we can think of the borrower as an entrepreneur who can bring collateral from his or her private wealth to secure the loan. In the case of default, lenders have no access to the non-collateralized private wealth of the borrower.

A consequence of the use of collateral is that if the loan defaults at maturity T, the lender not only obtains the liquidation value from the project of the subsidiary but also the liquidation value of the collateral. However, the liquidation process induces costs. The liquidation value $V_T \cdot (1 - \alpha_V)$ at maturity of the project is the project value V_T minus proportional bankruptcy costs $\alpha_V \cdot V_T$. Even if the firm is not liquidated but reorganized costs arise because e.g. some important employees quit or the firm worsens its reputation. Accordingly, a liquidation of the collateral with a value C_T at time T results in a liquidation value equal to $C_T \cdot (1 - \alpha_C)$. Again, even if the collateral is transferred to the lender, we can think of these costs in a way that the reservation value of the collateral is higher for the borrower than for the lender. Moreover, the fact that the holding company uses some of its assets as collateral causes costs even if no default occurs and the assets are not liquidated. We can think of these costs as a result of a limited flexibility to use these assets for other purposes. For example, after bringing them in as collateral, they cannot be sold until maturity T or utilized to back loans from other projects. The cost $\gamma \cdot C_0$ from the loss of flexibility are proportional to the size of collateral C_0 at time $t = 0$ when the loan is originated. Chan/Kanatas (1985) have also introduced cost for pledging collateral which were motivated as maintenance of the collateral according to an agreed level.

The optimal debt service at time T of a loan with collateral value C_T is to fully redeem the loan if the sum of project and collateral value $V_T + C_T$ exceeds the face value F. This strategy maximizes the equity value $E_T(V_T, C_T, F)$ of the borrower at time T. This is because if the face value is fully paid the equity value is $V_T + C_T - F$ but zero otherwise in the case of default. Regarding the fact that the lender bears bankruptcy costs in the case of default, the equity $E_T(V_T, C_T, F)$ and loan value $L_T(V_T, C_T, F)$ at maturity T are given by:

$$E_T(V_T, C_T, F) = \begin{cases} 0, & \text{if } V_T + C_T < F \\ V_T + C_T - F, & \text{if } V_T + C_T \geq F \end{cases}$$

$$L_T(V_T, C_T, F) = \begin{cases} (1 - \alpha_V) \cdot V_T + (1 - \alpha_C) \cdot C_T, & \text{if } V_T + C_T < F \\ F, & \text{if } V_T + C_T \geq F \end{cases}$$

The values of the loan and equity for a typical example are plotted in Figure 1. As long as the project value V_T is below the default barrier $F - C_T = 75$, the firm together with the collateral are liquidated in favor of the lender where the liquidation value equals $(1 - \alpha_C) \cdot C_T + (1 - \alpha_V) \cdot V_T$. Without a default, the loan value is equal to the face value $F = 100$ and the equity value comprises of the sum of project and collateral value $V_T + C_T$ minus face value F.

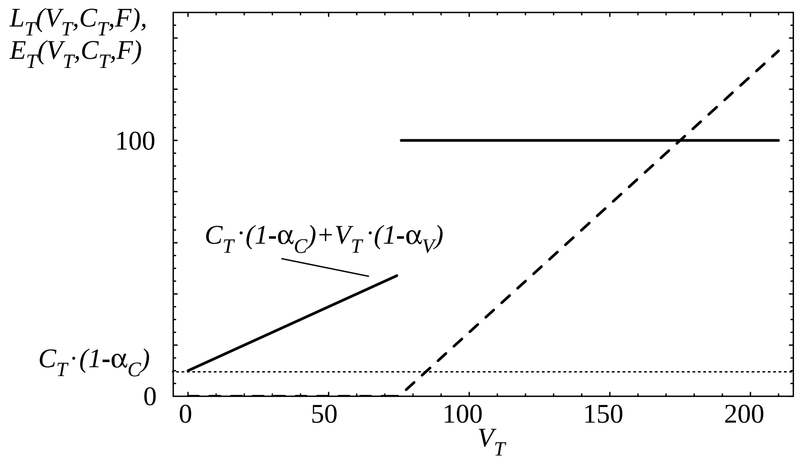
The project value V_T results from the project value V_0 at time $t = 0$ and depends on an uncertain return y. The return

$$y = \ln\left(\frac{V_T}{V_0}\right)/T \sim \text{normal distribution}\,(\mu, \sigma)$$

is normally distributed with mean μ and standard deviation σ. This assumption is consistent with a project value V_t following a geometric Brownian motion. The initial project value V_0 consists of two parts. The first part stems from the financing volume I raised by the loan. The remaining part $I_0 := V_0 - I$ is the contribution

Figure 1: Loan and Equity Value at Maturity

The diagram shows the loan value $L_T(V_T, C_T, F)$ (solid line) and the equity value $E_T(V_T, C_T, F)$ (dashed line) as a function of the project value V_T at maturity T. The parameter values are $C_T = 25$, $F = 100$, $\alpha_V = 0.5$, and $\alpha_C = 0.6$.



of the borrower which can be understood as the value of project know how or other immaterial funds from the borrower etc. The borrower can only participate on I_0 by running the project. If the borrower does not manage to raise a loan with financing volume I, the project does not succeed and therefore the value from the particular subsidiary/borrower for the holding company is zero.

In contrast to the project value, collateralizable assets exhibit a minor uncertainty. To keep the analysis simple, we assume that C_T given its value C_0 at time $t = 0$ is commonly known in advance. Moreover, we assume an arbitrage-free and complete market for claims on the project value V_t and for default-free zero-coupon bonds. The continuously compounding interest rate r remains constant over time. As a consequence, the collateral value C_t behaves like a money market account, i.e. the collateral value C_T equals $C_0 \cdot e^{r \cdot T}$ given that at the initial date $t = 0$ collateral C_0 was pledged. Using risk-neutral expectations $\mathbb{E}^{\mathbb{Q}}(\cdot)$, we obtain the equity value $E_0(V_0, C_0, F)$ and the loan value $L_0(V_0, C_0, F)$ conditional to the terms (F, C_0) of the loan contract as follows:

$$E_0(V_0, C_0, F) = e^{-r \cdot T} \cdot \mathbb{E}^{\mathbb{Q}}\left(E_T\left(V_0 \cdot e^{y \cdot T}, C_0 \cdot e^{r \cdot T}, F\right)\right),$$
$$L_0(V_0, C_0, F) = e^{-r \cdot T} \cdot \mathbb{E}^{\mathbb{Q}}\left(L_T\left(V_0 \cdot e^{y \cdot T}, C_0 \cdot e^{r \cdot T}, F\right)\right)$$

Since we are in a typical Black/Scholes pricing framework, the following closed-form solutions are valid for the values of equity and the loan:

$$\begin{aligned}E_0\left(V_0, C_0, F\right) &= V_0 \cdot N\left(d_1\right) - e^{-r \cdot T} \cdot \left(F - C_0 \cdot e^{r \cdot T}\right) \cdot N\left(d_2\right), \\ L_0\left(V_0, C_0, F\right) &= \left(1-\alpha_V\right) \cdot V_0 \cdot N\left(-d_1\right) + \left(1-\alpha_C\right) \cdot C_0 \cdot N\left(-d_2\right) \\ &\quad + e^{-r \cdot T} \cdot F \cdot N\left(d_2\right)\end{aligned} \tag{1}$$

where

$$\begin{aligned}d_1 &= \frac{\ln\left(\frac{V_0}{F - C_0 \cdot e^{r \cdot T}}\right) + \left(r + \frac{1}{2}\sigma^2\right) \cdot T}{\sigma\sqrt{T}}, \\ d_2 &= \frac{\ln\left(\frac{V_0}{F - C_0 \cdot e^{r \cdot T}}\right) + \left(r - \frac{1}{2}\sigma^2\right) \cdot T}{\sigma\sqrt{T}}\end{aligned}$$

holds and $N(\cdot)$ denotes the cumulative distribution function of the standard normal distribution. These formulae hold as long as the loan is not perfectly secured, i.e. $F > C_0 \cdot e^{r \cdot T}$. For $F \leq C_0 \cdot e^{r \cdot T}$, a default cannot occur and therefore $E_0\left(V_0, C_0, F\right) = V_0 - F \cdot e^{-r \cdot T} + C_0$ and $L_0\left(V_0, C_0, F\right) = F \cdot e^{-r \cdot T}$ are valid. Since this case will only represent an optimal solution in special cases, we will concentrate on the formulae in the standard case $F > C_0 \cdot e^{r \cdot T}$ and indicate when dealing with $F \leq C_0 \cdot e^{r \cdot T}$.

We assume that the borrower can raise the financing volume I by a loan with arbitrary conditions (F, C_0) as long as the loan value $L_0\left(V_0, C_0, F\right)$ is at least equal to I. Thus, we can think of perfect competition among lenders because the considered lender always accepts a loan contract if the value of the loan is at least as high as the payout I from the lender to the borrower.

Since the borrower acts as an agent of the holding company, he chooses the collateral volume C_0^* such that the net present value for the holding company from the project is maximized. The net present value $PV_0\left(V_0, C_0, F\right)$ comprises of the equity value $E_0\left(V_0, C_0, F\right)$ minus the collateral value C_0 and the cost $(\gamma) \cdot C_0$ for pledging collateral:

$$PV_0\left(V_0, C_0, F\right) = E_0\left(V_0, C_0, F\right) - (1+\gamma) \cdot C_0$$

Hence, the optimization condition reads

$$C_0^* = \arg\max_{C_0 \geq 0} PV_0\left(V_0, C_0, F\right) \tag{2}$$

subject to the participation condition

$$L_0\left(V_0, C_0^*, F\right) \geq I \tag{3}$$

and the option

$$PV_0\left(V_0, C_0^*, F\right) > 0 \tag{4}$$

not to run the project when it does not create value for the holding company. We note that an optimal solution C_0^* does not necessarily exist because the firm cannot or does not want to take a loan and run the project. This is because in certain cases for each C_0 one of the two constraints (3) and (4) is not satisfied. For example, if bankruptcy costs α_V are high, there might not be a face value F that satisfies $L_0(V_0, C_0, F) = I$ for low C_0. When F increases the loan value $L_0(V_0, C_0, F)$ might decline because the probability for a costly default rises (see e.g. Barro (1976)). This is the reason why $L_0(V_0, C_0, F)$ has a maximum in F. If this maximum is below I, the participation constraint (3) is violated. If the size of collateral C_0 is sufficiently high, then a face value F exists that satisfies the participation constraint. However, in this case the cost for pledging collateral $\gamma \cdot C_0$ might be such that that the present value $PV_0(V_0, C_0, F)$ from taking the loan and running the project is negative. Then Constraint (4) is violated.

If the borrower decides to run the project and selects a loan with the optimal amount of collateral C_0^* and a face value F, then

$$0 \leq C_0^* \leq F \cdot e^{-r \cdot T}$$

must hold. By definition C_0 cannot be negative. The second boundary is because in the special case that $C_0 = F \cdot e^{-r \cdot T}$ holds, the loan is perfectly secured and the lender will receive the face value with certainty. Thus, a higher collateral cannot provide a better securitization but it results in higher cost $\gamma \cdot C_0$ for pledging the collateral. As a consequence of the fact that the optimal collateral value C_0^* must lie in a compact set, we see that always a solution of the optimization problem (2) exists if only Constraint (3) rather than Constraint (4) has to hold. If this solution results in a positive value of the objective function, it is optimal to run the project and Constraint (4) is also valid. Otherwise, it is not optimal to take a loan and succeed the project as it results in negative wealth for the borrower.

Alternatively, we can represent the objective function of the borrower by the present value of all costs from pledging collateral and liquidation:

$$C_0^* = \arg\min_{C_0 \geq 0} BC^V(V_0, C_0, F) + BC^C(V_0, C_0, F) + \gamma \cdot C_0, \tag{5}$$

where $BC^V(V_0, C_0, F)$ and $BC^C(V_0, C_0, F)$ denote the present value of costs from liquidating the project and the collateral, respectively. These two representations are given by

$$\begin{aligned}BC^V(V_0, C_0, F) &= e^{-r\cdot T}\cdot \mathbb{E}^{\mathbb{Q}}\left(\alpha_V \cdot V_0 \cdot e^{y\cdot T}\cdot 1_{\{V_0\cdot e^{y\cdot T}+C_0\cdot e^{r\cdot T}<F\}}\right)\\ &= \alpha_V \cdot V_0 \cdot N(-d_1),\\ BC^C(V_0, C_0, F) &= e^{-r\cdot T}\cdot \mathbb{E}^{\mathbb{Q}}\left(\alpha_C \cdot C_0 \cdot e^{r\cdot T}\cdot 1_{\{V_0\cdot e^{y\cdot T}+C_0\cdot e^{r\cdot T}<F\}}\right)\\ &= \alpha_C \cdot C_0 \cdot N(-d_2).\end{aligned}$$

As usual $1_{\{\cdot\}}$ denotes the indicator function. The alternative objective function (5) indicates that the borrower has to carry all costs from pledging collateral and a possible liquidation. At first glance, this view might be surprising because in the case of default only the lender suffers from bankruptcy costs but the borrower is left with nothing. However, the lender already accounts for possible bankruptcy costs at time $t = 0$ by claiming fairly priced loan conditions (F, C_0) according to Equation (3). Clearly, in an optimum the loan value must equal I such that the equality in (3) holds. Hence, the present value that both borrower and lender obtain from this project conditional to the loan conditions (F, C_0) is

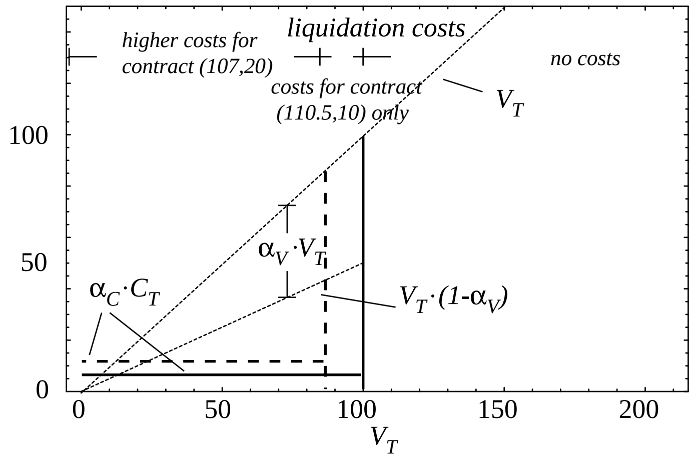
$$\begin{aligned}&E_0(V_0, C_0, F) + I - (1+\gamma)\cdot C_0 \\ &= V_0 - BC^V(V_0, C_0, F) - BC^C(V_0, C_0, F) - \gamma \cdot C_0.\end{aligned} \tag{6}$$

Hence, we find that the maximization according to the objective function (2) is equivalent to a maximization of the left-hand side of representation (6). Moreover, this maximization is equivalent to a minimization of Equation (5) as the right-hand side of (6) is a constant V_0 minus the term in (5). Thus, we see that the borrower uses collateral to reduce the aggregate value of all costs.

Figure 2 illustrates the costs that can be reduced by pledging more collateral and the required additional costs. The consequence of an increase of the collateral C_0 is that the default barrier $V_T - C_0 \cdot e^{r\cdot T} = V_T - C_T$ at time T declines such that a default becomes less likely. Therefore, in those states V_T for which a default is prevented by a higher C_0, bankruptcy costs equal to $\alpha_C \cdot V_C + \alpha_V \cdot V_T$ are saved. In the example of Figure 2, an increase of collateral from $C_T = 10$ to $C_T = 20$ prevents a default and associated bankruptcy costs for project values between 87 and 100.5. An additional important effect from bringing in collateral is that the required face value F to get the loan decreases with C_0 as in this example from 110.5 to 107. This is because the default probability declines with C_0 and the loss given default is lower the more collateral is pledged. However, if the project value V_T is such low that a default occurs at maturity, then a higher collateral C_0 results in higher bankruptcy costs $\alpha_V \cdot V_T + \alpha_C \cdot C_T$. Additionally, pledging more collateral increases the costs

Figure 2: Illustration of Bankruptcy Costs

The diagram illustrates the bankruptcy costs $\alpha_C \cdot C_T$ *of collateral and* $\alpha_V \cdot V_T$ *of the project value dependent on the project value* V_T *for two loan contracts* $(F = 107, C_T = 20)$ *(dashed line) and* $(F = 110.5, C_T = 10)$ *(solid line). The relative bankruptcy costs are* $\alpha_V = 0.5$ *and* $\alpha_C = 0.6$.



$\gamma \cdot C_0$ from losing flexibility of the collateralized assets. These effects are illustrated in Figure 2.

3 Optimal Size of Collateral

In this section, we consider the optimal choice of collateral C_0^* for the borrower. We denote this case as the first best solution, because the borrower and the lender have symmetric information; i.e. the lender perfectly knows the characteristics of the borrower comprising of the distribution of the project value and the bankruptcy costs. In Section 4, we will discard the assumption of symmetric information between the borrower and the lender to study the arising costs of a second best solution under asymmetric information.

In the presence of the symmetric information, an important consequence of pledging collateral is the reduction of the face value F. For a given level of collaterl C_0, the borrower only regards the loan contract $L_0(V_0, C_0, F) = I$ with the lowest feasible face value F as only this choice can maximize the objective function (2). We recall that $L_0(V_0, C_0, F) = I$ can have more than one solution. We can derive the slope by which the relevant face value $F = F(C_0)$ declines with C_0 as follows: Let F be the relevant face value for the collateral volume C_0. Thus,

$$L_0\left(V_0, C_0, F\left(C_0\right)\right) = I$$

holds. Applying the implicit function theorem, we can write for the derivative $\frac{\partial F(C_0)}{\partial C_0}$ of the face value function $F\left(C_0\right)$

$$\frac{\partial F\left(C_0\right)}{\partial C_0} = -\frac{\frac{\partial L_0(V_0,C_0,F)}{\partial C_0}}{\frac{\partial L_0(V_0,C_0,F)}{\partial F}} = e^{rT} \cdot \frac{\mathbf{A} - \left(1-\alpha_C\right) \cdot N\left(-d_2\right)}{\mathbf{A} + N\left(d_2\right)} \tag{7}$$

with

$$\mathbf{A} = \left(-\frac{1}{\sqrt{2\pi\sigma^2 T}} e^{-\frac{\left(2r-\sigma^2\right)^2 T^2 + 4\ln\left(\frac{V_0}{F-C_0e^{rT}}\right)^2}{8\sigma^2 T}} \frac{\left(\frac{V_0}{F-C_0e^{rT}}\right)^{\frac{1}{2}-\frac{r}{\sigma^2 T}}}{F - C_0e^{rT}} \right)$$
$$\cdot \left(\alpha_V \cdot \left(F - C_0e^{rT}\right) + \alpha_C \cdot C_0e^{rT}\right).$$

Figure 3 shows a typical example, of the face value $F\left(C_0\right)$ as a function of the collateral volume C_0. Clearly, the face value $F\left(C_0\right)$ declines with C_0 because a loan is better secured the higher C_0. Moreover, the decrease is convex. This observation is intuitive because the lender will benefit more from the first marginal unit of collateral than from further units. Therefore, the decline of $F\left(C_0\right)$ is especially pronounced for a collateral value C_0 close to zero. For a high size of collateral, the loan is almost default-free and therefore the face value is close to $I \cdot e^{r \cdot T}$ and does not vary strongly.

To determine the optimal amount of collateral C_0^*, the borrower regards the present value $PV_0\left(V_0, C_0, F\left(C_0\right)\right)$ of the project for the borrower depending on the collateral volume C_0. Here, the objective function only depends on the collateral volume C_0, because the size of collateral also specifies the face value $F = F\left(C_0\right)$. Hence, the borrower selects C_0^* that is related to the maximum of the objective function. Figure 4 shows an example of the borrower's objective function. This function first increases with C_0 and then declines. The reason for this shape is as follows. First the objective function benefits from a higher C_0 as the decrease of the face value $F\left(C_0\right)$ is especially pronounced which reduces the present value of bankruptcy costs. For higher volumes of collateral, the decrease of $F\left(C_0\right)$ is relatively low but further collateral causes additional costs, e.g. for pledging collateral γ. In this example, the optimal collateral volume C_0^* equals 26.8 and results in a present value $PV_0\left(V_0, C_0, F\left(C_0\right)\right)$ of the project for the borrower equal to 46.5. This value is considerably higher than the value $PV_0\left(V_0, 0, F\left(0\right)\right) = 38.1$ without any collateral and $PV_0\left(V_0, 100, F\left(100\right)\right) = 40$ with a perfect collateralization. As a general result,

Figure 3: Face Value of a Loan

The diagram shows the face value $F(C_0)$ of a loan as a function of the collateral volume C_0. The parameter values are $I_0 = 50$, $I = 100$, $\sigma = 0.3$, $r = 0.05$, $T = 1$, $\alpha_V = 0.5$, and $\alpha_C = 0.6$.

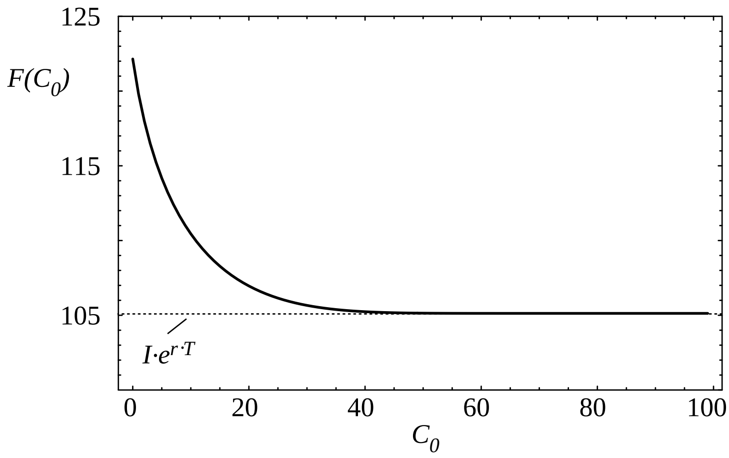
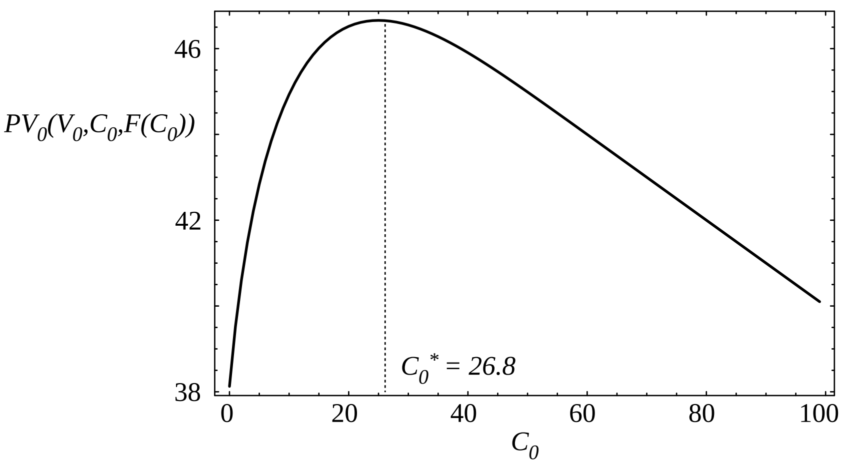


Figure 4: Objective Function of Borrower

The diagram shows the present value $PV_0(V_0, C_0, F(C_0))$ a borrower obtains with the project as a function of the collateral volume C_0. The parameter values are $I_0 = 50$, $I = 100$, $\sigma = 0.3$, $r = 0.05$, $T = 1$, $\alpha_V = 0.5$, $\alpha_C = 0.6$, and $\gamma = 0.1$.



we can conclude that the objective function declines with C_0 if the collateral volume C_0 is sufficiently high. For sufficiently high C_0 (at least if C_0 exceeds $F \cdot e^{-r \cdot T}$ and the loan is fully secured) the benefits from further collateral in form of a lower face value $F(C_0)$ and lower bankruptcy costs are marginal as the default probability is close to zero anyhow. Thus, an increase of C_0 primarily increases the marginal cost γ of pledging collateral which exceed the marginal benefits. The optimal size C_0^* of collateral is either equal to zero or follows from the first order condition. Given that $(F(C_0), C_0)$ is a feasible contract where the equality in (3) holds and profitable according to (4), the derivative of the objective function

$$\left.\frac{\partial \left(E_0 \left(V_0, C_0, F\left(C_0\right)\right) - (1+\gamma) \cdot C_0\right)}{\partial C_0}\right|_{C_0=C_0^*} = \tag{8}$$

$$\left. N\left(d_2\right) \cdot \left(1 - e^{-r \cdot T} \frac{\partial F\left(C_0\right)}{\partial C_0}\right) - (1+\gamma) \right|_{C_0=C_0^*} = 0$$

equals zero, if C_0^* is an inner solution. Otherwise, the boundary solution $C_0^* = 0$ can be optimal or it is not worthwhile for the borrower to run the project such that no solution for C_0^* exists.

In the following subsections, we analyze how the main parameters bankruptcy costs, α_V and α_C, cost for pledging collateral γ, the volatility of the firm value return σ, and the contribution I_0 of the borrower to the firm value affect the optimal choice C_0^* of collateral.

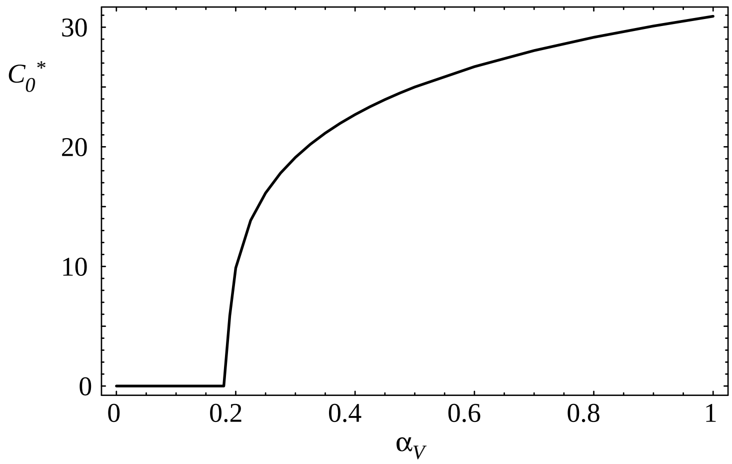
a) Bankruptcy Costs α_V of the Project Value

Higher bankruptcy costs α_V of the project value result in a lower loan value $L_0(V_0, C_0, F)$ when the terms of the loan (F, C_0) are fixed. Thus, the lender must claim a higher face value F for a given volume of collateral C_0 if α_V increases. If α_V is too high for a given C_0, no face value F might exist such that the loan value equals the investment volume I but lies below I for all F. Hence, F increases with α_V or a loan contract is not feasible anymore for a given C_0. As a consequence of the fact that F increases with α_V, the present value $PV_0(V_0, C_0, F)$ for the borrower also declines with α_V for every C_0. Thus, the present value under the optimal collateral volume C_0^* must also decline with α_V. However, the question now is how the size of optimal collateral C_0^* behaves with α_V.

If α_V is zero, a non-collalteralized loan $L_0(V_0, 0, F)$ does not cause any costs from liquidating assets or pledging collateral. Hence, $C_0^* = 0$ is always the optimal choice in the absence of bankruptcy costs α_V as this is the optimal outcome of the non-negative objective function (5). Even though the face value F is higher for $C_0 = 0$ than with a positive collateral value $C_0 > 0$ and therefore the capital costs are higher,

Figure 5: Optimal Collateral Volume

The diagram shows the optimal volume C_0^* of collateral as a function of bankruptcy costs α_V of the project value. The parameter values are $I_0 = 50$, $I = 100$, $\sigma = 0.3$, $r = 0.05$, $T = 1$, $\alpha_C = 0.6$, and $\gamma = 0.1$.



$C_0^* = 0$ is the optimal solution. However, it is not the size of the face value that matters but the size of the total costs given in Equation (5). As the cost are positive for $C_0 > 0$, it is not advantageous for the borrower to pledge collateral. Clearly, if $\alpha_V = 0$ holds, it is always possible to take a loan as $L_0(V_0, C_0, F)$ increases to the limit $V_0 = I_0 + I$ if F tends to infinity.

Figure 5 shows how the optimal collateral volume C_0^* evolves with α_V. We can see in this figure that C_0^* is an increasing function in α_V. If α_V is below a critical value equal to 0.18, no collateral is used but for higher values of α_V the collateral volume C_0^* strictly increases. The outcome that C_0^* increases with the bankruptcy costs α_V is intuitive because a higher α_V creates a higher potential to save costs by bringing in collateral. In other words, a marginal unit of collateral is especially worthwhile if the saved bankruptcy costs are high. In other cases, it can happen that for high bankruptcy costs α_V, it is not worthwhile for the borrower to run the project but Constraint (4) is violated for all C_0 for which a loan with value I can be raised.

To formally prove the monotonicity of C_0^* in α_V, we suppose the standard case that the objective function $PV_0(V_0, C_0, F(C_0))$ has one local maximum in C_0 which corresponds to the global maximum for all considered bankruptcy costs α_V. Let C_0' be the optimal collateral value for bankruptcy costs α_V'. In what follows, we show that for slightly higher bankruptcy costs $\alpha_V'' > \alpha_V'$, the derivative of the present value from the project for the borrower is still positive even though C_0 exceeds C_0'. This means that for higher bankruptcy costs α_V'' a local maximum of the objective function at a value higher than $C_0 > C_0'$ exists. The derivative of the objective function according to Equation (8) equals zero for C_0' when bankruptcy costs are α_V'. Now we consider the higher bankruptcy costs α_V'' and the same default barrier $F^{low}(C_0') - C_0' \cdot e^{r \cdot T}$ as with lower bankruptcy costs. Since the face value $F^{low}(C_0)$ given α_V' is lower than the face value $F^{high}(C_0)$ for the same collateral value C_0 but with a higher α_V'', we know that the coincidence of the default barrier

$$F^{low}(C_0') - C_0' \cdot e^{r \cdot T} = F^{high}(C_0'') - C_0'' \cdot e^{r \cdot T}$$

implies that the collateral volume under the higher bankruptcy costs is higher:

$$C_0'' > C_0'$$

One can easily verify that the derivative $\frac{\partial PV_0(V_0, C_0, F(C_0))}{\partial C_0}$ at $C_0'' > C_0'$ for high bankruptcy costs α_V'' is positive while the derivative equals zero for C_0' and α_V'. This is due to the fact that in Representation (8) only the term $\frac{\partial F(C_0)}{\partial C_0}$ is affected through the choice of higher bankruptcy costs $\alpha_V'' > \alpha_V'$ and collateral value $C_0'' > C_0'$ because the default barrier remains unaffected. The term $\frac{\partial F(C_0)}{\partial C_0}$ is lower under the higher bankruptcy costs and therefore the derivative $\frac{\partial PV_0(V_0, C_0, F(C_0))}{\partial C_0}$ at $C_0'' > C_0'$ is strictly positive. The reason that $\frac{\partial F(C_0)}{\partial C_0}$ is lower for high bankruptcy costs α_V is due to the fact that in Representation (7) only $\mathbf{A}$ is affected but d_2 is constant. Since $\mathbf{A}$ is negative and lower for α_V'', the whole derivative $\frac{\partial F(C_0)}{\partial C_0}$ is lower which reveals that $\frac{\partial (E_0(V_0, C_0, F(C_0)) - (1+\gamma) \cdot C_0)}{\partial C_0}$ is positive. Hence, the optimal collateral volume for α_V'' must be higher than that for $\alpha_V' < \alpha_V''$. We summarize our findings in the following result:

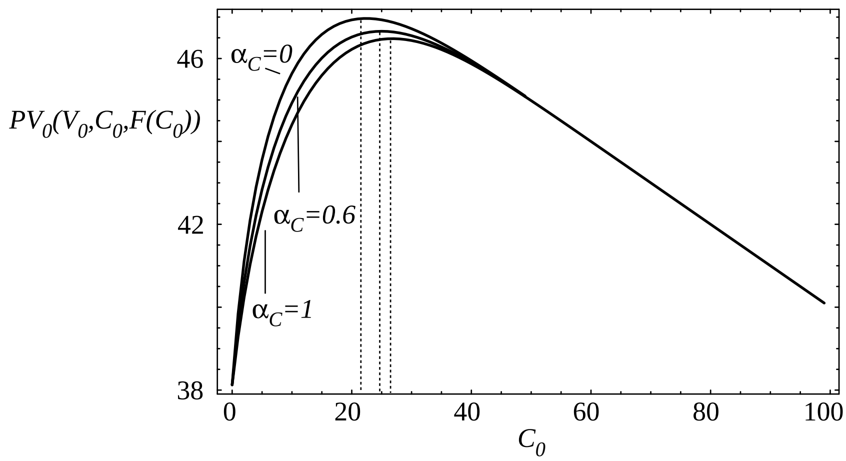
Result 1 (Bankruptcy Costs α_V) *The present value $PV_0(V_0, C_0^*, F(C_0^*))$ from the project for the borrower under the optimal volume of collateral C_0^* declines with α_V. If bankruptcy costs α_V are zero, then no collateral $C_0^* = 0$ is the optimal choice. Otherwise, the optimal choice of collateral C_0^* is non-decreasing in α_V (at least for those α_V for which the global maximum of the objective function $PV_0(V_0, C_0, F(C_0))$ is also the unique local maximum).*

b) Cost of Pledging γ and Liquidating Collateral α_C

An increase of the cost from pledging collateral γ results in a lower value $PV_0(V_0, C_0, F(C_0))$ of the project for the borrower for every size of collateral $C_0 > 0$.

Figure 6: Objective Function of Borrower

The diagram shows the present value $PV_0(V_0, C_0, F(C_0))$ a borrower obtains with the project as a function of the collateral volume C_0 for different bankruptcy costs α_C. The parameter values are $I_0 = 50$, $I = 100$, $\sigma = 0.3$, $r = 0.05$, $T = 1$, $\alpha_V = 0.5$, and $\gamma = 0.1$.



Therefore, $PV_0(V_0, C_0^*, F(C_0^*))$ under the optimal collateral strategy must also decline with γ. If pledging collateral is costless, $\gamma = 0$, the optimal amount C_0^* of collateral equals $F \cdot e^{-r \cdot T}$. In this case, the loan is fully collateralized and therefore the total costs involved are zero. This is the most favorable case for the borrower.

In addition, we can see from the objective function $PV_0(V_0, C_0, F(C_0))$ of the borrower that an increase of γ leads to a lower optimal collateral volume C_0^*. This is because the higher C_0, the more pronounced is the reduction of the objective function from a higher γ. Hence, the optimal strategy of the borrower is to choose a lower C_0^* when the cost for pledging collateral rise.

When γ becomes arbitrarily high, the costs for using collateral exceed the potential benefits and therefore $C_0^* = 0$ is the only possible solution. Whether or not the borrower will run the project with an uncollateralized loan depends on the fact that it is possible to raise a loan with payoff value I such that the value from the project for the borrower is still positive.

The higher the bankruptcy costs α_C of collateral, the lower the value $PV_0(V_0, C_0, F(C_0))$ of the project for the borrower. This is because a less favorable liquidation in the case of default forces a higher face value $F(C_0)$ for a given collateral volume. Hence, the value $PV_0(V_0, C_0^*, F(C_0^*))$ from the project for the borrower under the optimal collateral strategy declines with α_C.

Figure 6 shows the objective function $PV_0(V_0, C_0, F(C_0))$ over the collateral volume C_0 for different bankruptcy costs α_C. If α_C increases, the decrease of $PV_0(V_0, C_0, F(C_0))$ is primarily pronounced for intermediate values of collateral. For collateral values C_0 close to zero, there is only few collateral involved and therefore the change of $PV_0(V_0, C_0, F(C_0))$ with the bankruptcy costs of collateral is relatively low. For high sizes of collateral C_0 close to $F \cdot e^{-r \cdot T}$, the loan is almost fully secured. Therefore, the probability for a liquidation of the collateral is relatively low such that the bankruptcy costs $BC^C(V_0, C_0, F)$ are close to zero and do not severely depend on α_C. For intermediate collateral values C_0, a default is still probable and due to the severe amount of pledged collateral C_0 the liquidation value highly depends on α_C.

When α_C increases, we see that the optimal size C_0^* of collateral increases in the example of Figure 6. The reasoning behind this finding is that the ability of collateral to reduce the face value is less effective if α_C is high. Therefore, more collateral is required to optimally reduce the face value and the present value of bankruptcy costs.

It is also possible that the optimal collateral volume C_0^* declines with α_C. This case can be interpreted as follows. The higher α_C the lower are the benefits from using collateral. Since pledging collateral is costly, a lower collateral volume is used. Summing up, we obtain the following result:

Result 2 (Costs of Collateral α_V and γ) *The present value from the project $PV_0(V_0, C_0^*, F(C_0^*))$ for the borrower under the optimal volume of collateral C_0^* declines with γ and α_C. If the cost γ for pledging collateral are zero, then a full collateralization $C_0^* = F \cdot e^{-r \cdot T}$ is the optimal choice. Otherwise, the optimal choice of collateral C_0^* is non-increasing in γ. An increase of α_C can result in a higher and lower optimal collateral volume C_0^*.*

c) Volatility σ of Project Value Return

If the volatility σ of the project value return increases, the present value $PV_0(V_0, C_0, F(C_0))$ from the project for the borrower also increases for fixed C_0 and $F(C_0)$. This is a result of the call characteristic of the equity. Usually an increase of σ is associated with a decline of the loan value $L_0(V_0, C_0, F(C_0))$. In such cases the face value $F(C_0)$ increases with σ for every C_0 and the position of the borrower $PV_0(V_0, C_0, F(C_0))$ typically declines.

However, in special cases the loan value can also increase with σ because the loan value can have a convex shape in V_0 due to the bankruptcy costs. To see the convex shape regard a value V_T slightly below the default barrier $F - C_T$ and a slight change of $\pm\Delta V_T$. If V_T decreases by ΔV_T, the decrease of the size of the loan value decrease $\Delta V_T \cdot (1 - \alpha_V)$ can be much lower than the increase through a rise of V_T by ΔV_T. If a default is prevented and bankruptcy costs are saved when V_T increases, then the increase of the loan value is above $V_T \cdot \alpha_V$ which is much higher than the slight change of ΔV_T. In this case, $F(C_0)$ can decrease with σ at least for some C_0. Hence, we see that a higher σ can result in a higher and lower value $PV_0(V_0, C_0^*, F(C_0^*))$ of the position of the borrower.

If the volatility σ is zero, then the optimal strategy is to take no collateral. This is obvious as in the absence of uncertainty, $\sigma = 0$, the firm cannot default and therefore the total costs are zero even without collateral. Hence, further collateral would only increase the cost $\gamma \cdot C_0$ for pledging collateral which worsens the position of the borrower. In the opposite case that the volatility σ is sufficiently high, only fully collateralized loans $C_0^* = F \cdot e^{-r \cdot T}$ can be optimal or it is not worthwhile for the lender to grant any loan. This is because the value of a non-fully collateralized loan tends to the liquidation value $(1 - \alpha_C) \cdot C_0$ of the collateral if σ converges to infinity as Equation (1) indicates. Thus, to have a loan value equal to I, the relation $(1 - \alpha_C) \cdot C_0 = I$ must hold which implies $C_0 > I$ and contradicts the assumption $C_0 < I$ that non-fully collateralized can be granted. Hence, the only feasible loan contract is a default-free, fully-collateralized loan with $C_0 = I$ and face value $F = I \cdot e^{r \cdot T}$. Then, the cost $\gamma \cdot I$ for pledging the required collateral decide whether or not to run the project. As long as

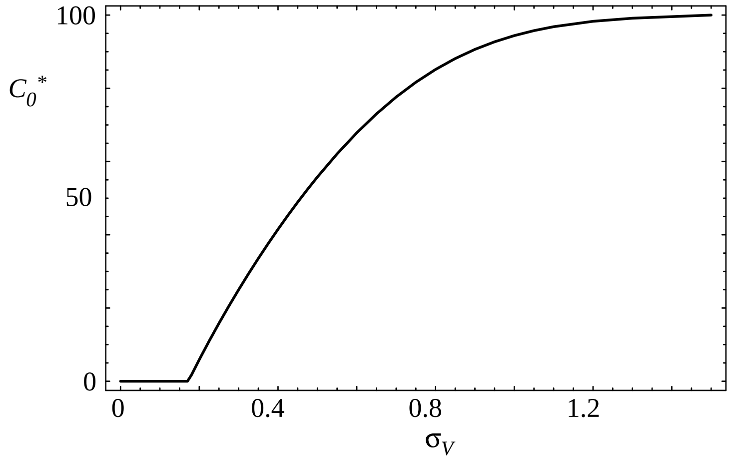
$$I_0 - \gamma \cdot I > 0$$

holds, the contribution of the borrower I_0 is higher than the cost $\gamma \cdot I$ from pledging a fully-collateralized loan with $C_0 = I$ and therefore the loan with $C_0^* = I$ is taken. Otherwise, for $I_0 - \gamma \cdot I < 0$, the project is not worthwhile for the borrower.

Figure 7 shows how the optimal size C_0^* of collateral behaves as a function of the volatility σ. For small volatilities below 0.17, it is optimal to use no collateral. For higher volatilities, C_0^* increases with σ until the loan is fully collateralized. This behavior is intuitive as a higher degree of uncertainty results in higher cost from liquidation which are reduced by bringing in more collateral. However, it is also possible that for intermediate volatilities $0 < \sigma < \infty$, the optimal size of collateral C_0^* can decrease with σ. This case can occur if e.g. bankruptcy costs α_V and α_C are close to one and the default barrier $F(C_0^*) - C_0^* \cdot e^{r \cdot T}$ is high relative to V_0. Then, a higher σ reduces the present value of bankruptcy costs as the probability that V_T exceeds the default barrier rises with σ and accordingly the default probability

Figure 7: Optimal Collateral Volume

The diagram shows the optimal volume C_0^* of collateral as a function of the volatility σ of return of the project value. The parameter values are $I_0 = 50$, $I = 100$, $r = 0.05$, $T = 1$, $\alpha_V = 0.5$, $\alpha_C = 0.6$, and $\gamma = 0.1$.



declines. Since the potential to save bankruptcy costs by bringing in collateral is lower for a higher value σ, we can understand why it is possible that C_0^* might decline with σ. These findings are summarized in the next result:

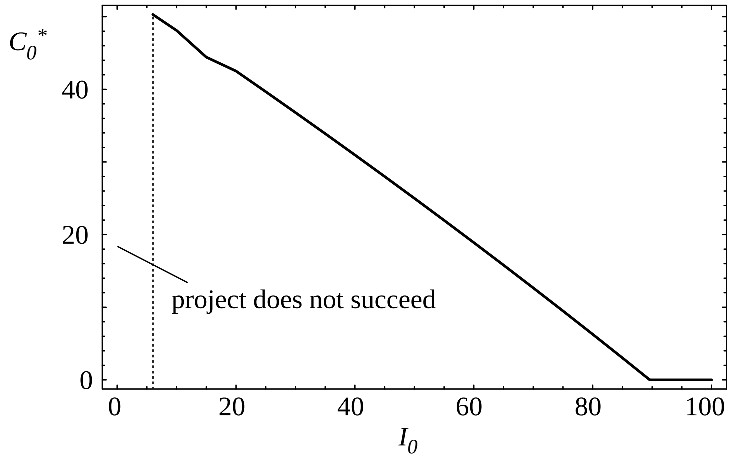
Result 3 (Volatility σ) *If the volatility σ of the project value return is zero, then no collateral, $C_0^* = 0$, is optimal for every loan contract. Conversely, if σ is sufficiently high, only a full collateralization $C_0^* = F \cdot e^{-r \cdot T}$ can be the optimal choice or it is worthwhile for the borrower not to run the project. The optimal choice of collateral C_0^* is usually non-decreasing in σ but it might also decrease in special cases. The present value $PV_0\left(V_0, C_0^*, F\left(C_0^*\right)\right)$ of the project for the borrower under the optimal volume of collateral C_0^* usually declines with σ but it can also increase.*

d) Contribution of the Borrower I_0

We consider a variation of the contribution of the borrower I_0 to the project for a fixed financing volume I. A higher contribution I_0 of the borrower to the project value V_0 results in a higher value $PV_0\left(V_0, C_0, F\left(C_0\right)\right)$ for every size of collateral C_0. This is obvious because the project value $V_0 = I + I_0$ benefits and the required loan redemption payment $F\left(C_0\right)$ declines. Therefore, $PV_0\left(V_0, C_0^*, F\left(C_0^*\right)\right)$ under the optimal collateral strategy must also increase with I_0. If the contribution I_0 by the borrower is sufficiently high, a loan contract without collateral, $C_0^* = 0$, is optimal

Figure 8: Optimal Collateral Volume

The diagram shows the optimal volume C_0^* of collateral as a function of the contribution I_0 of the borrower. The parameter values are $I = 100$, $r = 0.05$, $T = 1$, $\sigma = 0.3$, $\alpha_V = 0.5$, $\alpha_C = 0.6$, and $\gamma = 0.1$.



for the borrower. To see this optimal decision, we regard the costs of the borrower in Equation (5). For $C_0 = 0$ these costs tend to zero, when I_0 goes to infinity but the costs are positive if $C_0 > 0$ holds and pledging collateral is costly $\gamma > 0$. Conversely, if I_0 is close to zero and the firm bears positive bankruptcy costs α_V for the project and cost for pledging collateral γ, it is not optimal for the borrower to succeed the project. This is due to the fact that only collateralized loans are granted by lenders but the cost for pledging collateral exceed the contribution I_0 of the borrower to the project value if I_0 is sufficiently close to zero.

Figure 8 shows a typical example for the optimal size of collateral C_0^* depending on I_0. The higher I_0, the lower the optimal size C_0^* of collateral. This outcome is intuitive as a higher contribution I_0 to the project value results in a lower present value of bankruptcy costs and therefore the incentive to pledge collateral to save some of these bankruptcy costs diminishes. We observed this general property in all examples. For values of I_0 below 5.6, the present value $PV_0\left(V_0, C_0^*, F\left(C_0^*\right)\right)$ of the project for the borrower becomes negative and therefore the project is not succeeded. The next result summarizes our findings:

Result 4 (Contribution I_0 of the Borrower) *The present value $PV_0\left(V_0, C_0^*, F\left(C_0^*\right)\right)$ of the project for the borrower under the optimal volume of collateral C_0^* increases with I_0. If the contribution I_0 is sufficiently high, then* *no collateral $C_0^* = 0$ is the optimal choice. If I_0 is close to zero and the firm bears costs for liquidating assets α_V and pledging collateral γ, it is not worthwhile for the borrower to succeed the project. In all considered examples, the optimal choice of collateral C_0^* is non-increasing in I_0.*

In general, we observe that lower bankruptcy costs α_V and α_C, a lower volatility σ, and a higher contribution I_0 of the borrower improve the environment to raise a loan and reduce the required face value $F(C_0)$.[†] The consequence from more favorable conditions to raise a loan is that typically the optimal collateral volume C_0^* is lower.[‡] Hence, we can conclude that essentially bad borrowers use higher volumes C_0^* of collateral to reduce the costs from taking a loan. This finding is in contrast to the important class of models using collateral as a screening device under asymmetric information (see e.g. Bester (1985, 1987), Chan/Kanatas (1985) and Besanko/Thakor (1987a, 1987b)).

Nevertheless, Berger/Udell (1990) point out that these implications from models with asymmetric information are not in line with 'conventional wisdom' rather than our model. As a consequence, our model using collateral to reduce bankruptcy costs and interest rate payments is capable to provide an economic foundation for the 'conventional wisdom'. Empirical studies in this field presented by Berger/Udell (1990), Jimenez/Saurina (2003), Jimenez/Salas/Saurina (2004), Gonas/Highfield/Mullineax (2004), and Dey/Dunn (2004) confirm the fact that primarily bad rather than good borrowers use collateral.

4 Using Collateral to Separate Between Good and Bad Borrowers

One important reason in the literature for the usage of collateralized loans is the ability to mitigate information asymmetries. The standard argument is that pledging collateral creates costs if a loan defaults. Therefore, pledging collateral is cheaper for a good borrower with a low default probability compared to a bad borrower. This is the reason why equilibria can exist in which a good borrower prefers a loan with collateral and low face value while a bad borrower is better off with a loan with

[†] We recall that in very special cases, a higher volatility can be advantageous from the perspective of the lender and results in a lower face value.

[‡] In some cases lower bankruptcy costs α_C require more collateral C_0^* and in extreme cases a higher volatility σ might reduce the optimal collateral volume.

a higher face value but without collateral. Due to this separating device, lenders recognize the type of the borrower when the borrower makes a choice for a specific loan contract.

In our model framework with bankruptcy costs for the project value and collateral, there are different characteristics that distinguish a good from a bad borrower. A good borrower has lower bankruptcy costs, α_V and α_C, a lower volatility σ of the project value return, and a higher initial contribution I_0 to the project value compared to a bad borrower. Now the question is how a lender can separate between good and band borrowers in the presence of asymmetric information by offering collateralized loan contracts and which type of lender will prefer which contract. Then, it is interesting to see whether under asymmetric information good borrowers still pledge less collateral.

In what follows, we understand asymmetric information such that the lender and the borrower have perfect information in terms of all loan-relevant characteristics except for one particular characteristic. Concerning this characteristic the borrower can exhibit two attributes either a good one g or bad one b. The classification good or bad comes from the perspective of the lender. If c.p. an attribute results in a higher loan value (and therefore also in a lower face value function $F(C_0)$) we speak from a good borrower while in the case of the other attribute the borrower is denoted as bad. There is no information available about the probability of facing a good or bad borrower. If a bank cannot separate between a good and a bad borrower, the lender treats every borrower as a bad one. Otherwise, the lender bears the danger of running a loan portfolio with a negative present value as knowledge about the probability of having a good or bad borrower is not available. If the lender is sure to grant a loan to a good borrower, then the terms of the loan can be more favorable to the borrower, i.e. the required face value $F^g(C_0)$ is lower than the face value $F^b(C_0)$ a bad borrower must pay for the same collateral volume. In what follows we treat the lender as a social planner who follows Pareto optimal strategies for the borrowers under the constraint that only loan contracts with a value at least as high as the payoff I are granted.

In the ideal case that a separating equilibrium exists, the lender can learn about the type of the borrower by offering two different loan contracts $\left(F^{(1)}, C^{(1)}\right)$ and $\left(F^{(2)}, C^{(2)}\right)$ with identical payoff equal to the financing volume I of the borrower. The important condition for this set of contracts is that good borrowers decide for a different contract than bad borrowers. If the good borrower prefers contract one, then the lender accounts for the good type of the borrower and the face value $F^{(1)}$ is below that of a bad borrower $F^{(1)} < F^b\left(C^{(1)}\right)$. Additionally, the lender knows that only bad borrowers choose contract two such that the face value $F^{(2)}$ reflects the bad type of the borrower and equals $F^b\left(C^{(2)}\right)$. Now we want to examine whether suchlike separating equilibria exist in the presence of asymmetric information for the different characteristics of the borrower. Moreover, we will regard which costs arise from revealing the true type compared to the first best solution with symmetric information.

a) Bankruptcy Costs α_V and α_C

We start our analysis with the case that the lender cannot accurately observe the size of bankruptcy costs α_V of the project value. All other characteristics of a prospectus borrower are perfectly available. There are two possible states of the bankruptcy costs with

$$\alpha_V^g < \alpha_V^b.$$

Since the bankruptcy costs α_V^g of the project value from a good borrower are lower than the bankruptcy costs α_V^b from a bad borrower, the lender can grant loans to a good borrower with lower face value $F^g\left(C_0\right)$ than to a bad borrower $F^b\left(C_0\right)$ given that the lender knows about the borrower's type.

However, the net present value $PV_0^i\left(V_0, C_0, F\right)$ of the project from a borrower with type i does not depend on the bankruptcy costs α_V when the terms of the loan (F, C_0) are given:

$$PV_0^g\left(V_0, C_0, F\right) = PV_0^b\left(V_0, C_0, F\right)$$

This is because if the borrower defaults, he is left with nothing and the size of α_V only affects the wealth of the lender. This property brings in the problem that a good borrower prefers the loan contract $\left(F^{(1)}, C^{(1)}\right)$ relative to $\left(F^{(2)}, C^{(2)}\right)$ if and only if the bad borrower also prefers this contract because of

$$PV_0^i\left(V_0, C_0^{(1)}, F^{(1)}\right) > PV_0^i\left(V_0, C_0^{(2)}, F^{(2)}\right), \text{ for } i = g, b.$$

Hence, a set of two different loan contracts does not exist such that good and bad borrowers make different decisions. As a result, the lender will consider every type of borrower as a bad borrower. If every borrower can choose between any loan contract $\left(F^b\left(C_0\right), C_0\right)$ with collateral volume C_0 and the fair face value $F^b\left(C_0\right)$ for a bad borrower, the loan contract taken by both types of borrowers is the optimal loan contract of the bad borrower

$$\left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right).$$

Clearly, if the lender accepts any size of collateral, the bad borrower can follow his or her first best contract. In Section 3, we have seen that lower bankruptcy costs α_V result in a lower size of collateral. However, this effect arises if the lender accounts for the good type of the borrower by allowing for a lower face value $F^g(C_0)$. Since the good borrower must also pay the high face value $F^b(C_0)$, the objective function

$$\max_{C_0 \geq 0} PV_0^g\left(V_0, C_0, F^b(C_0)\right)$$

is identical to that of a bad borrower and therefore $\left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)$ is also the optimal loan contract for the good borrower.

Therefore, a bad borrower does not suffer under asymmetric information but the costs for the good borrower are that he or she obtains the same net present value from the project as with higher bankruptcy costs α_V^b.

If there is asymmetric information about the bankruptcy costs α_C of the collateral rather than α_V, it can be argued in an analogous way to show that the same findings are also true in the presence of asymmetric information about α_C. We summarize our findings in the next result:

Result 5 (Asymmetric Information about α_V or α_C) *In the presence of asymmetric information about α_V (or α_C), lenders cannot separate between good and bad borrowers by offering different loan contracts. Good and bad borrowers obtain the same net present value, $PV_0^i\left(V_0, C_0^{*,b}, F^b\left(C_0^{*,b}\right)\right)$, $i = g, b$, from the project and choose an identically collateralized loan contract $\left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)$. Therefore, the welfare loss from asymmetric information is fully carried by good borrowers.*

b) Volatility σ of Project Value Return

In this subsection, we consider the case that the lender cannot observe whether a borrower has a high σ^b or a low volatility σ^g of project value return. In what follows, we focus on the standard case in which a higher volatility destroys value because of an increase of the present value of bankruptcy costs. Hence, the sum of the net present value $PV_0^i(V_0, C_0, F)$ of the project for the borrower plus the loan value $L_0^i(V_0, C_0, F)$ are lower for a bad borrower $i = b$ than for a good borrower $i = g$:

$$PV_0^g(V_0, C_0, F) + L_0^g(V_0, C_0, F) > PV_0^b(V_0, C_0, F) + L_0^b(V_0, C_0, F) \tag{9}$$

In the first step we provide useful relations that are helpful for the following analysis. An important relation is that the borrower benefits from a bad state for given terms of the loan contract (F, C):

$$PV_0^g\left(V_0, C_0, F\right) < PV_0^b\left(V_0, C_0, F\right) \tag{10}$$

This is because a higher volatility only affects the equity value $E_0\left(V_0, C_0, F\right)$ of a borrower which can be represented by a standard call option written on the underlying V_t. Since call option values benefit from a higher volatility, we see why relation (10) holds. As a consequence of (9) and (10), we can conclude that the loan value for given terms suffers when the borrower's type is bad rather than good. Hence, a lender claims a higher face value $F^b\left(C_0\right)$ for a given collateral volume C_0 when the borrower is supposed to be bad rather than good. Only if the loan is fully collateralized, the required face values coincide:

$$F^b\left(C_0\right) \geq F^g\left(C_0\right)$$

Since the borrower suffers from a higher face value of the loan contract, both a good and a bad borrower are better off if they get a loan with face value $F^g\left(C_0\right)$ which is priced for the good borrower rather than a loan with same collateral but higher face value $F^b\left(C_0\right)$:

$$PV_0^i\left(V_0, C_0, F^g\left(C_0\right)\right) \geq PV_0^i\left(V_0, C_0, F^b\left(C_0\right)\right), \text{ for } i = g, b \tag{11}$$

In addition, the net present value of the project for the borrower is higher when the borrower is of a good type rather than of a bad type and the lender acknowledges the borrower's real type:

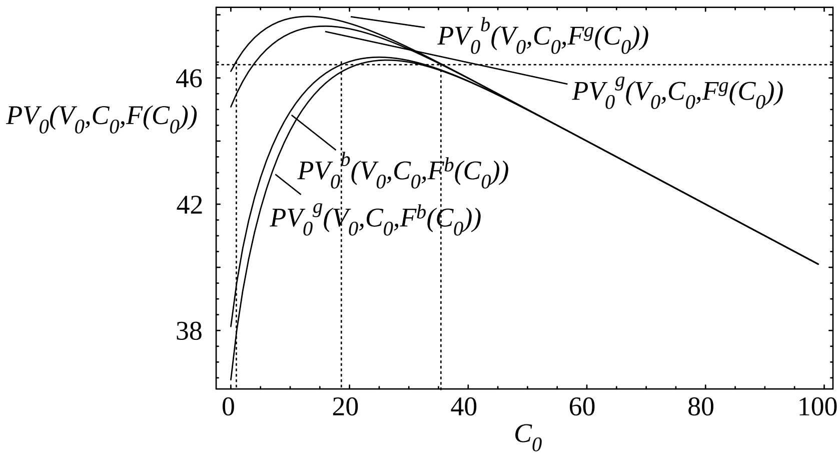
$$PV_0^g\left(V_0, C_0, F^g\left(C_0\right)\right) > PV_0^b\left(V_0, C_0, F^b\left(C_0\right)\right) \tag{12}$$

This is an immediate consequence of Relation (9) and the fact that the loan values are equal to the investment volume I.

Figure 9 shows the net present value of the project for a good and bad borrower if the borrower can raise a loan contract at the terms of a good or bad borrower. The good borrower differs from the bad one by a lower volatility $\sigma^g = 0.25 < 0.3 = \sigma^b$ of project value return. All these four functions are first increasing and then decreasing in C_0. For high collateral values, these four curves converge to the same asymptote because the loan is almost default free and the volatility does hardly matter. In the absence of asymmetric information the good borrower would take a loan contract with $\left(F^g\left(C_0^*\right), C_0^*\right)$ with $C_0^* = 16.8$ while the bad borrower would prefer a different contract $\left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)$ with more collateral $C_0^{*,b} = 25.0 > 16.8$ than the good borrower. In addition, we see in Figure 9 that — in line with (11) and (12) — the net

Figure 9: Present Value for Good and Bad Borrowers

The diagram shows the present value $PV_0(V_0, C_0, F(C_0))$ of the project for a good and bad borrower who can either take a loan at the terms of a good or bad borrower. The parameter values are $I_0 = 50$, $I = 100$, $r = 0.05$, $T = 1$, $\sigma^b = 0.3$ (bad type), $\sigma^g = 0.25$ (good type), $\alpha_V = 0.5$, $\alpha_C = 0.6$, and $\gamma = 0.1$.



present value $PV_0^b(V_0, C_0, F^g(C_0))$ of a bad borrower being treated as a good one is higher than that of a good borrower $PV_0^g(V_0, C_0, F^g(C_0))$ which is again higher than the value of a bad borrower $PV_0^b\left(V_0, C_0, F^b(C_0)\right)$ which is recognized as a bad borrower. The present value $PV_0^g\left(V_0, C_0, F^b(C_0)\right)$ is lowest if a good borrower is treated like a bad borrower by the lender. In other words, it is advantageous for a borrower to be of a bad type as long as the lender does not recognize it. However, if the lender knows about the type of the borrower, then a good type increases the net present value of the project for the borrower.

The problem for the lender is that the bad borrower has an incentive to pretend to be a good borrower in order to get a loan contract with a lower face value. However, if the bad borrower can choose between two loan contracts $\left(F^g\left(C^{(1)}\right), C^{(1)}\right)$ and $\left(F^b\left(C^{(2)}\right), C^{(2)}\right)$ with different collateral volumes but where the face value of the first contract is priced for a good borrower and the second contract is priced for a bad borrower, the bad borrower does not necessarily prefer contract one. In the example of Figure 9 the net present value $PV_0^b\left(V_0, C_0^{(2)}, F^b\left(C_0^{(2)}\right)\right)$ for a collateral volume $C_0^{(2)} = 19$ is higher than the present value $PV_0^b\left(V_0, C_0^{(1)}, F^g\left(C_0^{(1)}\right)\right)$ for other collateral volumes with $C_0^{(1)} < 0.7$ and $C_0^{(1)} > 35.4$ even though the face value $F^g\left(C_0^{(1)}\right)$ is that of a good borrower. The reason why the borrower prefers the loan contract $\left(F^b(19), 19\right)$, which is in fact priced for this type of borrower, is that $C^{(1)}$ with $C_0^{(1)} < 0.7$ or $C_0^{(1)} > 35.4$ is sufficiently far away from the optimal collateral volume and then the gains from a more favorable face value are less beneficial for the borrower than a favorable collateral volume.

Let

$$\mathbf{C}\left(F^b(C_0), C_0\right) = \left\{x \middle| PV_0^b(V_0, x, F^g(x)) \leq PV_0^b\left(V_0, C_0, F^b(C_0)\right)\right\}$$

be the set of collateral values x such that the bad borrower prefers a given loan contract $\left(F^b(C_0), C_0\right)$, which is fairly priced for the bad borrower, compared to loan contracts $(F^g(x), x)$ with a different collateral volume $x \neq C_0$ but with a face value as if the borrower were of a good type. We note that the set $\mathbf{C}\left(F^b(C_0), C_0\right)$ is always non-empty because $PV_0^b(V_0, x, F^g(x))$ becomes sufficiently small for high collateral volumes x. According to Figure 9, one can easily determine $\mathbf{C}\left(F^b(C_0), C_0\right)$ by drawing a parallel line to the abscissa through the value $PV_0^b\left(V_0, C_0, F^b(C_0)\right)$. Then all collateral volumes x belong to the set $\mathbf{C}\left(F^b(C_0), C_0\right)$ for which $PV_0^b(V_0, x, F^g(x))$ is below this parallel line. In the example of Figure 9, $\mathbf{C}\left(F^b(C_0), C_0\right)$ equals $[0, 0.7) \cap (35.4, \infty)$ for $C_0 = 19$. The critical values 0.7 and 35.4 result from the intersection of the parallel line through $PV_0^b\left(V_0, C_0, F^b(C_0)\right)$ with $PV_0^b(V_0, x, F^g(x))$.

Additionally, even for the remaining collateral values $x \in \left(\mathbb{R}^+/\mathbf{C}\left(F^b(C_0), C_0\right)\right)/\{C_0\}$ there are loan contracts $(F^m(x), x)$ with a medium face value $F^m(x)$ which can be offered to both borrowers by a lender in addition to the contract $\left(F^b(C_0), C_0\right)$. These additional contracts have two properties. First, a bad borrower will always prefer the former contract $\left(F^b(C_0), C_0\right)$. Second, the face value $F^m(x)$ is the lowest possible value such that granting this loan to a good borrower is still worthwhile for the lender, i.e. $F^m(x) \geq F^g(x)$ for all $x \in \left(\mathbb{R}^+/\mathbf{C}\left(F^b(C_0), C_0\right)\right)/\{C_0\}$. For the proposed collateral values $x \in \left(\mathbb{R}^+/\mathbf{C}\left(F^b(C_0), C_0\right)\right)/\{C_0\}$, the required face value $F^m(x)$ can be implicitly obtained from the following equation:

$$PV_0^b(V_0, x, F^m(x)) = PV_0^b\left(V_0, C_0, F^b(C_0)\right) \tag{13}$$

This equation reflects the first property that a bad borrower is not better off by switching from contract $\left(F^b(C_0), C_0\right)$ to the different contract $(F^m(x), x)$. The second property that $F^m(x) \geq F^g(x)$ holds is also valid because for $x \in \left(\mathbb{R}^+/\mathbf{C}\left(F^b(C_0), C_0\right)\right)/\{C_0\}$ a face value equal to $F^g(x)$ or lower would result in a net present value $PV_0^b(V_0, x, F^g(x)) > PV_0^b\left(V_0, C_0, F^b(C_0)\right)$ by definition of $\mathbf{C}\left(F^b(C_0), C_0\right)$. For notational convenience, we define the face value $F^m(x)$ for the further collateral values $x \in \mathbf{C}\left(F^b(C_0), C_0\right)$ as $F^g(x)$ and as $F^b(C_0)$ for $x = C_0$.

This definition of $F^m(x)$ allows the following interpretation. If a bad borrower can choose between a particular loan contract that is correctly priced for his or her type and further contracts $(F^m(x), x)$, the bad borrower has no incentive to decide for a different contract rather than $\left(F^b(C_0), C_0\right)$. Hence, we see that several separating equilibria exist for the example of Figure 9 that reveal the type of the borrower. In the example of Figure 9, there are collateral volumes x in $\mathbf{C}\left(F^b(C_0), C_0\right)$ for $C_0 = 19$ such that a good borrower obtains a higher net present value $PV_0^g(V_0, x, F^g(x))$ with a loan contract $(F^g(x), x)$ than with the contract $\left(F^b(C_0), C_0\right)$. E.g. for the critical collateral value $x = 35.4$, the net present value of a good borrower with the contract $(F^g(35.4), 35.4)$ is higher than with the contract $\left(F^b(19), 19\right)$. More than that even for $x \in \left(\mathbb{R}^+/\mathbf{C}\left(F^b(C_0), C_0\right)\right)/\{C_0\}$, there are contracts $(F^m(x), x)$ which might be more favorable for a good borrower than the contract $\left(F^b(C_0), C_0\right)$. In fact, every combination of such two loan contracts $(F^m(x), x)$ and $\left(F^b(C_0), C_0\right)$ with a collateral volume x not too far above $C_0 = 19$ characterizes a separating equilibrium. If a lender offers these two contracts to every borrower, then the bad borrower will always choose the contract $\left(F^b(C_0), C_0\right)$ while a good borrower prefers the other contract $(F^m(x), x)$. Since the type of the borrower is revealed by the individual choice, the lender can grant favorable conditions $F^m(x) < F^b(x)$ to the good borrower. However, the good borrower pays more than under symmetric information to ensure the separating property of these offers.

Hence, an infinite number of separating equilibria exists. In particular, the two contracts with an unsecured loan $\left(F^b(0), 0\right)$ for a bad borrower and an (over-) collateralized loan $(F^g(120), 120)$ for a good borrower are a separating equilibrium. However, this standard solution is not Pareto efficient. It can be improved if the good borrower chooses a contract $(F^m(x), x)$ which maximizes $PV_0^g(V_0, x, F^m(x))$ as

$$PV_0^g(V_0, x, F^m(x)) > PV_0^g(V_0, 120, F^g(120))$$

holds for some x.

Therefore, in the next step we characterize the Pareto optimal, socially-efficient strategy of the lender. Such a strategy means that a lender offers to every borrower a set of loan contracts from which he or she can select one to get the required payoff I. Since there are two types of borrowers, there can only be two non-redundant contracts at most which are selected by the borrowers. A Pareto optimal strategy of the lender means that no other set of loan contracts exists such that one type of borrower is better off without hurting another borrower. In addition, the loan contracts must be such that it is worthwhile for a lender to grant this loan. This condition means that a lender can — without further considerations — only grant loan contracts $\left(F^b(x), x\right)$ treating borrowers as bad. Only if the lender is sure that a contract is only selected by good rather than bad borrowers, the contract conditions can be more favorable. Every loan contract $(F^m(x), x)$ satisfies that no bad borrower will prefer one of these contracts over $\left(F^b(C_0), C_0\right)$.

To determine the efficient lending strategy it is important to note that a better loan contract for the bad borrower is also beneficial for good borrowers. If the bad borrower is supposed to take contract $\left(F^b(C_0), C_0\right)$, good borrowers can take one of the contracts $(F^m(x), x)$. If $PV_0^b\left(V_0, C_0, F^b(C_0)\right)$ increases due to a change of C_0 from $C_0^{(1)}$ to $C_0^{(2)}$, further elements are added to $\mathbf{C}\left(F^b\left(C_0^{(1)}\right), C_0^{(1)}\right)$ to obtain $\mathbf{C}\left(F^b\left(C_0^{(2)}\right), C_0^{(2)}\right)$, i.e.

$$\begin{aligned} \mathbf{C}\left(F^b\left(C_0^{(1)}\right), C_0^{(1)}\right) &\subset \mathbf{C}\left(F^b\left(C_0^{(2)}\right), C_0^{(2)}\right), \\ \text{for } PV_0^b\left(V_0, C_0^{(1)}, F^b\left(C_0^{(1)}\right)\right) &< PV_0^b\left(V_0, C_0^{(2)}, F^b\left(C_0^{(2)}\right)\right). \end{aligned}$$

Hence, with $\mathbf{C}\left(F^b\left(C_0^{(2)}\right), C_0^{(2)}\right)$ rather than $\mathbf{C}\left(F^b\left(C_0^{(1)}\right), C_0^{(1)}\right)$, a good borrower has more feasible collateral values x for which a loan at the favorable conditions $(F^g(x), x)$ can be taken. In addition for the remaining collateral values $x \in \left(\mathbb{R}^+/\mathbf{C}\left(F^b\left(C_0^{(2)}\right), C_0^{(2)}\right)\right)/\{C_0^{(2)}\}$, a higher value of $PV_0^b\left(V_0, C_0^{(2)}, F^b\left(C_0^{(2)}\right)\right)$ results in a lower face value $F^m(x)$ according to Equation (13). This is the reason why not only bad but also good borrowers benefit if bad borrowers are offered better contracts. As a consequence, the optimal contract for the bad borrower is the first best solution $\left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)$. Given that the optimal contract for a bad borrower is chosen, the face value $F^m(x)$ is never above $F^b(x)$ and it cannot be below $F^g(x)$:

$$F^g(x) \le F^m(x) \le F^b(x)$$

The socially-optimal collateral volume for the good borrower is given by

$$C_0^{*,g} = \arg\max_{C_0 \ge 0} PV_0^g\left(V_0, C_0, F^m(C_0)\right)$$

as this choice maximizes the net present value from the set of feasible contracts. In general, the efficient lending strategy $\left\{\left(F^m\left(C_0^{*,g}\right), C_0^{*,g}\right), \left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)\right\}$ is a separating equilibrium because the optimal collateral volumes $C_0^{*,g}$ for a good and $C_0^{*,b}$ for a bad borrower differ. Then, good borrowers can take a loan at better conditions $F^m\left(C_0^{*,g}\right)$ as if they were treated as bad borrowers. In special cases $C_0^{*,g}$ and $C_0^{*,b}$ coincide such that both borrowers take identical contracts priced for bad borrowers.

In the example of Figure 9 the socially-efficient set of loan contracts is $\left\{\left(F^{b}\left(25.0\right), 25.0\right), \left(F^{g}\left(33.2\right), 33.2\right)\right\}$ where the collateral volume 33.2 for the good borrower is the lowest collateral volume above $C_0^{*,b}$ such that the good borrower can borrow at the face value $F^{g}\left(x\right)$ for a good borrower.

The efficient lending strategy $\left\{\left(F^{m}\left(C_0^{*,g}\right), C_0^{*,g}\right), \left(F^{b}\left(C_0^{*,b}\right), C_0^{*,b}\right)\right\}$ has three important properties. First, we underline again that a bad borrower obtains his or her first best solution as in the case of symmetric information.[§] Second, the bad borrower obtains a higher net present value, i.e.

$$PV_0^g\left(V_0, C_0^{*,g}, F^m\left(C_0^{*,g}\right)\right) < PV_0^b\left(V_0, C_0^{*,b}, F^b\left(C_0^{*,b}\right)\right).$$

This is a result of the construction of the terms for the good borrower $\left(F^m\left(x\right), x\right)$. The net present value $PV_0^g\left(V_0, C_0^{*,g}, F^m\left(C_0^{*,g}\right)\right)$ a good borrower obtains cannot be higher than $PV_0^b\left(V_0, C_0^{*,b}, F^b\left(C_0^{*,b}\right)\right)$ because of the following inequalities:

$$PV_0^g\left(V_0, C_0^{*,g}, F^m\left(C_0^{*,g}\right)\right) < PV_0^b\left(V_0, C_0^{*,g}, F^m\left(C_0^{*,g}\right)\right) \leq PV_0^b\left(V_0, C_0^{*,b}, F^b\left(C_0^{*,b}\right)\right)$$

The first inequality is a results from Relation (10) that a bad borrower is better off than a good borrower for a given loan contract. The second inequality follows from the definition of $F^m\left(x\right)$ in (13).

This outcome is remarkable if we compare it to the case without asymmetric information. As indicated in (12) $PV_0^g\left(V_0, C_0, F^g\left(C_0\right)\right)$ is higher than $PV_0^b\left(V_0, C_0, F^b\left(C_0\right)\right)$ for every collateral volume C_0 and therefore a good borrower obtains a higher net present value from the project. However, this advantage of a good borrower is lost due to asymmetric information. Third, the collateral volume $C_0^{*,g}$, that the good borrower pledges in equilibrium is generally higher than the size of collateral $C_0^{*,b}$ from the bad borrower:

$$C_0^{*,g} \geq C_0^{*,b}$$

This property always holds under the reasonable condition that the difference between the four curves as in Figure 9 shrinks with C_0. This property is intuitive because the more collateral is pledged the less default risky are the loans and the positions of the borrower are more similar.

We note that if another contract is chosen for the bad borrower with a lower collateral volume $C_0 < C_0^{*,b}$, then the required collateral volume $C_0^{*,g}$ for the good borrower increases as we can see in Figure 9. This is notable because we would expect that

[§]This finding is consistent with a result by Boot/Thakor/Udell (1991). They show that if the manager's effort causes costs, then the introduction of asymmetric information does not affect the contract of the bad firm.

an equilibrium has a certain screening function if the collateral volumes for the good and bad borrower sufficiently differ. However, a lower collateral volume for the bad borrower requires a higher collateral volume for the good borrower.

The presence of asymmetric information causes costs. Since the bad borrower can still take a loan at the same conditions as under symmetric information, these costs are completely carried by the good borrower. Intuitively, we think of these costs as a restriction for good borrowers. With asymmetric information they can take a loan from the set $(F^m(x), x)$ which is less favorable than the set $(F^g(x), x)$ of loans out of which a contract without asymmetric information is chosen. In the example of Figure 9, a good borrower loses

$$PV_0^g(V_0, C_0^*, F^g(C_0^*)) - PV_0^g(V_0, C_0^{*,g}, F^m(C_0^{*,g})) = 1.0$$

from the existence of asymmetric information.

However, the good borrower is still better off compared to the case that the lender follows the efficient strategy and does not consider every borrower as a bad borrower. The gain from the separating strategy relative to being treated as a bad borrower is

$$PV_0^g(V_0, C_0^{*,g}, F^m(C_0^{*,g})) - PV_0^g\left(V_0, C_0^*, F^b(C_0^*)\right) = 0.1.$$

These findings provide us with the next result:

Result 6 (Asymmetric Information about σ) *In the presence of asymmetric information about σ, the Pareto efficient lending strategy is $\left\{(F^m(C_0^{*,g}), C_0^{*,g}), \left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)\right\}$ which is a separating strategy if $C_0^{*,g} \neq C_0^{*,b}$ holds. This strategy has the consequence that a bad borrower obtains his or her first best loan contract, the net present value from the project is higher for the bad borrower, and under reasonable conditions the good borrower pledges more collateral than the bad borrower.*

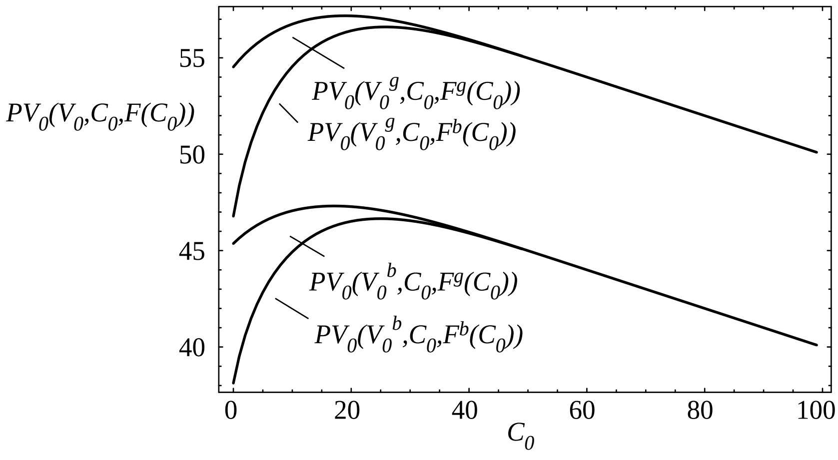
Therefore, we typically find that while in the absence of asymmetric information, good borrowers pledge less collateral than bad borrowers and obtain a higher net present value of the project, it is the other way round in the presence of asymmetric information, i.e. bad borrowers use less collateral and obtain a higher net present value of the project value.

c) Contribution I_0 to the Project Value by the Borrower

A further characteristic of the borrower, that can be subject to asymmetric information, is the contribution I_0 to the project value. Again, we assume a good borrower

Figure 10: Present Value for Good and Bad Borrowers

The diagram shows the present value $PV_0(V_0, C_0, F(C_0))$ of the project for a good and bad borrower who can either take a loan at the terms of a good or bad borrower. The parameter values are $I_0 = 50$ (bad type), $I_0 = 60$ (good type), $I = 100$, $r = 0.05$, $T = 1$, $\sigma = 0.3$, $\alpha_V = 0.5$, $\alpha_C = 0.6$, and $\gamma = 0.1$.



with a I_0^g above the contribution I_0^b of a bad borrower. Together with the financing volume I, the total project value of the good borrower is $V_0^g = I_0^g + I$ and $V_0^b = I_0^b + I$ for the bad borrower. Clearly, the higher V_0 the better secured is a loan contract and therefore the required face value is lower. Hence, the relations

$$\begin{aligned} F^b(C_0) &\geq F^g(C_0), \\ PV_0^i(V_0, C_0, F^g(C_0)) &\geq PV_0^i\left(V_0, C_0, F^b(C_0)\right), \text{ for } i = g, b, \end{aligned}$$

hold again. A fundamental difference to the case with asymmetric information about the volatility σ presented under b) is that not only the lender but also the borrower benefits from a good type and therefore

$$PV_0^g(V_0, C_0, F) > PV_0^b(V_0, C_0, F)$$

is valid for every arbitrary loan contract (F, C_0).

Figure 10 shows the values of the net present value from the project for a good and bad borrower conditional to being treated as a good or bad borrower. As in Figure 9, these values first increase with C_0 and then decline. For high collateral volumes C_0, the net present value of a particular borrower converges to a type-specific decreasing asymptote independent on the fact whether the borrower is treated as good or bad. Thus, for very high values of C_0, the opinion of the lender about the borrower hardly matters. A difference compared to Figure 9 is that for the borrower itself it makes a big difference between being good or bad. A good borrower always obtains a higher net present value from the project and the asymptote for high values of C_0 is considerably higher than for the bad borrower.

In the absence of asymmetric information the good borrower would decide for a loan contract $(F^g(C_0^*), C_0^*)$ with $C_0^* = 18.9$ while the bad borrower would take contract $\left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)$ with more collateral $C_0^{*,b} = 25.0 > 18.9$ than the good borrower.

In the next step, we want to analyze the socially-efficient lending strategy. A socially-efficient lending strategy comprises of two loan contracts between which each borrower can decide. A socially-efficient, Pareto optimal combination of contracts is — as presented under b) — such that borrowers obtain the highest possible net present values from the project while the lender can be sure not to grant a loan with a present value below the payoff I. Clearly, it would be helpful to have a separating strategy that detects the type of the borrower as otherwise both borrowers can only take loan contracts $\left(F^b(C_0), C_0\right)$ being treated as bad borrowers.

To obtain the efficient lending strategy, it is useful to first regard the set $\mathbf{C}\left(F^b(C_0), C_0\right)$ of collateral volumes for which a bad borrower prefers the loan contract $\left(F^b(C_0), C_0\right)$ even though the other contract treats the borrower as good:

$$\mathbf{C}\left(F^b(C_0), C_0\right) = \left\{x \middle| PV_0\left(V_0^b, x, F^g(x)\right) \leq PV_0\left(V_0^b, C_0, F^b(C_0)\right)\right\}$$

For the remaining sizes of collateral $\left(\mathbb{R}^+/\mathbf{C}\left(F^b(C_0), C_0\right)\right)/\{C_0\}$, the bad borrower is better off by pretending to be good and paying the lower face value $F^g(x)$. Nevertheless, there are lower face values F for those collateral volumes x such that there is no incentive for the bad borrower to deviate from the contract $\left(F^b(C_0), C_0\right)$. The lowest possible face value $F^m(x)$ for a collateral volume $x \in \left(\mathbb{R}^+/\mathbf{C}\left(F^b(C_0), C_0\right)\right)/\{C_0\}$, such that a bad borrower has no incentive to deviate from $\left(F^b(C_0), C_0\right)$, is implicitly defined by

$$PV_0\left(V_0^b, x, F^m(x)\right) = PV_0\left(V_0^b, C_0, F^b(C_0)\right).$$

In line with the findings under b), we can argue that the efficient lending strategy considers to attract bad borrowers by offering them their first best solution $\left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)$. This is because a higher value of $PV_0\left(V_0^b, C_0, F^b(C_0)\right)$ increases the set $\mathbf{C}\left(F^b(C_0), C_0\right)$ of possible loan contracts for the good borrower and the loan contracts for the remaining collateral volumes $x \in \left(\mathbb{R}^+/\mathbf{C}\left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)\right)/\left\{C_0^{*,b}\right\}$ have a lower face value $F^m(x)$. For notational convenience, we define again $F^m(x) := F^g(x)$ for all $x \in \mathbf{C}\left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)$ and

$F^m(x) := F^b(x)$ for $x = C_0^{*,b}$. According to this definition, $F^m(x)$ can be understood as the lowest possible face value for a collateral volume x that a good borrower can get such that the bad borrower has no incentive to deviate from the loan contract $\left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)$.

Hence, the optimal contract for the good borrower such that a bad borrower still chooses $\left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)$ results from the following optimization problem:

$$C_0^{*,g} = \arg\max_{C_0 \geq 0} PV_0\left(V_0^g, C_0, F^m\left(C_0\right)\right)$$

As long as $C_0^{*,g} \neq C_0^{*,b}$ holds, the two loan contracts $\left\{\left(F^m\left(C_0^{*,g}\right), C_0^{*,g}\right), \left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)\right\}$ form a separating equilibrium. Otherwise, only the contract $\left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)$ is offered. For example, if the net present value of the project sharply declines with C_0, then the contract for the good and bad borrower can coincide as it is optimal not to pledge any collateral. Nevertheless, if a good borrower who is restricted to take loans at the bad face value $F^b(C_0)$ optimally decides for a different collateral volume than a bad borrower, a separating equilibrium with two different contracts exists. Clearly, the contract for the good borrower in the separating equilibrium is better than to take a loan at $F^b(x)$ because he or she obtains the lower face value $F^m(x)$.

The efficient lending strategy $\left\{\left(F^m\left(C_0^{*,g}\right), C_0^{*,g}\right), \left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)\right\}$ has the following properties. First, bad borrowers obtain their first best solution. Second, the net present value of the good borrower is higher than that of the bad borrower. This is obvious because good borrowers have a higher project value V_0^g and can borrow at a non-higher face value $F^m(C_0)$ relative to the bad borrowers. Third, we usually observe that the optimal collateral volume $C_0^{*,g}$ of a good borrower under the socially-efficient strategy is higher than that of a bad borrower $C_0^{*,b}$. In the example of Figure 10, the separating equilibrium is $\left\{\left(F^b(25.0), 25.0\right), \left(F^g(31.8), 31.8\right)\right\}$. The intuition why $C_0^{*,g} = 31.8$ is higher than $C_0^* = 25.0$ is that for higher collateral values the face value $F^m(C_0)$ approaches $F^g(C_0)$. This effect can be seen in Figure 10 because the distance between the present value functions $PV_0\left(V_0^b, C_0, F^g(C_0)\right)$ and $PV_0\left(V_0^b, C_0, F^b(C_0)\right)$ of a bad borrower declines with the collateral value C_0.

Like under asymmetric information about the volatility of the project value, good borrowers have to carry the costs for a separating equilibrium. In contrast to the case with asymmetric information about the volatility of the project value, the costs are not so high that the present value of the project for a bad borrower is higher than that for a good borrower in equilibrium.

Summing up, we obtain the following result:

Result 7 (Asymmetric Information about I_0) *In the presence of asymmetric information about I_0, the Pareto efficient lending strategy is $\left\{\left(F^g\left(C_0^{*,g}\right), C_0^{*,g}\right), \left(F^b\left(C_0^{*,b}\right), C_0^{*,b}\right)\right\}$ which is a separating strategy if $C_0^{*,g} \neq C_0^{*,b}$ holds. This strategy has the consequence that a bad borrower obtains his or her first best loan contract, the net present value from the project is higher for the good borrower, and we always observe that the good borrower pledges more collateral than the bad borrower.*

5 Conclusion

The literature about collateralized loan contracts is characterized by a gap between theory and empirical observations. The standard models for the use of collateral, primarily those regarding asymmetric information, show that good borrowers which have less risky projects pledge more collateral than bad borrowers. Empirical studies, however, observe the opposite relation namely that rather bad borrowers pledge more collateral. Even though this behavior is consistent with 'conventional wisdom' as Berger/Udell (1990) state, a clear economic foundation for this observation is missing. For this purpose, we consider the simple motive to use collateral as a device to reduce bankruptcy costs in the case of default. If debt is secured by outside collateral, the borrower has a higher incentive to satisfy the debt obligation and therefore the probability of default reduces. Clearly, bringing in collateral reduces interest rate payments and the present value of costs from liquidating the assets of the firm but it also creates additional cost for pledging collateral. Therefore, the optimal use of collateral in order to reduce liquidation and collateral costs is a complex tradeoff.

We consider a typical framework similar to Merton (1974) but with bankruptcy costs. Moreover, the borrower has the ability to bring in additional collateral. Since lenders are under perfect competition, a higher collateral volume reduces interest rate payments for a given financing volume. Although pledging collateral and liquidating the collateralized assets in the case of default is costly, collateral acts as a powerful device to reduce bankruptcy costs and to increase the wealth of the borrower. In general, we see that a bad borrower, who has higher bankruptcy costs, riskier projects, and contributes less to the project, pledges more collateral than a good borrower.

Since asymmetric information is a prominent reason to justify to use of collateral, we examine whether these relations are robust even under asymmetric information.

In the presence of asymmetric information a good borrower cannot be distinguished from a bad borrower if asymmetric information refers to the bankruptcy costs. Otherwise, if information asymmetry refers to the riskiness of the project or the contribution of the borrower to the project, lenders can generally screen between good and bad borrowers. For the Pareto optimal equilibrium with information asymmetries, the following properties hold. A bad borrower always obtains his first best solution as with symmetric information but the contract for a good borrower typically changes. Due to asymmetric information, good borrowers tend to pledge more collateral than bad borrowers. Therefore, the relationship between the type of a borrower and the size of collateral inverts through the introduction of asymmetric information. A further remarkable finding is that under perfect information good borrowers obtain a higher present value from the project than bad borrowers but bad borrowers are better off than good borrowers under information asymmetries concerning the riskiness of the project.

As a result we can conclude that the observed pledging of collateral is consistent with the motive of borrowers to reduce bankruptcy costs. However, these relations are no longer valid if asymmetric information between borrowers and lenders is in effect.

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