

## **Valuation Methods of a Life Insurance Company**

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# 1 History

Traditionally life assurance companies have reported financial results to shareholders on the basis of the statutory requirements of the insurance companies' legislation. So the most common measure of a life insurance company's financial year was the statutory earnings from operation. This has been a convenient measure since it also represents the amount of money which can be paid to policyholder or paid in the form of dividends. The major disadvantage to relying upon statutory earnings as a measure of how well a company is doing, is that statutory accounting tends to be designed to protect against insolvency and, therefore, by its very nature, suffers from over conservatism. Statutory earnings do not measure "how well" a company is doing on a going concern basis. For example, capital invested in acquiring business (acquisition expenses and valuation strain) is immediately written off. Successful acquisition of profitable new business results in an immediate "loss" followed by a subsequent enhanced series of profits. Although suitable for solvency testing, the statutory approach, by charging the "capital" cost of new business to revenue and ignoring the future surplus stream attributable to new business, fails to display in any accounting period a meaningful account of the trading activity of that period. For most products, a slowdown in sales will result in an immediate increase in statutory earnings and generally, most would not regard a slowdown in sales as being a sign of a healthy company! So, it is clear that statutory earnings are the wrong method to measure the health of the company.

Largely as a result of the inadequacies of statutory accounting, US insurers were required by the Securities Exchange Commission in the early 1970's to begin to report earnings to shareholders on a Generally Accepted Accounting Principles (GAAP) basis. The major advantage to GAAP accounting is that it does attempt to produce earnings that reflect how well or how badly the insurance company had performed in a form, which is useful to management. With GAAP, generally an increase in sales will not depress GAAP earnings to the same degree, as it would statutory earnings. Unfortunately, because 100% of acquisition costs are not deferred, increased sales will still depress GAAP earnings to some extent. Additionally, margins for conservatism are normally introduced into the assumptions, and GAAP might suffer from the lock-in principle. Once assumptions are set for a particular generation or branch of business, the assumptions cannot be changed unless future losses are likely. Another major disadvantage to GAAP is that GAAP earnings may vary significantly between two identical companies depending on the objectiveness of management in establishing assumptions. Therefore, overall, GAAP is not a good prognosticator for how well a company is doing.

During the periods of fluctuation in interest rates, which occurred in the US during the mid-1970's and early 1980's, some US companies began to look at cash flows as a measure of "how well" their companies were doing. The real advantage to using cash flows as a measurement tool is that it is the only basis that looks at "real" money. But it is really only an immediate solvency test. Cash flow is not a measure of "how well" a company is doing.

In the 80's, there were new products (for example unit linked products), the increase in competition, mergers and restructuring, decrease in profit margins due to increased competition etc. The result has been a greater need to be able to manage and control insurance operations in the face of increasing fluctuations and uncertainty.

It is interesting to note that solvency and profitability cannot be showed by the same method. Solvency is a constraint, which determines the security margins and methods to use. Profitability tries to show how well a company is doing in eliminating the cost effect of the first year and taking the future profits into account.

In short, we can say that four points have contributed to the adoption of valuation methods

- Hard competition between insurers
- Investors' pressure to have comprehensive results
- Products' evolution towards greater flexibility
- Deregulation, financial control of solvency and no more tariff approval such as standard mortality tables

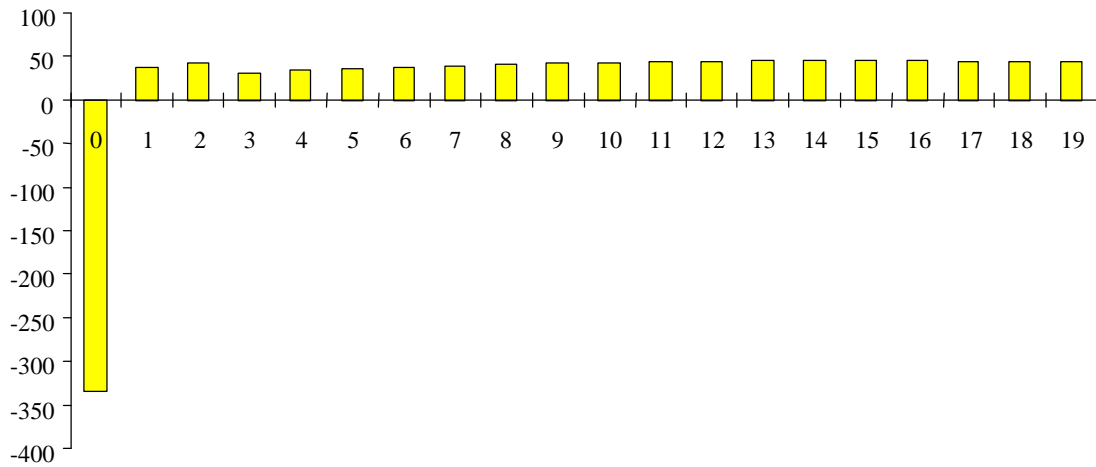
## 2 Product assessment : Profit Testing

We know that the acquisition of profitable new business results in an immediate loss due to the acquisition expenses and the first payment to technical reserve. Afterwards, we hope that a series of profits will follow. The Profit Testing takes future profits into account and then decreases the effect of the investment of the first year. To calculate or to estimate these future profits, we use expected values. In fact these future profits come from the profit and loss account. Here is an example of an insurance account. We have on the left the charges and on the right the income.

Profit and loss account	
Death claims paid Amounts paid on maturity Surrender values paid Expenses incurred and commissions paid Bonus paid Increase in reserves Taxes <b>Profit during the year</b>	Release of reserves Premiums received Interest and gains received

Typically, we have the following curve for the Profit Testing, which represents the expected profits of the profit and loss account each year.

### Profit Testing



It is very common to have a loss followed by a series of profits.

But where does it come from? Is it the Profit Testing related with the actuarial rules? Yes, it is and we will see this in the next chapter.

## 2.1 The Profit Testing in 3 steps

We will see the relationship between the Equivalence Principle and the Profit Testing in 3 steps. Starting from the Equivalence Principle, we obtain the Traditional Margin and then after some modifications we obtain the Profit Testing.

### 2.1.1 Equivalence Principle

Equivalence Principle has the following definition:

Actuarial Present Value (APV) of Premiums equals to APV of Benefits plus APV of Charges.

Using a traditional endowment for a male aged  $x$  of duration  $n$  years, we have the following application

$$P' \cdot \ddot{a}_{xn} = A_{xn} + \mathbf{a} + \mathbf{g} \cdot \ddot{a}_{xn}$$

Where

$\ddot{a}_{xn}$  = the net single premium for a  $n$ -year temporary life annuity-due which provides for annual payments of 1 unit as long as the beneficiary lives.

$A_{xn}$  = the net single premium for an endowment which provides for a payment of 1 at the end of the year of death if it occurs within the  $n$  first years otherwise at the end of the  $n$ th year.

$a$  = acquisition expenses

$g$  = administration expenses

Then, we have in our example that the total premium equals to the endowment benefit plus the acquisition expenses (only the first year) plus the total administration expenses. We suppose that the collection expenses are integrated in the administration expenses.

### 2.1.2 Traditional Margin

Using the best estimate instead of prudent assumptions, we can rewrite the formula as follows (we symbolise the best estimate basis with the star).

$$P'' \cdot \ddot{a}_{\overline{xn}|}^* - TM = A_{\overline{xn}|}^* + a^* + g^* \cdot \ddot{a}_{\overline{xn}|}^* \\ + \sum_{k=0}^{n-1} v^{*(k+1)} \cdot L_{x+k} \cdot u_{x+k}^* \cdot {}_k p_x^* + \sum_{k=0}^{n-1} v^{*(k)} \cdot B_k \cdot {}_k p_x^*$$

Where

$TM$  = Traditional Margin

$u_{x+k}^*$  = probability to lapse

$L_{x+k}$  = Lapse benefit

$B_k$  = Bonus

The two last expressions are the surrender benefit and the bonus.

This is traditionally how the profit is calculated. The Profit is not recognised yearly but at the beginning of the contract for the whole duration.

### 2.1.3 Profit Testing

Inserting reserves, rewriting the formula with sums on the time  $k$  and regrouping, we obtain

$$\sum_{k=0}^{n-1} \left\{ \begin{array}{l} P'' \cdot {}_k p_x^* - g^* \cdot {}_k p_x^* - v^* \cdot q_{x+k}^* \cdot {}_k p_x^* \\ - v^* \cdot L_{x+k} \cdot u_{x+k}^* \cdot {}_k p_x^* - B_k \cdot {}_k p_x^* \\ - (v^* \cdot {}_{k+1} V \cdot {}_{k+1} p_x^* - {}_k V \cdot {}_k p_x^*) + I_k - G_k \end{array} \right\} \cdot v^{*(k)} \\ - v^{*(n)} \cdot {}_n p_x^* - a^* = 0$$

Where

$G_k$  = Profit

${}_k V$  = Reserves

$I_k$  = interest earned

In the sum, we can see the profit from the accounting  $G_k$ . The insurance profits are discounted with the factor  $v^*$ .

The element in brackets at the end of the sum is the difference between the reserve at the end of the year and the other at the beginning of the year.

In fact each term of the sum is a line of the Profit and Loss account. So, the Profit Testing uses the traditional elements of the actuarial science. Of course, any new element can be inserted like, for example, a reinsurance premium.

We can see that the Profit Testing can be used as profitability tool or as a pricing tool.

## **2.2 Common criteria to evaluate the profitability**

### **2.2.1 Net Present Value**

The Net Present Value with Risk Discount Rate is defined as follows:

$$\sum_{k=0}^{n-1} G_k \cdot v_{RDR}^k = NPV_{RDR}$$

If the Net Present Value is positive, the product is profitable. Profitability occurs when the product generates at least  $RDR$  on the first year strain.

#### **Risk Discount Rate (RDR)**

The Risk Discount Rate is defined as the rate of return linked to business risk of the insurance company.

In reality, this rate is very subjective. It depends on the management, the shareholders and the investors.

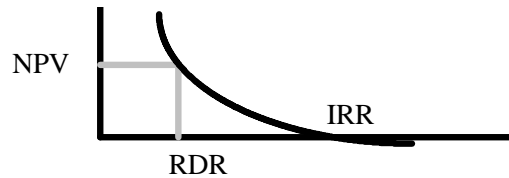
### **2.2.2 Internal Rate of Return**

The Internal Rate of Return is defined as the rate, which gives a Net Present Value of zero.

$$\sum_{k=0}^{n-1} G_k \cdot v_{IRR}^k = 0$$

If the Internal Rate of Return is higher than the Risk Discount Rate, then the product is profitable.

### **2.2.3 Relation between IRR and RDR**



$$NPV_{RDR} \geq 0 \Leftrightarrow i_{IRR} \geq i_{RDR}$$

### 3 Company assessment : Methods

We will study five methods, which evaluate an insurance company. In this chapter, all the numerical examples are based on the data given in the annexe "Numerical application 1".

#### 3.1 Embedded Value

Embedded Value represents, at the valuation date, an estimate of the asset value and the stock of the insurance company.

More precisely, the Embedded Value of a life office, at a particular valuation date, is taken to be the sum of the shareholders' net assets and the value of the business in-force at the valuation date. The value of in-force business at the valuation date is the present value of future profits expected to emerge from policies already written.

$$\begin{aligned} \text{Embedded Value} &= \text{Shareholders' Net Assets} \\ &+ \\ &\text{The Stock: the value of the business in-force at the valuation date} \end{aligned}$$

Where

$$\text{Net Assets} = \underbrace{\text{share capital} + \text{solvency capital} + \text{profit} + \text{hidden reserves}}_{\text{surplus}}$$

Let us see now this formula more in detail. If we denote

$t$  = valuation year

$EV_t$  = EV at the end of the year  $t$

$NA_t$  = Net Assets at the end of the year  $t$

$S_t$  = Stock evaluates at time  $t$

$k$  = year of profit



$$H_k^{(t)} = \begin{cases} \text{for } k > t, \text{ statutory profit expected for the year } k, \text{ evaluated in } t, \text{ for the whole in} \\ \text{force business at the time } t \\ \text{for } k = t, \text{ statutory profit realised for the year } k \end{cases}$$

$v^{(z-t)}_{RDR}$  = discount factor with the Risk Discount Rate

We have the following formula for Embedded Value

$$EV_t = NA_t + \underbrace{\sum_{z \geq t+1} H_z^{(t)} \cdot v_{RDR}^{z-t}}_{stock_t}$$

We know that  $H_z^{(t)}$  represents the future profit expected to be generated in respect of presently in force business and to be transferable after allowing for all relevant taxes to the profit and loss account. In the sum we use (t+1) to take only the future profits into account. But we can write it more explicitly.

$s$  = new business year  
 $t$  = valuation year  
 $k$  = statutory profit year

$$G(s, t, k) = \begin{cases} \text{for } s \leq t < k, & \text{statutory profit expected for the year } k, \text{ for the business written} \\ & \text{the year } s, \text{ evaluated in } t \\ \text{for } s < t \text{ et } k = t, & \text{statutory profit realised the year } t \text{ for the business written the year } s \\ \text{for } s = t = k, & \text{statutory profit realised the year } t \text{ for the new business} \end{cases}$$

Summing the  $G(s, t, k)$  on the whole years  $s$ , we obtain the statutory profit of the profit and loss account:

$$H_k^{(t)} = \sum_{s \leq t} G(s, t, k) \quad \text{for } k \geq t$$

If we introduce a tax rate that is paid when the statutory profits are positive

$$i_t = \text{tax rate}$$

and

$$(H_k^{(t)})^+ = \begin{cases} H_k^{(t)} & \text{if } H_k^{(t)} > 0 \\ 0 & \text{otherwise} \end{cases}$$

We get the following formula for Embedded Value

$$EV_t = FP_t + \sum_{k>t} [H_k^{(t)} - (H_k^{(t)})^+ \cdot i_T] \cdot v_{RDR}^{k-t}$$

where

$$H_k^{(t)} - (H_k^{(t)})^+ \cdot i_T$$

is called distributable profit of the year  $k$ . This is the statutory profit after taxes.

### 3.1.1 Numerical example

Let us take the following example in order to illustrate the Embedded Value.

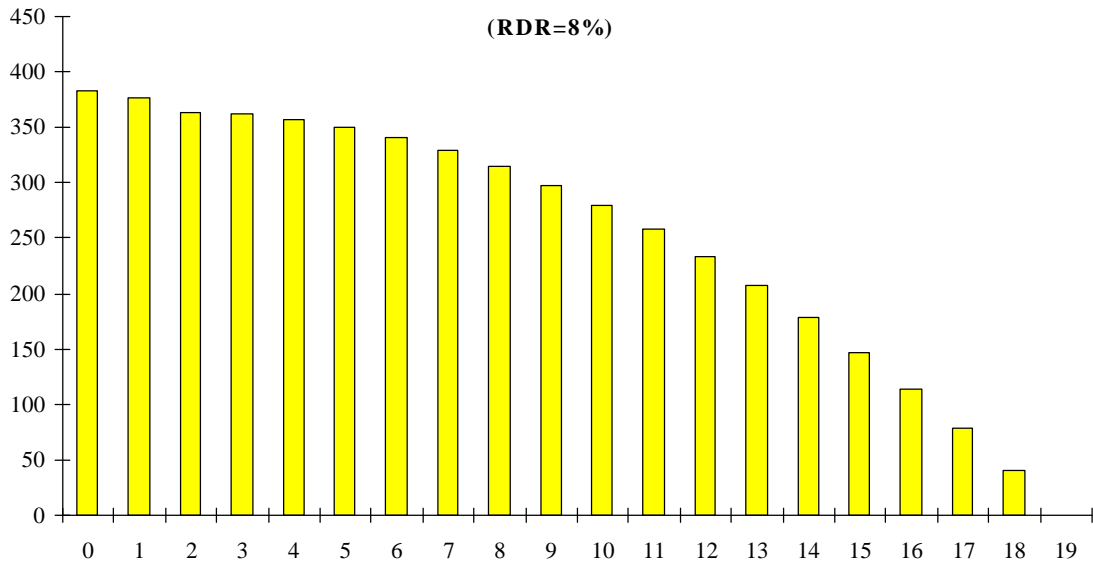
#### Endowment insurance

$x = 30$	$a = 4\%$ on Capital	SM 1978/83
$n = 20$	$b = 1.5\%$ on Premium	$i = 4\%$
$C = 100'000$	$g = 0.3\%$ on Capital	$P'' = 3970$
$a^* = 3.8\%$	$b^* = 1\%$	$g^* = 0.2\%$
$i^* = 6\%$	$i_{RDR} = 8\%$	Tax rate = 10%
Interest on net assets = 1.5%		

The interest on net assets is the after tax investment return earned by the assets that support shareholders' net assets. It is not unusual for this rate of return to be substantially below the return earned on assets supporting insurance liabilities because of assets allocation techniques used by companies. It is common to assign non-interest-bearing assets or lower-yielding assets to support shareholders' net assets and to allow the higher-yielding assets to back policyholder liabilities.

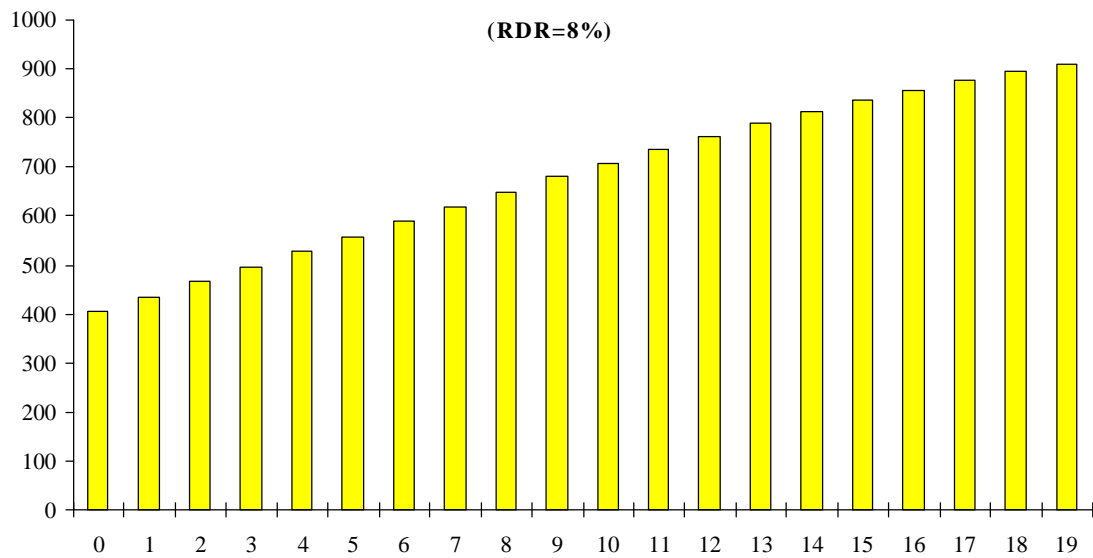
Before showing the charts, we have to notice that we will present a projection of the valuation methods. In reality these methods are calculated each year with the data of that year and used to compare it with figures of previous years. In our example, we are interested in the development of the results through time and then we suppose that we do not change the assumptions, that the assumptions match with the reality and that there is no new business. With these assumptions, the real profit equals the expected profit.

Stock profile



We notice that the profile is decreasing. This is because the annual profits are nearly constant and then each year the stock has one profit less (because there is no new business).

#### Embedded Value profile



Each year the Embedded Value increases because of the interest earned on net assets and the annual profit.

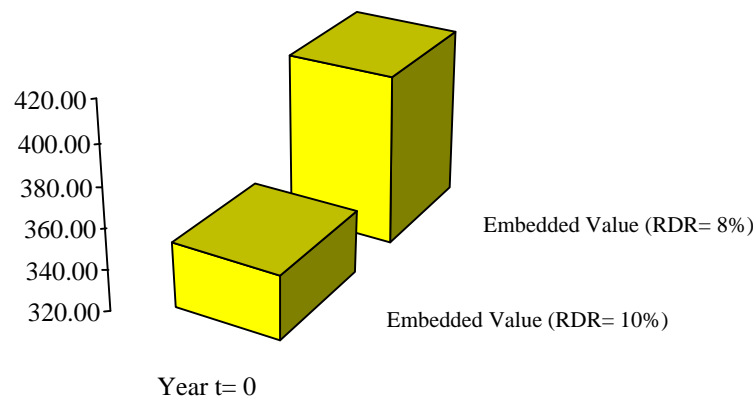
### 3.1.2 Embedded Value advantages

- Embedded Value takes the future profits of the Profit Testing into account, and then the value is not only influenced by the loss of the first year.
- Embedded Value allows comparing financial results of different kinds of activities.
- Visual representation, which seems to be easy to understand.

### 3.1.3 Embedded Value disadvantages

- Without publishing the assumptions, the result of the method has no real sense.
- Risk Discount Rate is not a general concept and furthermore Embedded Value is especially sensitive to Risk Discount Rate. In traditional book keeping, we never use this rate. This rate is only used in Profit Testing and Embedded Value.

Embedded Value's sensitivity to RDR



Embedded Value with RDR of 8% equals 403.6  
Embedded Value with RDR of 10% equals 351.7

## 3.2 Appraisal Value

Appraisal Value is the extension of the Embedded Value to the market value. So to obtain this value we have to consider the goodwill. But the goodwill is a subjective value, which represents the value of the clients, the future new business etc. So this value is very difficult to evaluate.

$$\text{Appraisal Value} = \text{Embedded Value} + \text{Goodwill}$$

## 3.3 Value Added

Value Added is a dynamic value. It is the increase in the Embedded Value between two time periods.

$$VA_t = EV_t - EV_{t-1}$$

We saw that the Embedded Value is the value of the company, the Appraisal Value is the market value of the company and now we see the Value Added which is a dynamic measure considered to show how well the company is doing during a specified period. We will use now only Embedded Value because Appraisal Value is more complicated to value because of the goodwill. But of course this method and the following can also be calculated with Appraisal Value.

To illustrate the Value Added we can take the following example. Suppose that a company has 100 of equity and invest everything in new business, we could have for example:

	Beginning of year	End of year
Net assets	100	0
Business in force	0	115

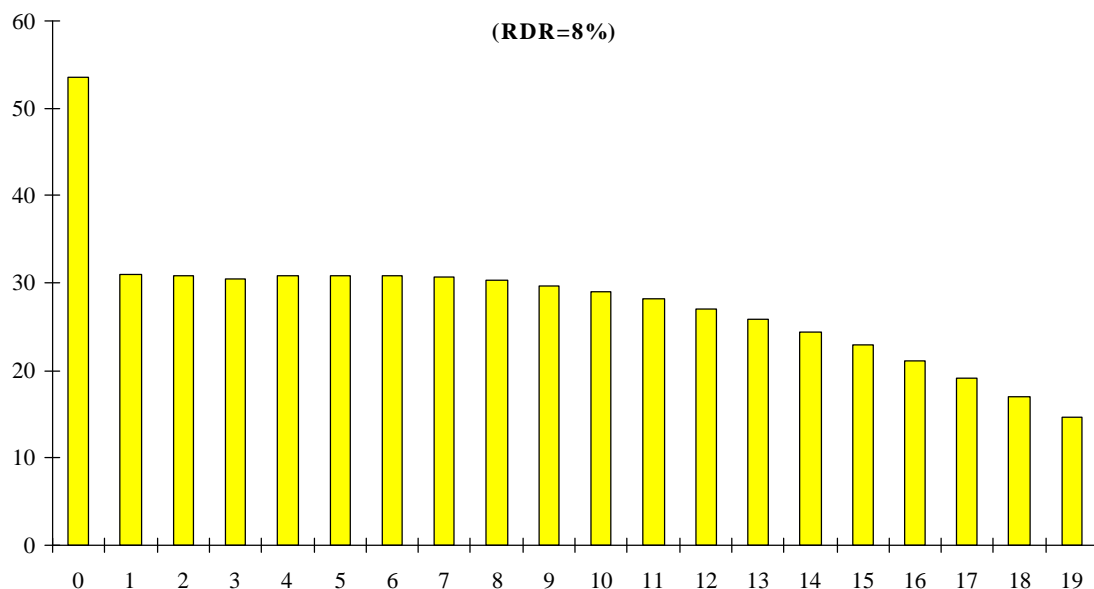
$$\text{Value Added} = 115 - 100 = 15$$

Profit and loss account could have shown a loss of 100.

Through this example, we notice that the Value Added seems to be more able to show the profitability than the statutory profits.

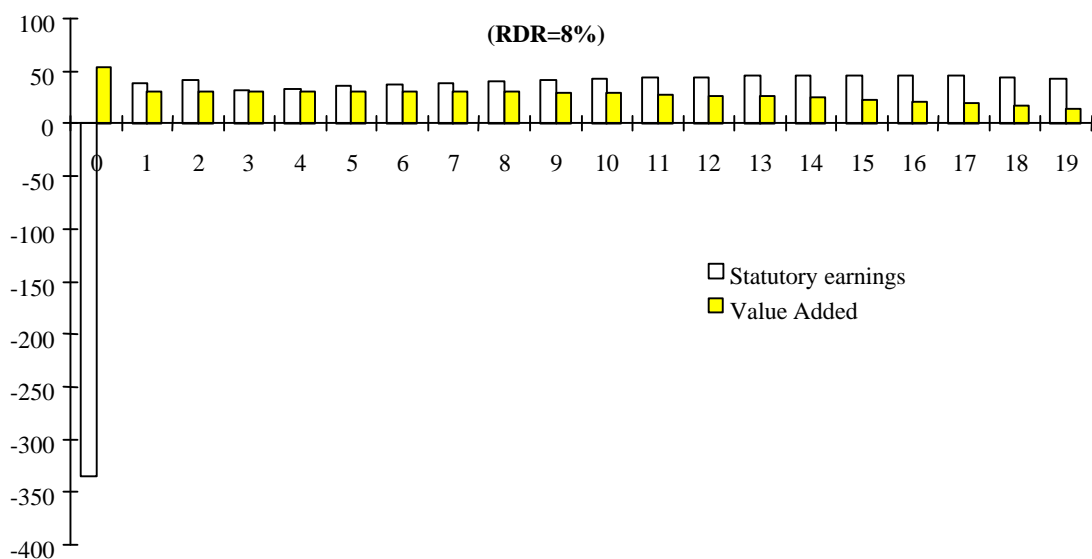
### 3.3.1 Numerical example

Value Added profile



First year, the Value Added is especially high because the company sells a profitable product at this time. Afterwards, there is no new business and the Value Added decreases.

Comparison between Value Added and statutory earnings:



This graph is very interesting because we can see the difference between Value Added and statutory earnings. In this example, the two methods behave completely differently. The statutory earnings show first a loss followed by a series of gains and the Value Added shows first a gain followed by a series of smaller gains. The Value Added seems much more able to show profitability than the statutory earnings.

### 3.3.2 Value Added components

Starting with the definition of the Value Added, we will split the formula in five components

$$VA_t = EV_t - EV_{t-1}$$

$$VA_t = FP_t - FP_{t-1} + S_t - S_{t-1}$$

we note

$i_{FP}$  = interest on net assets

$i_{RDR}$  = Risk Discount Rate

$i_T$  = tax rate

$i_D$  = dividend rate

and we rewrite the stock at time  $t-1$  on another way in order to prepare the calculation

$$VA_t = \underbrace{FP_{t-1} + i_{FP} \cdot FP_{t-1} - i_{FP} \cdot (FP_{t-1})^+ \cdot i_T + H_t^{(t)} - (H_t^{(t)})^+ \cdot i_T - (H_t^{(t)})^+ \cdot (1 - i_T) \cdot i_D}_{FP_t} - FP_{t-1} + S_t - \underbrace{\left( (1 + i_{RDR}) \cdot S_{t-1} - i_{RDR} \cdot S_{t-1} \right)}_{S_{t-1}}$$

we use the formula of the stock and we obtain

$$VA_t = i_{FP} \cdot FP_{t-1} - i_{FP} \cdot (FP_{t-1})^+ \cdot i_T + i_{RDR} \cdot S_{t-1} - (H_t^{(t)})^+ \cdot (1 - i_T) \cdot i_D + H_t^{(t)} - (H_t^{(t)})^+ \cdot i_T \\ + \sum_{k>t} \left( H_k^{(t)} - (H_k^{(t)})^+ \cdot i_T \right) \cdot v_{RDR}^{k-t} - \sum_{k>t-1} \left( H_k^{(t-1)} - (H_k^{(t-1)})^+ \cdot i_T \right) \cdot v_{RDR}^{k-t}$$

and regrouping

$$VA_t = i_{FP} \cdot FP_{t-1} - i_{FP} \cdot (FP_{t-1})^+ \cdot i_T + i_{RDR} \cdot S_{t-1} - (H_t^{(t)})^+ \cdot (1 - i_T) \cdot i_D \\ + \sum_{k \geq t} \left( H_k^{(t)} - (H_k^{(t)})^+ \cdot i_T - H_k^{(t-1)} + (H_k^{(t-1)})^+ \cdot i_T \right) \cdot v_{RDR}^{k-t}$$

We use the formula of statutory profit and we isolate the taxes

$$VA_t = i_{FP} \cdot FP_{t-1} - i_{FP} \cdot (FP_{t-1})^+ \cdot i_T + i_{RDR} \cdot S_{t-1} - (H_t^{(t)})^+ \cdot (1 - i_T) \cdot i_D \\ + \sum_{k \geq t} \left( (H_k^{(t-1)})^+ \cdot i_T - (H_k^{(t)})^+ \cdot i_T \right) \cdot v_{RDR}^{k-t} + \sum_{k \geq t} \left( \sum_{s \leq t} G(s, t, k) - \sum_{s \leq t-1} G(s, t-1, k) \right) \cdot v_{RDR}^{k-t}$$

$$VA_t = i_{FP} \cdot FP_{t-1} - i_{FP} \cdot (FP_{t-1})^+ \cdot i_T + i_{RDR} \cdot S_{t-1} - (H_t^{(t)})^+ \cdot (1 - i_T) \cdot i_D \\ + \sum_{k \geq t+1} \left( (H_k^{(t-1)})^+ \cdot i_T - (H_k^{(t)})^+ \cdot i_T \right) \cdot v_{RDR}^{k-t} + (H_t^{(t-1)})^+ \cdot i_T - (H_t^{(t)})^+ \cdot i_T \\ + \sum_{k \geq t} \left( G(t, t, k) + \sum_{s \leq t-1} (G(s, t, k) - G(s, t-1, k)) \right) \cdot v_{RDR}^{k-t}$$

we arrange the elements

$$VA_t = i_{FP} \cdot FP_{t-1} - i_{FP} \cdot (FP_{t-1})^+ \cdot i_T + i_{RDR} \cdot S_{t-1} - (H_t^{(t)})^+ \cdot (1 - i_T) \cdot i_D \\ + \sum_{k \geq t+1} \left[ \left( (H_k^{(t-1)})^+ - (H_k^{(t)})^+ \right) \cdot i_T \cdot v_{RDR}^{k-t} \right] + \left( (H_t^{(t-1)})^+ - (H_t^{(t)})^+ \right) \cdot i_T \\ + G(t, t, t) + \sum_{k \geq t+1} G(t, t, k) \cdot v_{RDR}^{k-t} + \sum_{k \geq t} \left( \sum_{s \leq t-1} (G(s, t, k) - G(s, t-1, k)) \right) \cdot v_{RDR}^{k-t}$$

and we get finally this formula

$$VA_t = i_{FP} \cdot FP_{t-1} - i_{FP} \cdot (FP_{t-1})^+ \cdot i_T + i_{RDR} \cdot S_{t-1} \\ + G(t, t, t) + \sum_{k \geq t+1} \left[ G(t, t, k) + \left( (H_k^{(t-1)})^+ - (H_k^{(t)})^+ \right) \cdot i_T \right] \cdot v_{RDR}^{k-t} + \left( (H_t^{(t-1)})^+ - (H_t^{(t)})^+ \right) \cdot i_T \\ - (H_t^{(t)})^+ \cdot (1 - i_T) \cdot i_D + \sum_{k \geq t} \left( \sum_{s \leq t-1} (G(s, t, k) - G(s, t-1, k)) \right) \cdot v_{RDR}^{k-t}$$

where the five components of the Value Added can be seen

$$\begin{aligned}
VA_t = & \underbrace{i_{FP} \cdot FP_{t-1} - i_{FP} \cdot (FP_{t-1})^+ \cdot i_T}_{\text{interest on net assets}} + \underbrace{i_{RDR} \cdot S_{t-1}}_{\text{contribution from in force business}} \\
& + \underbrace{G(t, t, t)}_{\substack{\text{statutory profit of year } t \\ \text{for the new business of year } t \\ \text{(investissement)}}} + \underbrace{\sum_{k \geq t+1} \left[ G(t, t, k) + \left( (H_k^{(t-1)})^+ - (H_k^{(t)})^+ \right) \cdot i_T \right] \cdot v_{RDR}^{k-t}}_{\substack{\text{stock of new business of year } t \\ \text{contribution from new business}}} + \underbrace{\left( (H_t^{(t-1)})^+ - (H_t^{(t)})^+ \right) \cdot i_T}_{\text{taxe relief}} \\
& - \underbrace{(H_t^{(t)})^+ \cdot (1 - i_T) \cdot i_D}_{\text{capital adjustments}} + \underbrace{\sum_{k \geq t} \left( \sum_{s \leq t-1} (G(s, t, k) - G(s, t-1, k)) \right) \cdot v_{RDR}^{k-t}}_{\text{change of assumptions and emergence of actual experience}}
\end{aligned}$$

We have found five components of the Value Added, which are the following:

- Interest on net assets
- Contribution from in force business
- Contribution from new business
- Capital adjustments
- Change of assumptions and emergence of actual experience

### 3.4 Total Rate of Return

Total Rate of Return expresses the increase of Embedded Value as a percentage.

$$TRR_t = \frac{VA_t + D_t - C_t}{EV_{t-1}}$$

where  $D_t$  = shareholders dividends at the end of time  $t$

$C_t$  = shareholders contributions at the end of time  $t$

It is necessary to do corrections for dividends and shareholder contribution because the Embedded Value is changed by these values.

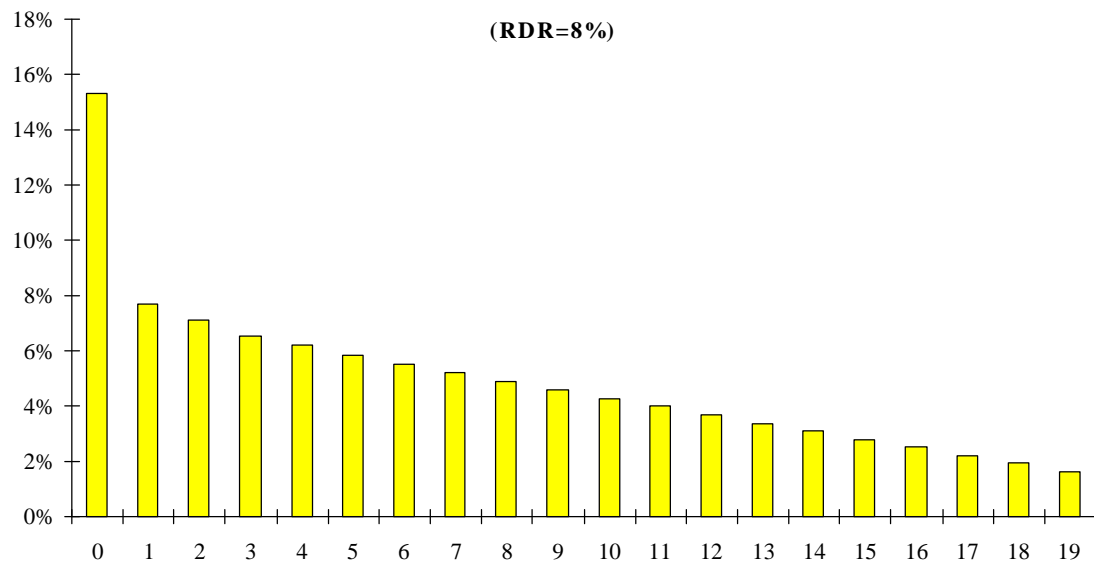
When dividend is paid, the net assets decrease and the Embedded Value too. The shareholder's view is that, he receives dividend and the Value Added. Then we have to take account of the dividend in the Total Rate of Return.

When shareholders make contributions, the Embedded Value increases but the shareholder's view is that, he receives the Value Added less what he contributes.



### 3.4.1 Numerical example

Total Rate of Return profile



This profile seems to be easy to understand and allow fixing a simple objective. For example, one could say next year we have to reach a Total Rate of Return equal to the RDR.

## 4 Critical Approach

Until now, we have seen the valuation methods. Everything seems to be simple and we are able to show the profitability of a company. But some aspects of these methods could be criticised. In this chapter, all the numerical examples are based on the data given in the annexe "Numerical application 2".

### 4.1 The problem

The results of the valuation methods depend on the model and the parameters like the return on investment, the best estimate mortality etc. These methods use always the mean of future risks. So the advantage is to obtain a unique value but the disadvantage is to hide the fluctuations. In reality the fluctuations are due to the model, the randomness and the uncertainty.

To choose a model with a view to conceptualising real phenomena is a simple approach which cannot reflect exactly the reality.

The randomness is the unforeseeability of events, which follow a probability function. Even when we perfectly know the probability law, the effective realisation of the events cannot be determined. In our model in particular, the randomness corresponds

to the fluctuations of the return on investment and the fluctuations of the real number of deaths. The estimations we calculate today are only mean value of future realisation. The uncertainty comes from the imperfect knowledge of the parameters and their possible evolution through the time. For example we do not know exactly the real amount of expenses and this amount can follow a future unexpected evolution.

Besides these fluctuations problems, there is an interpretation problem. The Value Added and the Total Rate of Return are often considered as profitability tools even though these methods can give a false signal to the profitability of the company. To discuss this interpretation problem, we take the following numerical example

### Endowment insurance

$x = 40$	$a = 5\%$ on Capital	SM 1988/93
$n = 25$	$g = 0.4\%$ on Capital	$i = 4\%$
Capital = 100'000	$b$ included in $g$	$P' = 3321$
$a^* = 5800$		$g^* = 80$
Mutation expenses = 333		$i^* = 6\%$
$i_{RDR} = 8\%$		Taxe rate = 10%
Interest on net assets = 1.5%		

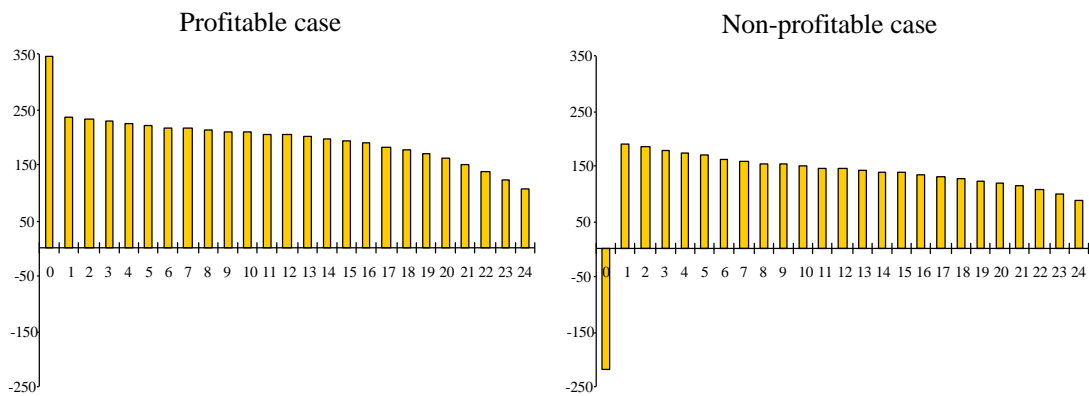
We will use two special cases as follows:

- **A profitable case:** Proportion of profit retained 13%
- **A non profitable case:** Proportion of profit retained 5%

The proportion of profit retained is the percentage of profit that the insurance company wants to keep. This means the part which is not distributed to the insured.

Let us see now the charts in these two cases.

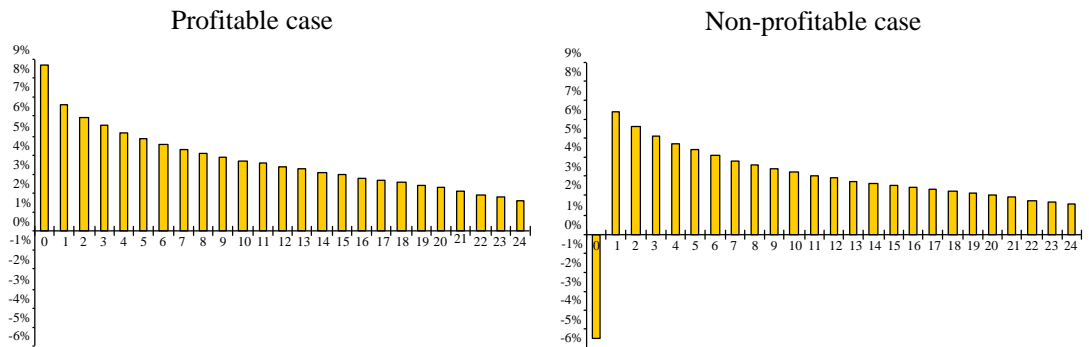
## Value Added



The first year looks sensible there is no interpretation problem. The insurance company sells a profitable contract, which increases the value of the company in the first case. In the second case the contract is non-profitable and the value of the company decreases.

Second year and later. For the first case, we do not notice any problem. The Value Added decreases because the company does not sell any other contract. In the second case, the company does not sell any other contract and the Value Added is positive although it had sold previously a non-profitable contract. The step especially between the first and the second year is not easy to understand for the purpose of profitability.

## Total Rate of Return



We have exactly the same interpretation problem with the Total Rate of Return.

In order to understand what happens, let us analyse the figures of the Value Added.

Profitable case

t	Interest on net assets	Contribution from in force business	Contribution from new business	Capital adjustment (dividend)	Value Added
0	54,00	0,00	292,94	0,00	346,94
1	12,28	274,99		52,87	234,40

Non-profitable case

t	Interest on net assets	Contribution from in force business	Contribution from new business	Capital adjustment (dividend)	Value Added
0	54,00	0,00	-273,04	0,00	-219,04
1	12,28	229,71		52,87	189,12

For the first year, there is no interpretation problem. The difference comes from the new business, positive for the profitable case and negative for the non-profitable case.

For the second year we have an interpretation problem. Let us analyse each component of the Value Added:

-Interest on net assets is the same because bonus begins only the third year in our example.

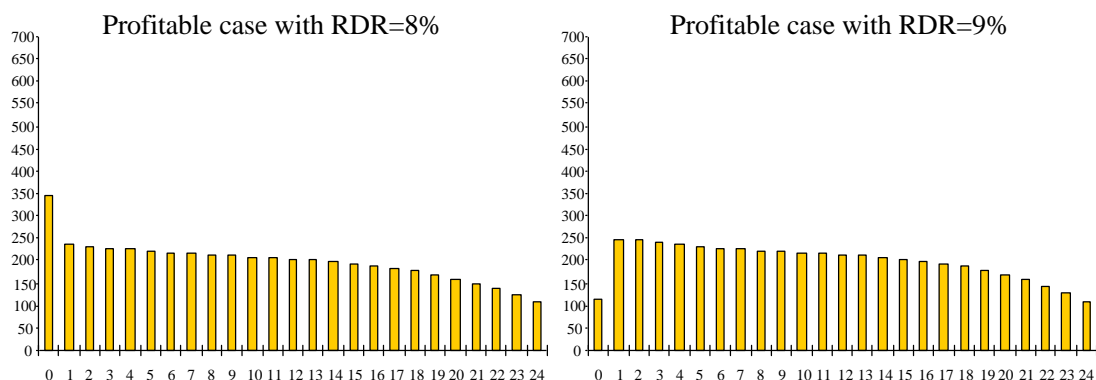
-Contribution from new business is the same because we do not sell any new contracts.

-Capital adjustment is the same because we do not give any bonus before the third year.

-The difference comes from the contribution from in force business. What is surprising is that in the non-profitable case this contribution is positive and this is like a return (effectively in the calculation). That's why the Value Added is positive in the non-profitable case.

Finally, the problem of interpretation is due to the contribution from in force business and more especially to the Risk Discount Rate.

Let us see now the influence of Risk Discount Rate on Value Added.



The Risk Discount Rate is a requirement of return in the first year but afterwards this rate becomes a return on the stock. After the first year, the Value Added calculated with a RDR of 9% is higher than with a RDR of 8%. This is really the source of the interpretation problem.

## **4.2 Three kinds of valuation**

We have to admit that the Value Added and the Total Rate of Return are not able to show directly the profitability of the company. But the interpretation problem disappears if we use the valuation methods in the correct context.

- Value of an insurance company
- Increase of the value of a company
- Profitability of a company

### **4.2.1 Value of a company**

Of course, the value of a company is directly given by Embedded Value or Appraisal Value.

### **4.2.2 Increase in the value of a company**

For measuring the increase in the value of a company we have to analyse the Value Added and its components (interest on net assets, contribution from in force business, contribution from new business, capital adjustments, change of assumptions and emergence of actual experience) and the Total Rate of Return.

We can analyse each point to see if it is positive or negative. And then we know which component increase, decrease or stagnate the value of the company.

The Value Added gives the real increase. The Total Rate of Return gives the relative increase and the components of the Value Added allow separation of the composition of the increase.

In our previous example, despite the fact that the contracts sold do not meet the criteria of Internal Rate of Return, the return is positive but smaller than the RDR. That is why the company increases its value with this product.

### **4.2.3 Profitability of a company**

The problem of interpretation comes also because we do not have a clear definition of profitability. We will use the following definition of profitability. In our context, the profitability is the measure of the activity of a company in connection with the new business, evaluated through the future profits expected to emerge from the new business, during a specified period of time.

When we are interested in the profitability of an insurance company, it seems to be logical to be especially interested in the new business created during the specified period of time.

In our context, we can analyse the Internal Rate of Return of the new business and its NPV. We can then have the following conclusions:

<b>IRR</b>	<b>NPV of new business</b>	<b>Conclusion</b>
$>RDR$	$>0$	Aim of return reached
$0 < IRR < RDR$	$<0$	Aim not reached but positive return
$<0$	$<0$	Loss

New business has also an influence on the administrative expenses, on the dividends etc but we concentrate on the taxes.

### **4.3 Final remarks about critical approach**

We have seen that the Value Added and the Total Rate of Return cannot be directly considered as tools for measuring the profitability. We have seen that the valuation methods must be used in the right context. We have to use the components of the Value Added. This is why we have defined what we call value, increase of the value and profitability of an insurance company or a profit centre. In our context, we give a definition of profitability. We have not found any new method, we just explain that the valuation methods do not always directly give the right answer and that we have to use it in the right context.

## **5 Sensitivity Analysis**

As we have already seen, the results of the valuation methods depend on the model and the parameters like the return on investment, the best estimate mortality etc. These methods use always the mean of future risks. So the advantage is to obtain a unique value but the disadvantage is to hide the fluctuations. In reality the fluctuations are due to the model, the randomness and the uncertainty.

The uncertainty comes from the imperfect knowledge of the parameters and their possible evolution through the time. For example we do not know exactly the real amount of expenses and this amount can follow an unexpected future evolution.

### **5.1 Approach**

The sensitivity analysis is used to analyse the influence of the uncertainty in the parameters' values. For this purpose, we create some scenarios with optimistic and pessimistic values of parameters. We suppose that certain ranges of parameter value are possible and we test the reactions of the result of the method with those modifications of parameter value.

Two ways are possible, the successive sensitivity analysis and the simultaneous sensitivity analysis.

In the successive sensitivity analysis, the value of a unique parameter is changed. The value of the other parameters does not change. The aim of this analysis is to extract the more influent parameters on the result of the valuation method.

In the simultaneous sensitivity analysis, the value of the whole parameters is changed. The aim of this analysis is to analyse the worst and the best possible situation in taking respectively the pessimistic value of several parameters and in taking the optimistic value of several parameters.

But we have to notice that some interdependencies are possible between the parameters. So, we have to model it, when it is possible and relevant.

It is true that the level of variation of the parameters value will influence the variation in the result of the valuation method. But to have a homogeneous comparison, we can take a constant percentage of variation for each parameter value. Of course, this percentage is nevertheless arbitrary.

## 5.2 Sensitivity estimation

We will use current economic notion: the elasticity

$$\text{Theoretic elasticity } E_p(R) = \frac{\partial R}{R} \bigg/ \frac{\partial p}{p} \quad \text{Empirical approximation } \hat{E}_p(R) = \frac{\Delta R}{R} \bigg/ \frac{\Delta p}{p}$$

This is the elasticity of the result  $R(p)$  of the valuation method in keeping with the parameter  $p$ . Afterwards we will only use the empirical approximation of the elasticity but to simplify the term, we will only speak about elasticity and no more about empirical approximation of the elasticity.

We say that the parameter  $p$  is elastic if the result of this formula is higher than 1 and respectively inelastic if the result of this formula is lower than 1.

This elasticity formula is used in the successive sensitivity analysis, but it is possible to show that we can generalise this formula for successive sensitivity analysis.

## 5.3 Interdependence between parameters

In our model, we have taken the following interdependencies between parameters.

- We suppose that the expenses depend on the inflation and are interdependent.
- The interest rate depends on inflation.
- The Risk Discount Rate and the interest on net assets depend on the interest rate.
- The bonus depends on the best estimate basis.

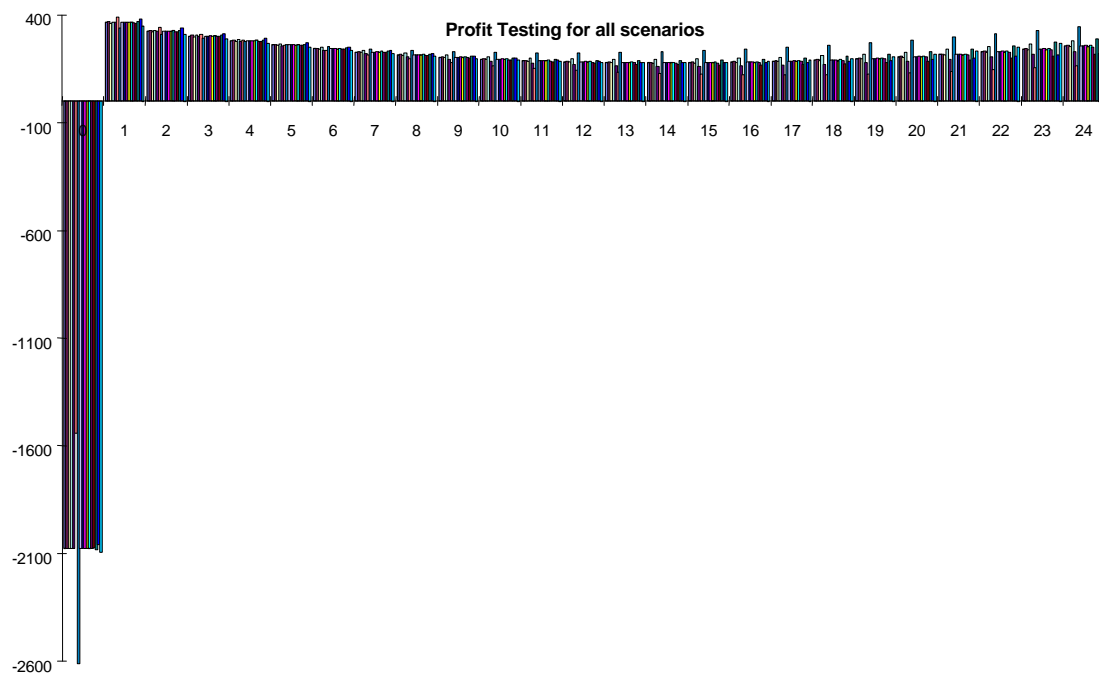
The following table shows the parameters, which are changed in the scenarios.

Parameter	Symbol	Interdependence formula
Risk Discount Rate	$i_{RDR}$	$i_{RDR} = I \cdot i^*$
Interest on net assets	$i_{FP}$	$i_{FP} = r \cdot i^*$
Proportion of profit retained	$i_R$	$J(x, n) = \frac{(1-i_R)}{c} \cdot (CP_{xn}^* - CP_{xn}^{**}) \cdot \frac{a^{**}}{H^*(x, n)}$
Acquisition expenses at time $k$	$FA_k$	$FA_k = d \cdot (MFA + CA) \cdot (1+i_l)^k$
Administration expenses at time $k$	$FG_k$	$FG_k = d \cdot (MFG) \cdot (1+i_l)^k$
Mutation expenses at time $k$	$FM_k$	$FM_k = d \cdot (MFM) \cdot (1+i_l)^k$
Surrender value at the end of year $k$	$S_{x+k}$	
Tax rate	$i_T$	
Dividend rate	$i_D$	
Best estimate mortality	$q_x^*$	
Annual inflation rate	$i_l$	
Surrender rate	$u_x^*$	

## 5.4 Application

A sensitivity analysis is applied to the Profit Testing, Embedded Value and its increase with four kinds of products, an endowment with annual premiums, an endowment with one single premium, a term insurance with annual premiums and a temporary immediate life annuity. In this chapter, all the numerical examples are based on the data given in the annexe "Numerical application 3". A successive and a simultaneous sensitivity analysis are done with variations of parameter value of  $\pm 10\%$ .

### 5.4.1 Endowment with annual premiums



Firstly, we cannot see any significant variation in this chart. This is the traditional profile of an endowment with annual premiums. The only scenario which is clearly



different from the others is the expense scenario. The others seem to be close to the reference scenario.

But if we analyse the NPV we have another situation.

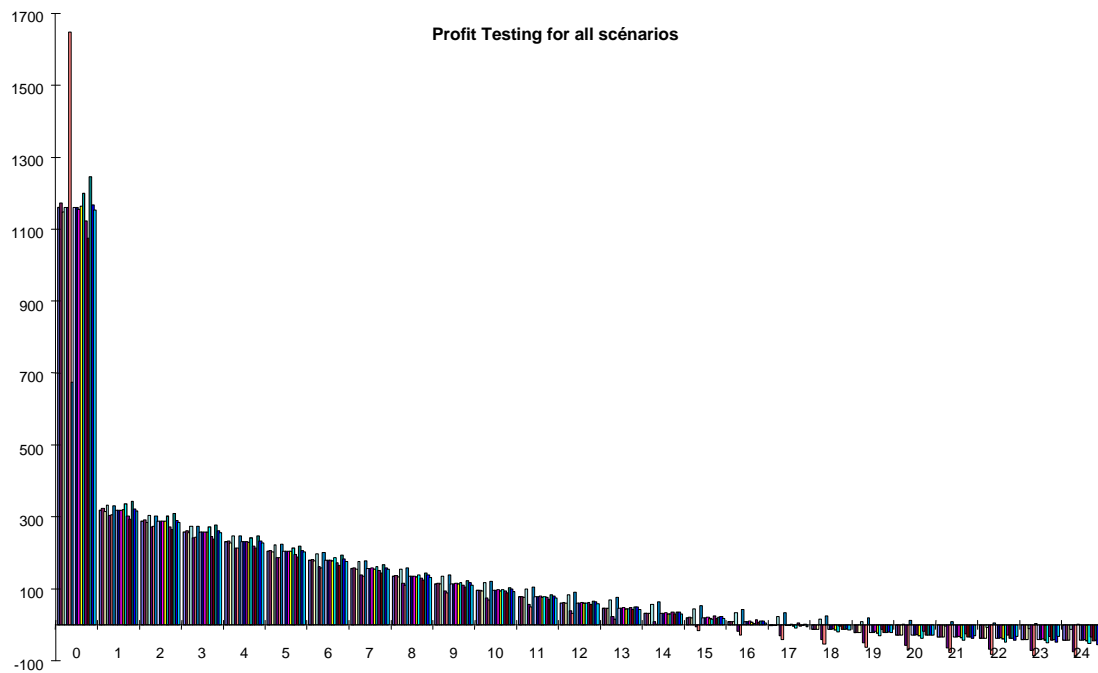
	NPV	NPV Elasticity	IRR	IRR Elasticity
Reference scenario	462		11.2%	
Optimistic expenses	743	-6.06	16.2%	-4.38
Pessimistic expenses	182	-6.06	8.9%	-2.06
Optimistic Risk Discount Rate	328	-2.91	11.2%	0.00
Pessimistic Risk Discount Rate	614	-3.27	11.2%	0.00
Optimistic proportion of profit retained	558	2.06	11.8%	0.48
Pessimistic proportion of profit retained	367	2.06	10.7%	0.51
Optimistic mortality rate	515	-1.14	11.7%	-0.45
Pessimistic mortality rate	410	-1.13	10.8%	-0.42
Optimistic taxes	489	-0.57	11.4%	-0.1626
Pessimistic taxes	436	-0.57	11.1%	-0.1628
Optimistic inflation	438	0.53	10.8%	0.37
Pessimistic inflation	483	0.45	11.6%	0.35
Optimistic surrender value	486	0.50	11.4%	-0.1368
Pessimistic surrender value	439	-0.50	11.1%	-0.1373
Optimistic surrender rate	471	-0.18	11.3%	-0.0435
Pessimistic surrender rate	454	-0.17	11.2%	-0.0433

This elasticity of expenses means that for example a variation of +10% in the expenses causes a variation of -60.6% in the NPV!

In this table, the expenses are clearly the parameter to which the NPV is most sensitive in the case of an endowment with annual premiums. As the acquisition expenses are important in the first year and because only a part of it is zillmerised, the difference is particularly relevant for the first year. Afterwards, the difference is smaller but is becomes greater through time. At the end of the contract the difference is again higher because inflation increases the expenses and at the same time the annual premiums remains always the same.

The Risk Discount Rate, the proportion of profit retained and the mortality are also considered elastic parameters.

## 5.4.2 Endowment with single premium



The profile of this single premium endowment product is unusual. We do not have any initial investment and we have some losses at the end of the contract.

As before, we cannot see any high difference in this chart. The only scenario that is clearly different from the others is the expense scenario. The others seem to be closed to the reference scenario.

	NPV	NPV Elasticity
Reference scenario	2452	
Optimistic expenses	2661	-0.85
Pessimistic expenses	2240	-0.87
Optimistic proportion of profit retained	2651	0.81
Pessimistic proportion of profit retained	2252	0.82
Optimistic inflation	2282	0.69
Pessimistic inflation	2616	0.67
Optimistic surrender value	2532	-0.3235
Pessimistic surrender value	2373	-0.3240
Optimistic Risk Discount Rate	2395	-0.23
Pessimistic Risk Discount Rate	2513	-0.25
Optimistic taxes	2480	-0.1128
Pessimistic taxes	2425	-0.1128
Optimistic mortality rate	2481	-0.1148
Pessimistic mortality rate	2424	-0.1143
Optimistic surrender rate	2457	-0.02018
Pessimistic surrender rate	2448	-0.02017

In this case we see that no parameter can be considered elastic. This is because the NPV is very high in the reference scenario. Furthermore the profile of the statutory earnings are for example not sensitive to the RDR because we have a high gain at the beginning and less high amounts afterwards.

### 5.4.3 Term insurance with annual premiums

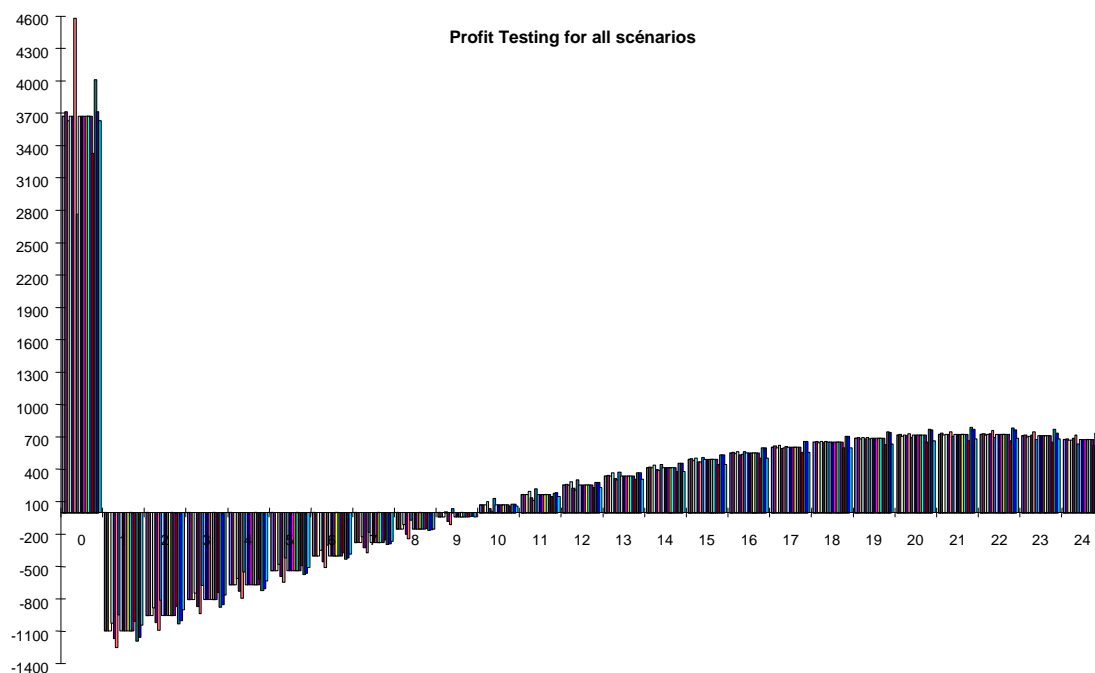


In this case, we have again a traditional profile of statutory earnings. The expense scenario is again the most influential.

	NPV	NPV Elasticity	IRR	IRR Elasticity
Reference scenario	1149		12.1%	
Optimistic expenses	1787	-5.54	15.4%	-2.67
Pessimistic expenses	512	-5.54	9.6%	-2.06
Optimistic Risk Discount Rate	873	-2.40	12.1%	0.00
Pessimistic Risk Discount Rate	1462	-2.72	12.1%	0.00
Optimistic proportion of profit retained	1354	1.78	12.8%	0.51
Pessimistic proportion of profit retained	945	1.78	11.5%	0.54
Optimistic mortality rate	1347	-1.72	12.7%	-0.48
Pessimistic mortality rate	953	-1.71	11.5%	-0.50
Optimistic inflation	1264	-1.00	12.2%	-0.026
Pessimistic inflation	1039	-0.96	12.1%	-0.028
Optimistic taxes	1205	-0.49	12.3%	-0.16562
Pessimistic taxes	1093	-0.49	11.9%	-0.16557
Optimistic surrender rate	1178	-0.25	12.2%	-0.06944
Pessimistic surrender rate	1122	-0.24	12.0%	-0.06937
Optimistic surrender value	1154	-0.04	12.1%	-0.00996
Pessimistic surrender value	1145	-0.04	12.1%	-0.00998

We have exactly the same elastic parameters than in the endowment product with annual premiums.

#### 5.4.4 Temporary immediate life annuity



The profile of statutory earnings is once again different. A high gain the first year follows by a series of losses and then a series of gains. This is principally due to the bonus, which decreases.

The expense scenario is always the remarkable one.

	NPV	NPV Elasticity
<b>Reference scenario</b>	<b>1643</b>	
Optimistic proportion of profit retained	2046	2.453
Pessimistic proportion of profit retained	1239	2.455
Optimistic Risk Discount Rate	1494	-0.90
Pessimistic Risk Discount Rate	1825	-1.11
Optimistic inflation	1508	0.82
Pessimistic inflation	1771	0.78
Optimistic expenses	1740	-0.60
Pessimistic expenses	1543	-0.61
Optimistic taxes	1700	-0.35
Pessimistic taxes	1585	-0.35
Optimistic mortality rate	1640	-0.018
Pessimistic mortality rate	1646	-0.022

In this case, the proportion of profit retained and the Risk Discount Rate are elastic parameter.

We notice that here the expenses scenario is not the most influential. This is because the administration expenses are lower and there is no mutation expense because surrender is impossible.

## **5.5 Simultaneous sensitivity**

We consider now a profit centre, which sells the first year a portfolio of identical contracts. Afterwards, the profit centre sells each year a new portfolio of the same product with a volume increasing by 5% pa. We consider that there are no stochastic fluctuations (randomness) and that we do not change the assumptions.

So, the first year is the year where the profit centre launches the product and the fifth year corresponds to the situation in which the profit centre is after five years of selling the product with the assumptions of the given scenario.

We notice that all the different scenarios diverge from the reference scenario. Each year the new contracts sold accentuate this difference. The following general conclusion can be made on the simultaneous analysis:

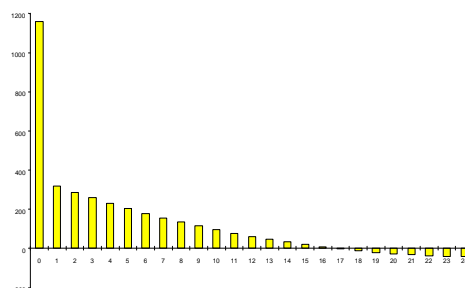
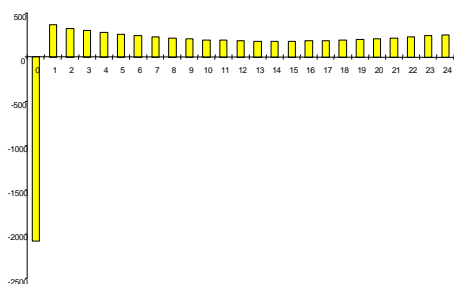
- The influential parameters in Profit Testing are also influential on the Embedded Value and its increase.
- No new parameter is considered as elastic.
- In the pessimistic scenario of the simultaneous analysis the Embedded Value decreases.

## **5.6 Conclusion on the sensitivity analysis**

Through this analysis we notice three influential parameters: the expenses, the RDR and the proportion of profit retained.

There are also affects on the Net Present Value and as the Embedded Value and its increase.

In particular the product with annual premiums and product with one single premium do not have the same sensitivity to the parameters. The sensitivity of the product with annual premiums is always higher than the product with one single premium. This is due to the profile of statutory earnings.



### **Products with annual premiums**

With annual premiums we have a loss in the first year followed by a series of gains. So, this profile is more risky than the one of the single premium. For example, The RDR is of course more influent on such a profile because the gains come in the future to recover the first year loss. The same consideration is available for the proportion of profit retained. This is more surprising for the mortality. We may have thought that the product (a term insurance which insured only the risk of death or an endowment) have more influence on the result of the valuation method than the way of paying the premiums. But in fact, this is logical. This risk taken by the company is higher with annual premiums product than with single premium product and this is clear in the chart of the statutory earnings (of course with a whole life annuity the sensitivity could be high too).

### **Products with single premium**

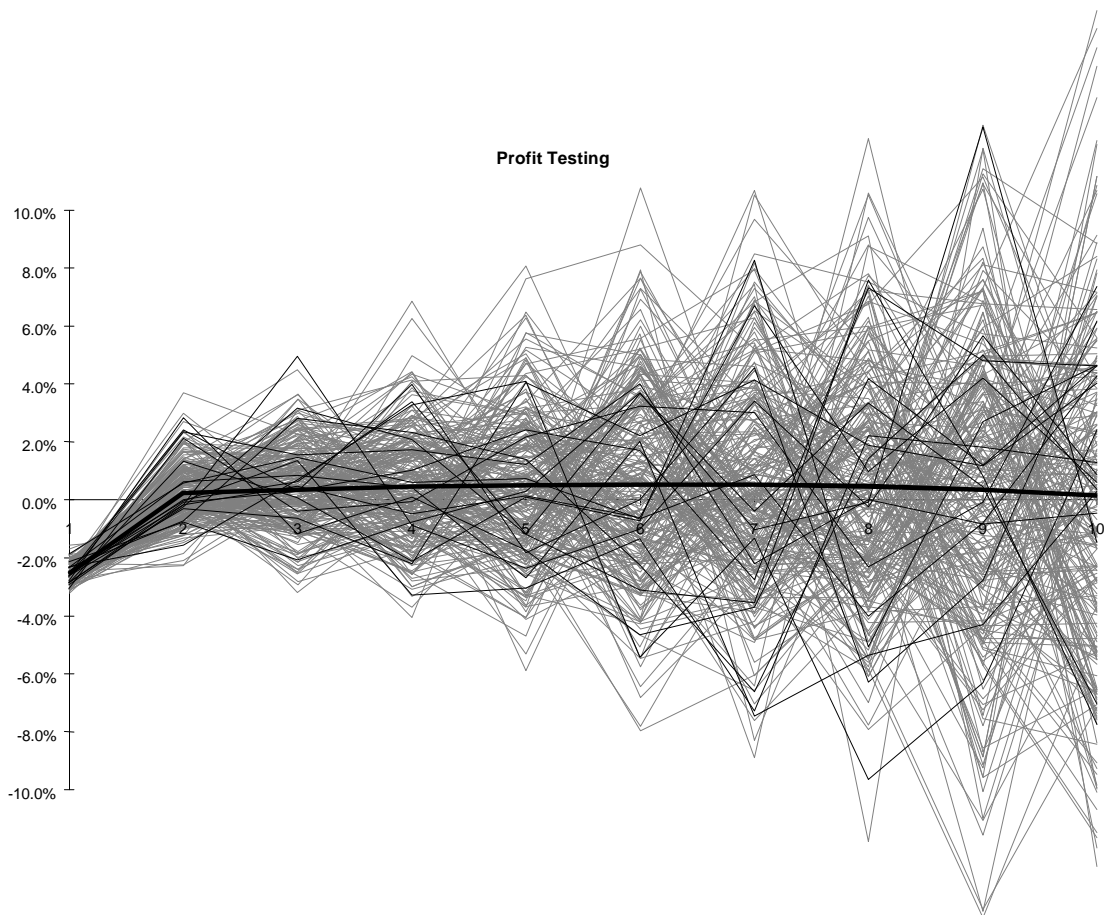
With a single premium we have a gain the first year followed by a series of profits.

Finally, the influence of the parameters depends especially about the profile of the statutory earnings.

One could have thought that the interest rate is an influent parameter. We are in a deterministic model where no stochastic fluctuations exist. And in our model there is interdependence between the interest rate, the inflation and the RDR, which reduces the effect of changing the interest rate.

This analysis shows that the valuation methods are sensitive to the changes in the parameter value. This is particularly clear in the simultaneous analysis: A modification of 10% of the value of parameters in the simultaneous analysis could completely change the view that we have of a product, of a profit centre or even of a company. The sensitivity is so high that we can doubt about the reliability to use the Profit Testing as a pricing tool!

Furthermore, in this analysis, we have only analysed the fluctuations due to the uncertainty of the parameter value. Certainly taking together the uncertainty, the randomness and the error of the model into account would accentuate the problem of reliability of the method. A study shows that the fluctuations due to the randomness are highly relevant. For example to simulate the interest rate instead of using a mean interest rate and to simulate also the number of death and surrenders gives the following chart.



This is an example of a traditional endowment. We suppose that the interest rate follows a lognormal law and that the number of death and surrenders follow both a Bernoulli law. In bold, in the middle, we see the curve of the Profit Testing calculated with the mean value of the parameters. With this chart we really see that even if we know very well the law of the events, the reality is completely different from the mean. This is the effect of the randomness. The outcomes are completely different from the expected value.

## 6 Conclusion

We have seen what the valuation methods are.

We have seen that we can use these methods as pricing or profitability tools.

We have seen that these methods are not always easy to understand.

We have also seen that variations due to the uncertainty are relevant. It means that using for example the Profit Testing as a pricing tool to obtain the most competitive premium is very dangerous. Of course, the randomness we spoke about before accentuates the non-reliability of the valuation methods.

## 7 Annexe

### 7.1 Basic actuarial formulae

- $v = \frac{1}{1+i}$  discount factor
- ${}_t p_x$  = probability that a life aged  $x$  will survive between age  $x$  and  $x+t$  (pricing basis)
- $q_x$  = probability that a life aged  $x$  will die between age  $x$  and  $x+1$  (pricing basis)
- $q_x^*$  = probability that a life aged  $x$  will die between age  $x$  and  $x+1$  (best estimate basis)
- $u_x^*$  = probability to surrender between age  $x$  and  $x+1$  (best estimate basis)
- $p_x^* = (1 - u_x^*) \cdot (1 - q_x^*)$  = probability for a lived aged  $x$  to stay in the portfolio at the age  $x+1$  (best estimate basis)
- ${}_t p_x^* = p_x^* \cdot p_{x+1}^* \cdot p_{x+2}^* \cdot \dots \cdot p_{x+t-1}^*$  = probability that a life aged  $x$  will survive between age  $x$  and  $x+t$  (best estimate basis)
- $A_{\overline{xn}|}$  = Net present value of the payment of 1 if the insured is alive at the end of  $n$  years and also of the payment of 1 unit at the end of the year of death if death occurs within  $n$  years (endowment insurance)
- $A_{\overline{xn}|} = \sum_{k=0}^{n-1} v^{k+1} \cdot {}_k p_x \cdot q_{x+k} + v^n \cdot {}_n p_x$
- $\ddot{a}_{\overline{xn}|}$  = Net present value of annual payments of 1 unit during  $n$  years as long as the beneficiary lives (A  $n$ -years temporary life annuity-due).
- $\ddot{a}_{\overline{xn}|} = \sum_{k=0}^{n-1} v^k \cdot {}_k p_x$



## 7.2 Numerical application 1

This annexe shows the data used in chapters 2 and 3.

### Benefits and data on the assured

A male aged 30 buys an endowment for duration of 20 years. An increasing bonus is paid each year as a deduction from the gross premium. In case of surrender, the assured receives 90% of the Zillmerised reserve.

### Pricing basis

- Acquisition expenses, at the beginning of the first year:  $\mathbf{a} = 4\%$  on the sum assured
- Collection expenses, beginning of year:  $\mathbf{b} = 1,5\%$  on the gross premium
- Administration expenses, beginning of year:  $\mathbf{g} = 0,3\%$  on the sum assured
- Technical rate:  $i = 4\%$
- Mortality table: SM 1978/83
- Actuarial formula of an endowment:

$$P'' \cdot \ddot{a}_{\overline{xn}|} = A_{\overline{xn}|} + \mathbf{a} + \mathbf{b} \cdot P'' \cdot \ddot{a}_{\overline{xn}|} + \mathbf{g} \cdot \ddot{a}_{\overline{xn}|} \qquad P'' = 0,0397$$

### Best estimate basis

- Best estimate basis are symbolised by \*
- Collection costs, beginning of the year:  $\mathbf{b}^* = 1\%$  on the gross premium
- Acquisition costs, beginning of the first year:  $\mathbf{a}^* = 3,8\%$  on the sum assured
- Administration costs, beginning of the year:  $\mathbf{g}^* = 0,2\%$  on the sum assured
- Expected return rate on the assets:  $i^* = 6\%$
- Best estimate mortality rate:  $q_x^* = 85\% \cdot q_x$  of the pricing mortality calculated with the mortality of the table: SM 1978/83

- Probability of surrender

$$u_x^* =$$

$$\begin{cases} 0,05 & \text{the first three years} \\ 0,03 & \text{the following years} \end{cases}$$

Surrender benefit  $S_{x+t}$ , paid at the end of year  $t$ , is as follows  $S_{x+t} = 90\% \cdot {}_tV_x''$

where  ${}_tV_x''$  is the Zillmerised reserve

$${}_tV_x'' = \begin{cases} 0 & \text{if } {}_tV_x'' \leq 0 \\ (A_{\overline{x+t:n-t}|} - P \cdot \ddot{a}_{\overline{x+t:n-t}|}) - \frac{a}{\ddot{a}_{\overline{xn}|}} \cdot \ddot{a}_{\overline{x+t:n-t}|} & \text{otherwise} \end{cases}$$

where  $P$  = net premium

- The bonus is increasing as follows  $(PE)_t = 0,0045 \cdot (1,08)^t$ . It is paid annually in deduction of the gross premium, at the beginning of the year from the second premium.

### Other assumptions

- A portfolio of 10000 lives is considered
- The reserves used are the reserves with the administration expenses
- The initial net assets equal to 350
- $i_{RDR}$  (Risk Discount Rate) = 8 %
- $i_T$  (tax rate) = 10%
- $i_{FP}$  (return rate on the net assets) = 1,5%

### 7.3 Numerical example 2

The charts and numerical applications showed in the chapter 4 come from a unique example which is an endowment with the following specifications.

#### Benefits and date on the assured

A male aged 40 buys an endowment for duration of 25 years. An increasing bonus is paid each as a deduction from the gross premium. In case of surrender, the assured receives 90% of the Zillmerised reserve.

#### Pricing basis

- Sum assured:  $C = 100'000$
- Acquisition expenses, at the beginning of the first year:  $\mathbf{a} = 5\%$  on the sum assured
- Administration expenses, beginning of year:  $\mathbf{g} = 0,4\%$  on the sum assured
- Collection expenses :  $\mathbf{b}$  included in  $\mathbf{g}$
- Technical rate:  $i = 4\%$
- Mortality table: SM 1978/83
- Actuarial formula of an endowment:

$$CP'' \cdot \ddot{a}_{\overline{xn}|} = C \cdot A_{\overline{xn}|} + \mathbf{a} \cdot C + \mathbf{g} \cdot C \cdot \ddot{a}_{\overline{xn}|}$$

$$\text{where } A_{\overline{xn}|} = 0,40321 \qquad \ddot{a}_{\overline{xn}|} = 15,517$$

$$\text{Then the gross premium is : } P'' = 0,033208 \qquad CP'' = 3320,80$$

$$P'' = \text{gross premium for a sum assured of 1}$$

#### Best estimate basis

- Best estimate basis are symbolised by \*
- Acquisition costs, beginning of the first year = fixed costs + commission to the agents = 5974
- Administration costs, beginning of the year = 80
- Surrender costs, beginning of the year = 333,33
- Expected return rate on the assets:  $i^* = 6\%$

- Best estimate mortality rate:  $q_x^* = 85\% \cdot q_x$
- Surrender probability  $u_x^* = \text{MAX} \left\{ \frac{0,2}{x-20}; 0,005 \right\}$
- Surrender benefit  $S_{x+t}$ , paid at the end of year t, is as follows  $S_{x+t} = 90\% \cdot {}_tV_x''$   
if  ${}_tV_x'' > 0$ , otherwise  $S_{x+t} = 0$

where

${}_tV_x'' =$  Zillmerised reserve

$${}_tV_x'' = ((A_{\overline{x+tn-t}|} - P \cdot \ddot{a}_{\overline{x+tn-t}|}) - \frac{0,5 \cdot a}{\ddot{a}_{\overline{xn}|}} \cdot \ddot{a}_{\overline{x+tn-t}|}) \cdot C \quad \text{where } P = \text{net premium}$$

- The bonus  $(PE)_t$  is paid annually in deduction of the gross premium. The bonus increased yearly and is paid from the third premium. We use the following formula where  $J(x, n)$  represents the yearly bonus in percentage of the gross premiums

$$J(x, n) = \frac{(1 - i_R)}{CP''} \cdot (CP'' - CP''^*) \cdot \frac{\ddot{a}_{\overline{xn}|}^*}{H^*(x, n)}$$

$$\text{where } H^*(x, n) = b_1 \cdot {}_{j-1|}\ddot{a}_{\overline{xn-j+1}|}^* + b_2 \cdot {}_{j-1|}(I\ddot{a})_{\overline{xn-j+1}|}^*$$

$P''^* =$  gross premium calculated with the best estimate basis

\* symbolised the best estimate basis

j = is the number of premiums from which the bonus is paid

We choose  $b_1 = 4$  and  $b_2 = 3$

and we have

third premium deducted by:  $(b_1 + b_2) \cdot J(x, n)$

fourth premium deducted by:  $(b_1 + 2b_2) \cdot J(x, n)$

etc.

## Other assumptions

- We consider a portfolio of identical policies, benefits and expenses
- We use the Zillmerised reserves
- Zillmerised reserves : 50% of the acquisition expenses are Zillmerised
- Initial net assets equal to 4000
- $i_{RDR}$  (Risk Discount Rate) = 8 %
- $i_T$  (tax rate) = 10%
- $i_{FP}$  (return rate on the net assets) = 1,5%
- $i_R$  (proportion of profit retained, profitable case) = 13%
- $i_R$  (proportion of profit retained, non profitable case) = 5%
- $i_D$  (dividend rate in percentage of the distributable profits) = 10%

## 7.4 Numerical application 3

This annexe shows the data used in chapter 4. Four kinds of product are used, an endowment with annual premiums, an endowment with single premium, a term insurance with annual premiums and an immediate temporary annuity-due. The assured are male aged 40 and the duration of the contract is 25 years

Let us start with the data of the endowment with annual premiums and then we will only indicate for the other kinds of product the differences compared with the endowment with annual premiums.

### Endowment with annual premiums

#### Benefits

- Sum assured equals to 100'000
- A bonus is paid annually in deduction of the gross premium, at the beginning of the year from the second premium.
- Surrender benefit  $S_{x+t}$ , paid at the end of year  $t$ , is as follows  $S_{x+t} = 90\% \cdot {}_tV_x''$  if  ${}_tV_x'' > 0$ , otherwise  $S_{x+t} = 0$ .

#### Pricing basis

- Acquisition expenses, at the beginning of the first year:  $\mathbf{a} = 5\%$  on the sum assured
- Administration expenses, beginning of year:  $\mathbf{g} = 0,4\%$  on the sum assured
- Collection expenses :  $\mathbf{b}$  included in  $\mathbf{g}$
- Technical rate:  $i = 4\%$
- Mortality table: SM 1988/83
- Actuarial formula of an endowment:

$$CP'' \cdot \ddot{a}_{\overline{xn}|} = C \cdot A_{\overline{xn}|} + \mathbf{a} \cdot C + \mathbf{g} \cdot \ddot{a}_{\overline{xn}|}$$

$$\text{where } A_{\overline{xn}|} = 0,39873 \quad \ddot{a}_{\overline{xn}|} = 15,633$$

Then the gross premium is :  $P'' = 0,0327043$  for a sum assured of 1

and  $CP'' = 3270,43$  for the sum assured of 100'000

## Best estimate basis

- Best estimate basis are symbolised by \*
- Acquisition costs, beginning of the first year: fixed costs ( $MFA$ ) = 1000 and commission to the agents ( $CA$ ) = 3,8% of the sum assured
- Administration costs, beginning of the year ( $MFG$ ) = 250
- Surrender cost, beginning of the year ( $MFM$ ) = 50
- Expected return on the net assets :  $i^* = 6\%$
- Best estimate mortality rate:  $q_x^* = 85\% \cdot q_x$
- Surrender probability  $u_x^* = \text{MAX} \left\{ \frac{0,2}{x-20}; 0,005 \right\}$

Surrender benefit  $S_{x+t}$ , paid at the end of year t, is as follows  $S_{x+t} = 90\% \cdot {}_tV_x''$

if  ${}_tV_x'' > 0$ , otherwise  $S_{x+t} = 0$  where :  ${}_tV_x'' = \text{Zillmerised reserve}$

$${}_tV_x'' = ((A_{\overline{x+tn-t}|} - P \cdot \ddot{a}_{\overline{x+tn-t}|}) - \frac{0,5 \cdot a}{\ddot{a}_{\overline{xn}|}} \cdot \ddot{a}_{\overline{x+tn-t}|}) \cdot C \quad \text{where } P = \text{net premiums}$$

- The bonus  $(PE)_t$  is paid annually in deduction of the gross premium. The bonus increased yearly and is paid from the third premium. We use the following formula where  $J(x, n)$  represents the yearly bonus in percentage of the gross premiums

$$J(x, n) = \frac{(1 - i_R)}{CP''} \cdot (CP'' - CP''^*) \cdot \frac{\ddot{a}_{\overline{xn}|}^*}{H^*(x, n)}$$

$$\text{where } H^*(x, n) = b_1 \cdot {}_{j-1|}\ddot{a}_{\overline{xn-j+1}|}^* + b_2 \cdot {}_{j-1|}(I\ddot{a})_{\overline{xn-j+1}|}^*$$

$P''^*$  = gross premium calculated with the best estimate basis

\* symbolised the best estimate basis

j = is the number of premiums from which the bonus is paid

We choose  $b_1 = 1$  and  $b_2 = 5$

and we have

third premium deducted by:  $(b_1 + b_2) \cdot J(x, n)$

fourth premium deducted by:  $(b_1 + 2b_2) \cdot J(x, n)$

etc.

### **Other assumptions**

- We consider a portfolio with identical policies, benefits and expenses
- We use the Zillmerised reserves
- Zillmerised reserves : 50% of the acquisition expenses are Zillmerised
- Initial net assets equal to 4000
- The interdependence between the parameters are given in the numerical application
- $i_{RDR}$  (Risk Discount Rate) = 8 %
- $i_T$  (tax rate) = 10%
- $i_{FP}$  (return rate on the net assets) = 1,5%
- $i_R$  (proportion of profit retained, profitable case) = 17%
- $i_I$  (annual inflation rate) = 2%
- $i_D$  (dividend rate in percentage of the distributable profits) = 10%

Let us see now the other kinds of products. We only indicate the parameters which do not have the same value than in the endowment with annual premiums.

### **Endowment with single premium**

- Acquisition costs, beginning of the first year: fixed costs ( $MFA$ ) = 54 and commission to the agents ( $CA$ ) = 4,8% of the sum assured
- The bonus is calculated in percentage of the sum assured and increased with the following factors  $b_1 = 15$  and  $b_2 = 1$

### **Term insurance with annual premiums**



- $i_R$  (proportion of profit retained, profitable case) = 80%
- Acquisition costs, beginning of the first year: fixed costs ( $MFA$ ) = 3054 and commission to the agents ( $CA$ ) = 1,8% of the sum assured
- The bonus increased with the following factors  $b_1 = 10$  and  $b_2 = 1$

### Temporary immediate life annuity-due

- Acquisition expenses, at the beginning of the first year:  $\mathbf{a} = 5\%$  on the gross premium
- Administration expenses, beginning of year:  $\mathbf{g} = 2\%$  on the annual annuity paid
- $i_R$  (proportion of profit retained, profitable case) = 13%
- Best estimate mortality rate:  $q_x^* = 120\% \cdot q_x$
- Acquisition costs, beginning of the first year: fixed costs ( $MFA$ ) = 104 and commission to the agents ( $CA$ ) = 4,8% of the gross single premium
- Administration costs, beginning of the year ( $MFG$ ) = 180
- No surrender cost, because to surrender is impossible in this product
- The bonus is calculated in percentage of the annual annuity paid and decreased with the following factors  $b_1 = 25$  and  $b_2 = -1$ . It begins at the end of the second insurance year.

## 7.5 Bonus formula

In the numerical application 2 and 3, we use a bonus formula, which is explained more in detail now.

The net present value of the future bonus paid is as follows:

$$(1 - i_R) \cdot \left( P''_{xn} - P''^*_{xn} \right) \cdot C \cdot \ddot{a}^*_{xn}$$

$i_R$  = proportion of profit retained

$x$  = age of the assured at the beginning of the contract

$n$  = duration of the contract

\* symbolised the best estimate basis

In this formula  $\left( P''_{xn} - P''^*_{xn} \right) \cdot C$  represent the difference between the gross premium and the premium calculated with the best estimate basis. By multiplying by  $(1 - i_R)$ , we get the part given to the assured and using the net present value ( $\ddot{a}^*_{xn}$ ) to discount it, we get the net present value at the beginning of the contract of the future bonus . We want now to distribute these bonus during the whole duration of the contract and express the bonus in percentage of a reference (called characteristic) as for example the premium etc.

$$J(x, n) \cdot H^*(x, n) = \frac{(1 - i_R)}{c} \cdot \left( CP''_{xn} - CP''^*_{xn} \right) \cdot \ddot{a}^*_{xn}$$

where

$J(x, n)$  = bonus rate function of age at the inception  $x$  and duration  $n$

$H^*(x, n)$  = net present value representing the way of distributing the bonus (scheme for 1 unit)

$c$  = characteristic. Value on which we want to apply the bonus rate (for example premium, sum assured etc)

Now we can isolate the bonus rate.

$$J(x, n) = \frac{(1 - i_R)}{c} \cdot \left( CP''_{xn} - CP''^*_{xn} \right) \cdot \frac{\ddot{a}^*_{xn}}{H^*(x, n)}$$

Let us see more in detail this formula in the case of the endowment with annual premiums.

$$J(x, n) = \frac{(1 - i_R)}{c} \cdot \left[ \left( \frac{A_{xn}}{\ddot{a}_{xn}} + \frac{\mathbf{a}}{\ddot{a}_{xn}} + \mathbf{g} \right) \cdot C - CP''^*_{xn} \right] \cdot \frac{\ddot{a}^*_{xn}}{H^*(x, n)}$$

where  $\mathbf{a}$  = acquisition expenses

$\mathbf{g}$  = administration expenses

In this formula we have

$$\ddot{a}_{\overline{xn}|} = \sum_{k=0}^{n-1} v^k \cdot {}_k p_x$$

$$A_{\overline{xn}|} = \sum_{k=0}^{n-1} v^{k+1} \cdot {}_k p_x \cdot q_{x+k} + v^n \cdot {}_n p_x$$

$$P_{\overline{xn}|}^{**} = \frac{1}{\ddot{a}_{\overline{xn}|}} \cdot [A_{\overline{xn}|}^{**} + Z^*(x, n)]$$

$$\text{where } A_{\overline{xn}|}^{**} = \left( \sum_{k=0}^{n-1} v^{k+1} \cdot {}_k p_x^* \cdot q_{x+k}^* + v^n \cdot {}_n p_x^* \right) \cdot C$$

$$+ \underbrace{IC}_{\text{Initial cost}} + \underbrace{CA}_{\text{Commission to the agents}} + \underbrace{\sum_{k=0}^{n-1} v^{*k} \cdot {}_k p_x^* \cdot FG_k}_{\text{Net present value of the administration costs}}$$

$$\ddot{a}_{\overline{xn}|}^* = v^{*k} \cdot {}_k p_x^*$$

$$Z^*(x, n) = \sum_{k=1}^{n-1} v^{*k} \cdot {}_{k-1} p_x^* \cdot u_{x+k-1}^* \cdot S_{x+k}$$

where  $S_{x+k}$  = surrender benefit at the end of year  $k$

In our model, we consider that the bonus  $(PE)_t$  is paid annually in deduction of the gross premium. The bonus increased yearly and is paid from the third premium.

So, we have

third premium deducted by:  $(b_1 + b_2) \cdot J(x, n)$

fourth premium deducted by:  $(b_1 + 2b_2) \cdot J(x, n)$

etc.

In the formula we have :

$$H^*(x, n) = b_1 \cdot {}_{j-1|} \ddot{a}_{\overline{xn-j+1}|}^* + b_2 \cdot {}_{j-1|} (I\ddot{a})_{\overline{xn-j+1}|}^*$$

$$= b_1 \cdot {}_{j-1} E_x^* \cdot \ddot{a}_{\overline{x+j-1n-j+1}|}^* + b_2 \cdot {}_{j-1} E_x^* \cdot (I\ddot{a})_{\overline{x+j-1n-j+1}|}^*$$

$$= b_1 \cdot v^{*j-1} \cdot {}_{j-1} p_x^* \cdot \sum_{k=0}^{n-j} v^{*k} \cdot {}_k p_{x+j-1}^*$$

$$+ b_2 \cdot v^{*j-1} \cdot {}_{j-1} p_x^* \cdot \sum_{k=0}^{n-j} {}_k p_{x+j-1}^* \cdot v^{*k} \cdot \ddot{a}_{\overline{x+j-1+kn-j+1-k}|}^*$$

where  $j$  = represents the number of premiums from which the bonus is paid

## 8 References

- [1] Bangert R. M., "Valuation of a Life Assurance Company for Purchase", Journal of the Institute of Actuaries, vol. 99 , part 2, 1973
- [2] Baumol W. J., Blinder A. S., Scarth W. M., "L'économique Principes et politiques micro-économie", pages 115-123, Edition Etudes Vivantes, Montréal, 1990
- [3] Burrows R. P. and Fickes S. W., "Measuring the Financial Performance of Insurance Companies", International Congress of Actuaries, vol 1, 1988
- [4] Burrows R. P. and Whitehead G. H., "The Determination of Life Office Appraisal Values", Journal of the Institute of Actuaries, vol 114, part 3, 1987
- [5] Chuard M., "Wert- und Erfolgsmessung in der Lebensversicherung", DGVM-Seminar Lenggries, 23.-25.3.1994, non publié
- [6] Chuard P., "Analyse du résultat et répartition de l'excédent dans l'assurance sur la vie", Institut de sciences actuarielles de l'Université de Lausanne, 1988
- [7] Collins S. A., "Counting The Wrong Beans", the Actuarial Digest, Volume 16, No. 1, Feb. /Mar. 1997
- [8] Collins S. A. and Keeler D. J., "Analysis of Life Company Financial Performance" Presented to: Staple Inn Actuarial Society on 2nd November 1993, non publié
- [9] Darbellay P.-A., "Evaluation d'une compagnie d'assurances vie et de ses produits: méthodes, approche critique et analyse de sensibilité Thèse présentée à l'Ecole des Hautes Etudes Commerciales de l'Université de Lausanne, 1999
- [10] Delvaux T. et Magnée M., "Les nouveaux produits d'assurance-vie", Editions de l'Université de Bruxelles, Collection Actuariat, 1991
- [11] Fagan C., "Profit Recognition, Reporting and Analysis in Life Assurance", Presented to Staple Inn Actuarial Society; London, January 22 1991, non publié
- [12] Gardner A. M., "G.A.A.P. Accounting for Life Offices", Transactions of the 21st International Congress of Actuaries, vol 4, 1980
- [13] Geddes T., "Recognition of Life Assurance Profits - the Embedded Value Approach", Report by Institute of Actuaries Working Party, One day seminar - The Albany Hotel, Birmingham, 10 July 1990, non publié
- [14] Gerber H. U., "Life Insurance Mathematics", Springer-Verlag, 1990
- [15] Hare D. J. P. et McCutcheon J. J., "An introduction to Profit-Testing", publication of the University Heriot-Watt, Edinburgh, U. K., 1991

- [16] Hauser H., "Profit Testing von Lebensversicherungsprodukten", Versicherungswirtschaft Heft 18/1993
- [17] Kamieniecki J., "Evaluer une compagnie d'assurances", L'argus 11 juin 1993
- [18] Lahme D., "Ertragswertanalyse-Wandel in der Tarifentwicklung der deutschen Lebens-versicherung", Versicherungswirtschaft Heft 2/1994
- [19] Levasseur M. et Quintart A., "Finance", Edition economica, Paris, 1990
- [20] Merdian G. A., "Value-Based Financial Measurement", Het Verzekerings-Archief LXX, 1993-3
- [21] Shapiro A. C., "Modern Corporate Finance", pages 250-268, Business & Economics, New York, 1991
- [22] Smart I. C., "Pricing and Profitability in a Life Office", Journal of the Institute of Actuaries, vol 104, part 2, 1977
- [23] Smith P. R. , "More Informative Disclosure of Life Office Affairs", Journal of the Institute of Actuaries, vol 99, part 3, 1973
- [24] Veraguth C., "Etude de la variabilité des méthodes d'évaluation de l'assurance vie dans un contexte stochastique", Thèse présentée à l'Ecole des Hautes Etudes Commerciales de l'Université de Lausanne, 1999
- [25] Watson R & Sons, "Life Profit Recognition-The Next Stage", Watson Comment, 1991, non publié

## Other references

- Bardola J., "Embedded Value", présentation Zurich Assurances 1996, non publié
- Berliet J.-P. and Collins S. A. , "Insurance Strategy Group : Managing Economic Value Added For Life and Property/Casualty Insurers", February 7, 1997, non publié
- Bowers N. L., Gerber H. U., Hickman J. C., Jones D. A., Nesbitt C. J., "Actuarial Mathematics", Society of Actuaries, 1986
- Collins S. A. , "Using core skills in value added analysis", the Newsletter of the Society of Actuaries, Volume 29, No. 2, February, 1995
- Edwards C., "Profit Testing in practice", Swiss Actuaries Working Session, June 5, 1997, non publié
- Hertz D. B., "Risk Analysis in Capital Investment", Harvard Business Review January-February 1964
- Hertz D. B., "Investment policies that pay off", Harvard Business Review January-February 1968
- Rohde M., "Den Erfolg im voraus bestimmen-Kennzahlengestütztes Profit Testing", Produktcontrolling und Tarifierung im deregulierten Versicherungsmarkt, 11. und 12. Dezember 1995, non publié
- Köhler M., "Embedded Value", séminaire Winterthur Assurances 1995, non publié
- Metha J. B., "Allowing for Asset, Liability and Business Risk in the Valuation of a Life Office", Journal of the Institute of Actuaries, vol 119, part 3, 1992
- Prautzsch A., "Shareholder value : Révolution ou pétard mouillé ?", Journal Bancassurance, Fév./Mars 1998
- Ross S. M., "Initiation aux probabilités", Presses polytechniques romandes, 1987
- Reed O. A., "Financial Reporting In The U.K.", A memo, October 12, 1993, non publié
- Wyss R., " Wert und Erfolgsmessung in der Lebensversicherung", DGVM-Seminar, 12. September 1996, non publié