Can Prospect Theory explain it? Home bias phenomena and the notion of risk

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Abstract

This paper attempts to provide an explanation to the "home bias puzzle" in portfolio allocation and consumption choice, using a nonstandard specification of agents' preferences, which draws from recent experimental evidence, and in particular from Prospect Theory as in Kahneman and Tversky (1979, 1991). It argues that with a specification of preferences in which the mere appearance of a shock matters, in addition to, or even instead of, the size of the shock itself, risk sharing may be useless and even counterproductive.

This may in turn help explain why in many important settings risk diversification may not be a "natural" inclination of human behaviour, enshrined in preferences. Moreover, the paper argues that the presence of "reference values" and of incomplete and asymmetric information may support such specification. This seems particularly appropriate to study consumption and investment decisions, and hence explain the emergence of (rational) home bias phenomena for these.

1 Introduction

Finance theory predicts that diversification is worthwhile - by risk sharing, a representative agent may accomplish an upward shift in the risk-return frontier and enjoy a higher level of utility. Nevertheless, a large

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body of literature has flourished on the so called "home bias puzzle": agents tend to hold financial wealth concentrated in assets issued in their own country, without exploiting the benefits of international diversification (French and Poterba, 1991; Tesar and Werner, 1992). Furthermore, a similar (and partly related) tendency has been observed in consumption, which is also expression of lack of international risk sharing due to poor diversification[1] (van Wincoop, 1994). In a recent review, Lewis (1999) argues that no thorough explanation of the observed home bias phenomena can be fully endorsed for the time being, although the literature has at least thinned the range of the possible explanations from many non viable options.

The literature on portfolio and consumption risk sharing (and lack thereof) is based on a particular specification of agents' attitude toward risk, which is in most cases the standard mean-variance approach in the expected utility framework. However, recent research, especially in the field of economic psychology and experimental economics, has shown that expected utility, and the mean-variance loss function, may be severely flawed in a number of important respects (see Machina, 1987, and Rabin, 1997, for excellent surveys). Therefore, this paper sets out to argue that a significant portion of the home bias puzzles may be explained by a nonstandard, and yet plausible and consistent with recent research, specification of agents' preferences and in particular of agents' attitude toward risk.

The analysis of this paper attempts to demonstrate that the incentives to agents to undertake risk sharing may not be necessarily "natural" and risk diversification shall not be necessarily rational. For instance, French and Poterba (1991), whilst focusing on the portfolio bias, argue that lack of diversification does not seem to be an exclusive feature of financial investment, but rather a relatively common behavior. They report that:

> The evidence of incomplete diversification presented here is consistent with evidence from many other markets. [...] Perhaps the most striking example of incomplete diversification is the tendency of most households to own residential real estate near where they work. The returns on their human and physical capital may consequently be highly correlated. This generates a much less diversified portfolio than

[1] In the following of this paper, "diversification" and "risk sharing" will be used as synomyms. Whilst the meaning of the two terms may differ, the former being more used in financial investment allocation, and the second in comsumption allocation, there is no possibility of confusion in the analyses carried out in this paper.

holding, for example, a real estate investment trust with a national estate portfolio.

A similar view is that of Tesar and Werner (1992):

> It may be that the observed lack of international diversification has less to do with "international" investment choices but simply the issue of why people in general tend not to hold well-diversified portfolios, even within a given national market.

Thus, we may want to explain not only a lack of diversification in international financial markets and a lack in consumption risk sharing, but also in other markets and situations. As a consequence, the odds are that what pushes agents to refrain from diversification is something relatively independent of the specific features of a particular decision problem, but rather linked to some general trait of human behavior, that is, related to preferences. At the same time, however, an additional goal of this paper is trying to draw a distinction between, and rationalize, those situations when risk diversification is indeed normally undertaken (e.g. typically health and life insurance) and those when in most cases it is not (e.g. portfolio allocation and consumption).

To understand the core of the argument, it should be observed first that, according to the mean-variance tradition, diversification is always the right option as long as returns (or in general the relevant stochastic processes) are not perfectly correlated. This is described in the most simple way in a Central Limit Theorem setting, where the average of a large number n of independent shocks converges to a Normal distribution with variance equal to $1/n$ the individual variances (hence the reduction in risk). Clearly, however, the Central Limit Theorem, and in general the averaging effect, do not imply that the "noise" (that is, the mass of probability of outcomes different from the mean) of a series of given stochastic processes disappears, but simply that it is shifted toward the mean. Thus, assume that agents are risk averse (in the loose sense that they dislike noise), but care about the probability of the occurrence of shocks in addition to, or instead of, the size of the shocks themselves. Then, it is easily found that they may be better off with a single independent stochastic process, than with an average of a large number of processes, which converges to the Central Limit distribution. Intuitively, it may not be true that large shocks with small probability

(negatively) matter more than small shocks with high probability (as the mean-variance approach *de facto* assumes): it may be right the opposite. In more formal terms, this paper relaxes the traditional assumption of concavity of the utility function, while holding that agents are risk averse. This directly leads to the conclusion that risk diversification may not be sought by agents characterized by such preferences.

To some extent, this paper builds on the literature on Prospect Theory (Kahneman and Tversky, 1979), and especially on the diminishing sensitivity to gains and losses in the notion of loss aversion as argued for in Tversky and Kahneman (1991). In this paper, this feature of loss aversion is named "resignation effect", because this term more intuitively conveys the concept that agents, as they learn that a shock has taken place, are less concerned about the magnitude of the shock itself. Kahneman and Tversky report that such a specification of agents' attitude toward risk is supported by a large body of experimental evidence; very few applications, however, have been attempted with problems outside the laboratory (Benartzi and Thaler, 1994, being a notable exception). Therefore, given the non trivial leap implicit in applying experimental findings to real world problems, the application of Prospect Theory to the home bias puzzles argued for in this paper should not be seen as obvious nor safe. In fact, laboratory experiments may be conducted in settings, which importantly differ from real world situations. As a consequence, this paper also tries to provide additional insights and analysis, in order to demonstrate why loss aversion and diminishing sensitivity to gains and losses (resignation effect) make sense to model agents' preferences in some important situations. In particular, additional insights are given for portfolio allocation choice problems, where imperfect information and/or information asymmetries might lead to justify a specification of loss function, which resembles loss aversion. Moreover, the presence of "reference values" or "focal points" (Thaler, 1980), like the *status quo* bias as in Samuelson and Zeckhauser (1998), is called for in the case of consumption choice.

The main conclusion out of this study is that the incentives to risk diversification are not a "natural" feature embedded in agents' preferences and the lack of risk sharing is not necessarily the result of irrational behavior. Instead, such incentives depend on the specific features of the problem agents face, and the observed behavior may be used as an *ex post* check on the nature of preferences. This seems a mere application of the Occam's razor: the simplest solution has the greatest likelihood to be the true one. As a trivial example, suppose your kid does not eat apples: the first conclusion you reach is that they do not enter positively in his utility function, so that he rationally chooses to eat, say, bananas

instead. Similarly, in this paper a conclusion is drawn in this paper on the structure of preferences of agents in portfolio allocation and in consumption decision problems, given the stylized fact that they often abstain from risk diversification. An important consequence is that the large weight given in the mean-variance literature to the magnitude of the shocks, as opposed to the probability that *any* shock occurs, may be flawed. Moreover, if the speculations of these paper have some truth, the whole notion of risk and the theory on how to deal with risk could be importantly affected.

The paper is organized as follows. In Section 2 the main argument of the paper is introduced with a simple example; the analysis is made slightly more complicated and general in Section 3. The plausibility of the specification of preferences such as those proposed in Sections 2 and 3 for portfolio allocation and consumption choice is discussed in Section 4. Finally, Section 5 contains some concluding remarks.

2 A simple example of resignation effect: risk diversification does not pay

Suppose some scientists wish to send a rocket on the moon and there is a relevant risk that the rocket may miss the target and go lost in space. For instance, let x be the stochastic process representing the result of the rocket reaching the moon ($x = 0$) with probability $P > 0$, or not reaching the moon ($x \neq 0$) with probability $1 - P > 0$. For simplicity, it is assumed that the stochastic variable x has zero mean (it does not matter if the moon is missed left or right). Suppose further that the scientists pay a cost of C if the rocket misses the moon. We assume that they evaluate uncertain outcomes on the basis of expected utility. Therefore, the expected loss of this undertaking will be:

$$E(U) = CP(x \neq 0) = C(1 - P) \tag{1}$$

Now the same scientists are given the chance that the stochastic process driving the result of the launch of the rocket may be the average of a large number n of independent stochastic processes x_i, all distributed identically to the process x. Call this $Y = \sum x_i/n$, which – according to the Central Limit Theorem – converges to a Normal distribution with zero mean (because all the x_i have zero mean) and variance σ_x^2/n.

Clearly, this is the situation when, according to the mean-variance literature, diversification is a free lunch, because the average process Y has a smaller variance than any x_i, while maintaining the same mean.

However, the probability that a rocket driven by the stochastic law Y reaches the moon is actually zero. Therefore, the expected loss of the scientists is:

$$E(U) = CP(x \neq 0) = C > C(1 - P) \tag{2}$$

As a consequence, risk sharing results in a net loss for our scientists.

The example above[2] is representative of a tendency we refer to as the "resignation effect". This effect arises when the disutility coming from a shock to some reference point rises *less* than proportionally with the magnitude of the shock. In the extreme case, like that of the scientists mentioned before, the magnitude does not matter altogether, as only the probability of the misses is relevant. In other, more intermediate cases, the size of the innovation will be important, but the agent will suffer proportionately less (hence the term "resignation").

To illustrate it in formal terms, take two stochastic processes X and Y. Suppose that the first is a lottery which on average gives a certain payoff π, but with an oscillation of ϵ up and down occurring with a probability of P. Suppose also that the second process has exactly the same probability distribution and is also independent of the first.

Now take any convex linear combination $Z = \theta X + (1-\theta)Y$ with any finite θ. For instance, if the considered problem is a portfolio allocation choice, X and Y may represent the returns of two financial assets and Z is the return on a portfolio with weights θ and $1 - \theta$ to X and Y respectively.

It is assumed here that the value function (Kahneman and Tversky, 1979), in this case a loss function, takes the form:

$$L = \lambda \| Z - \pi \|^{\phi}, \phi \geq 0 \tag{3}$$

[2] The example of the rocket to the moon may seem *ad hoc*, but it does not appear to be a very special case. Many other similar examples may be found. For instance, take a politician, whose objective is the win the next elections. The outcome of the elections depends on the result of this policy in the term, which should excess a certain benchmark. Thus, our politician cares about *any* deviation in the outcome of his policy, irrespective of its size. Similarly, a theorist may care about any deviation of his theory from reality, irrespective of how important the deviation is. Finally, arriving late may be costly (e.g. in terms of reputation) *per se*, in addition to, and even without relation with, the magnitude of the delay. In all these cases, the expected loss function will be very similar to that of the rocket to the moon.

Where $\|$ stays for absolute value, ϕ is a nonnegative real scalar and λ is a strictly positive real scalar. An important feature of this loss function, unlike that in Tversky and Kahneman (1991), is that gains and losses are treated symmetrically. This captures, intuitively, the notion that any deviation from the reference value π entails a cost, as in the simple example above of the rocket to the moon. However, all the results of this analysis and this paper could be generalized allowing an asymmetric treatment of gains and losses in the value function (for instance, inserting a λ_G for gains and a bigger λ_L for losses as Tversky and Kahneman, 1991, did). It is important to notice that, given that $\lambda > 0$, a representative agent with the loss function in (3) is always characterized by risk aversion, because she would turn down any lottery against its expected value. This would hold even if $\phi < 0$, although such specification would lead to the – highly not intuitive – feature that the bigger the shocks, the smaller the loss associated to them; we therefore stick to a nonnegative ϕ.

The "prospective loss" of a lottery Z is expressed in a very general form, which encompasses, but not necessarily assumes, expected loss. This resembles the prospective utility function as in Kahneman and Tversky (1979):

$$PL = \int \Phi(Z)\lambda \|Z - \pi\|^{\phi} \partial Z \tag{4}$$

Where $\Phi(Z) = F(P(Z))$ is a whatever function F of the probability distribution of the Z, such that its values are always nonnegative. When F is the identity function ($F(P(Z)) = P(Z)$), the traditional expected loss (or expected utility) formulation is recovered. We allow for a more general specification here, given the frequent violations of the expected utility in experimental studies (see Rabin, 1997). For instance, overweighting of small probabilities with respect to the expected utility is a relatively common feature of the literature on Prospect Theory.

Some further remarks on the prospective loss as in (4) may be useful. Clearly, such specification mimics the loss function of the scientists of the example with $\phi = 0$; in this case, the prospective loss function collapses to:

$$PL = \int_{Z \neq \pi} \Phi(Z) \partial Z \tag{5}$$

The integral is defined only over the region of the probability space where $Z \neq \pi$. With $\Phi(Z)$ equal to $P(Z)$ and λ to C, the formulation as

in (1) is exactly recovered. More in general, when ϕ is strictly positive, the term $\|Z-\pi\|^{\phi}$ will appear in the prospective loss, and its importance will grow with ϕ. Thus, when ϕ approaches zero, the relative importance of the size of the shock (captured by $\|Z-\pi\|^{\phi}$) with respect to $\Phi(Z)$, for $Z \neq \pi$, will decline. The "resignation effect" will therefore prevail. The opposite happens when ϕ goes to infinite: the importance of the size of the shocks will overcome that of the mere occurrence of it (that is, of expression (5)). Given that, if $0 \leq \phi < 1$, the importance of the size will grow at a pace slower than that of the size itself, we technically refer to a resignation effect only in this case. In the standard mean-variance case, λ is the Arrow-Pratt coefficient of risk aversion, $\phi = 2$ (no resignation effect exists) and $\Phi(Z) = P(Z)$.

Let us turn to our decision problem. Agents want to minimise their prospective loss for their control variable θ:

$$Min_{\theta} PL = \int \Phi(X,Y)\lambda\|\theta X + (1-\theta)Y - \pi\|^{\phi}\partial X \partial Y \tag{6}$$

From the assumption of independence and identical distribution of X and Y, the joint probability distribution $P(X,Y)$ is given by:

	$\mathbf{X} = \boldsymbol{\pi}$	$\mathbf{X} = \boldsymbol{\pi} \pm \boldsymbol{\epsilon}$
$\mathbf{Y} = \boldsymbol{\pi}$	$(1-P)^2$	$P(1-P)$
$\mathbf{Y} = \boldsymbol{\pi} \pm \boldsymbol{\epsilon}$	$P(1-P)$	P^2

Thus, the prospective loss may be rewritten as:

$$PL = F(P(1-P))(\theta\epsilon)^{\phi} + F(P(1-P))((1-\theta)\epsilon)^{\phi} + K \tag{7}$$

Where K is a constant term (that is, it is not a function of θ: the intuition is that when both process are shocked or when neither of the two is, then the outcome is independent of θ).

The first order condition of the above minimization problem is:

$$\partial PL/\partial\theta = [F(P(1-P))]\phi\epsilon(\theta\epsilon)^{\phi-1} - [F(P(1-P)]\phi\epsilon((1-\theta)\epsilon)^{\phi-1} = 0 \tag{8}$$

Then, after very simple algebra, the value θ^* satisfying the first order condition may be found as:

$$\partial PL/\partial\theta = 0 \Leftrightarrow \theta^* = 1/2 \tag{9}$$

Therefore, the first order condition identifies a unique global solution, which in turn corresponds to complete risk diversification. For instance, if X and Y represent stochastic processes driving returns of two financial assets, and if our agent wants to allocate wealth optimally between the two, the first order condition suggests her invest in the two in equal parts. This is also the finding of the mean-variance literature (as early as in Markowitz, 1952).

To check the optimality of the solution of the first order condition, the second derivative is easily calculated as:

$$\partial^2 PL/\partial\theta^2 = [F(P(1-P))]\phi\epsilon^2(\phi-1)(\theta\epsilon)^{\phi-2}+[F(P(1-P))]\phi\epsilon^2(\phi-1)((1-\theta)\epsilon)^{\phi-2} \tag{10}$$

And, for $\theta^* = 1/2$, it is finally found:

$$\partial^2 PL/\partial\theta^2 < 0 \Longleftrightarrow \phi > 1 \tag{11}$$

$$\partial^2 PL/\partial\theta^2 = 0 \Longleftrightarrow \phi = 1 \tag{12}$$

$$\partial^2 PL/\partial\theta^2 > 0 \Longleftrightarrow \phi < 1 \tag{13}$$

As a consequence, the complete diversification solution coming out of the first order condition is a minimum only if $\phi > 1$; if $\phi = 1$ the first order condition identifies a saddle, whereas and if $\phi < 1$ it finds a maximum instead of a minimum. Given that the two stochastic processes are independent and identically distributed, and given the fact that only positive shocks are allowed in this model, we may restrict the search for maxima and minima in the interval $0 \leq \theta \leq 1$. The intuition here is that, for instance, short sales in portfolio allocation are not allowed nor helpful for our agent. Given that the prospective loss function is continuous and differentiable, the minimum in the case of $\phi < 1$ will be reached at the extremes of the interval, that is either $\theta^* = 1$ or $\theta^* = 0$. This situation corresponds to risk concentration.

We then come to the following:

Proposition 1 *Given two stochastic processes X and Y, independent and identically distributed, with possible outcome either the expected value π with probability P, or the expected value plus or minus some shock ϵ with probability $1-P$, and given a prospective value function as in (4), then risk diversification ($\theta = 1/2$) is a minimum if $\phi > 1$, it is a saddle if $\phi = 1$, it is a maximum if $\phi < 1$. In the latter case (resignation effect), the optimum is reached at either $\theta = 1$ or $\theta = 0$ (risk concentration).*

For $\phi > 1$ the prospective loss function is a parabolic with minimum at $1/2$ and maximum in 0 and 1. If $\phi < 1$, the function becomes an inverse parabolic, with a maximum in $1/2$ and a minimum in 0 and 1. The steeper the slope of the function in both cases (for $\phi \to \infty$ and for $\phi \to 0$ respectively), the greater the loss in selecting risk concentration and risk diversification in the two cases respectively. We have therefore the following:

Corollary 2 *The incentives to risk diversification rise if $\phi \to \infty$; conversely, the incentives to risk concentration rise if $\phi \to 0$. The latter incentives are reach a maximum at $\phi = 0$. This result holds whatever function F is used to determine the weights in the prospective loss function.*

This corollary states that the price to pay to forgo risk diversification grows as long as ϕ goes to infinite, but exactly the opposite holds if ϕ goes to zero. This result is independent on the weights $\Phi(Z)$, that is on the particular function used in the prospective loss. This property gives a remarkable degree of generality to the results in Proposition 1.

Qualitatively, the main conclusion stemming from the analysis above is the relation between the resignation effect and the incentives to risk concentration. The more pronounced the former, the stronger the latter (and the more likely abstaining from risk diversification). Conversely, with $\phi > 1$, risk diversification recovers its optimality (for example, in the mean-variance specification, which is a very particular case of the function in (4)). Therefore, this analysis sets out to argue that a justification of the observed lack of risk sharing may, and ultimately *must*, lie in the prevalence of some form of resignation effect in agents' preferences. Whilst the predominance of the mean-variance specification (which assumes $\phi = 2 > 1$) in the literature suggests that an "alternative" specification with $\phi < 1$ is somewhat nonstandard, there are nevertheless contributions in the opposite direction. Most notably, Tversky

and Kahneman (1991) estimated the parameter ϕ, albeit in a specification of preferences not exactly equal to (4), at 0.88, a value at which the incentives are slightly against risk diversification (that is, slightly in favor of risk concentration, either $\theta = 0$ or $\theta = 1$). At a minimum, the finding of Tversky and Kahneman, if taken together with the analysis in this paper, is suggestive that Prospect Theory, and loss aversion, hold the potential to contribute to the explanation of the observed home bias phenomena, although with the necessary caution and qualifications (which will be discussed in more detail in the following).

3 A more general case

In this section a slightly more complicated and general case is studied, when the two considered processes are not independent. Therefore, the optimal allocation will not be necessarily symmetrical, and will in principle differ from the polar cases of complete risk concentration ($\theta = 0$ or $\theta = 1$) or complete risk diversification ($\theta = 1/2$). While this setting is somewhat more complicated than that in the previous section, the conclusion is nonetheless the same, that is, incentives to risk diversification exist only if $\phi > 1$. For the sake of simplicity, the analysis maintained a bivariate specification, which is anyway complex enough to capture all the relevant aspects of the problem.

Take X and Y as in the previous section, both having as possible outcomes either the expected value π or a shock ϵ, which is treated symmetrically in the loss function (both positive and negative shocks being losses for our representative agent). As before, the agents must choose an optimal allocation θ and $1 - \theta$ between the two stochastic processes, the loss function being as in (3), and the prospective loss as in (4). Here it is also assumed that the joint probability distribution $P(X, Y)$ is more general than in the previous section:

	$\mathbf{Y} = \boldsymbol{\pi}$	$\mathbf{Y} = \boldsymbol{\epsilon}$
$\mathbf{X} = \boldsymbol{\pi}$	$P_{\pi\pi}$	$P_{\pi\epsilon}$
$\mathbf{X} = \boldsymbol{\epsilon}$	$P_{\epsilon\pi}$	$P_{\epsilon\epsilon}$

With $P_{\pi\epsilon}$ not necessarily equal to $P_{\epsilon\pi}$, so that the probability distribution may not be symmetrical. (Of course, it must also hold that $P_{\pi\pi} + P_{\pi\epsilon} + P_{\epsilon\pi} + P_{\epsilon\epsilon} = 1$). As before, we assume that agents weigh probabilities through a function F so that the weights in the prospective loss will be $\Phi_{\pi\pi} = F(P_{\pi\pi})$ and so on with the other three combinations. All the observations made in the previous section also apply here: the

only difference is that the assumption of independence of the two processes is relaxed, so that a more general distribution prevails. This seems particularly reasonable as, in actual situations, the joint probability distribution will have a general form and the hypothesis of independence will not hold.

We then move to compute the first order condition with respect to θ (for the sake of simplicity the expected value π is assumed to be zero, and the shocks ϵ always taken with the positive sign):

$$\partial PL/\partial\theta = \int \Phi(X,Y)\lambda\partial/\partial\theta\{(\theta X + (1-\theta)Y)^{\phi}\}\partial X\partial Y = 0 \qquad (14)$$

We can clearly neglect the states of the world in which both processes are π and ϵ, as the allocation (that is, the parameter θ) necessarily does not matter on the outcome and the value of the loss function. Therefore, it is possible to look only at the case when either $X = \pi$ and $Y = \epsilon$ or $X = \pi$ and $Y = \epsilon$. The first order condition is then obtained as:

$$\Phi_{\epsilon\pi}\epsilon\phi(\theta\epsilon)^{\phi-1} - \Phi_{\pi\epsilon}\epsilon\phi((1-\theta)\epsilon)^{\phi-1} = 0 \qquad (15)$$

Since $\epsilon > 0$ and $\phi > 0$, after some simple algebra it is obtained:

$$\theta^* = \Phi^{\alpha}_{\pi\epsilon}/(\Phi^{\alpha}_{\pi\epsilon} + \Phi^{\alpha}_{\epsilon\pi}) \qquad (16)$$

Where $\alpha = 1/(\phi - 1)$. For a finite first order condition to exist, it must hold that $\phi \neq 1$, in which case the prospective loss is completely flat as a function of θ. The intuition behind the above result is that the share allocated to the first stochastic process should depend *positively* on the probability that it the process X is not shocked when the other process is, and *negatively* on the probability of the opposite case. Clearly, moreover, the probabilities are not taken directly, but weighted through the function F. This, again, resembles the insurance provided by statistical averaging in the mean-variance literature, as also found for the first order condition in the previous section. In fact, if $P_{\pi\epsilon} = P_{\pi\epsilon}$, the general case studied here collapses to the simplest risk diversification possible, that is $\theta^* = 1/2$.

Coming to the assessment of the optimality of the first order solution, it may be noted that the second derivative of the prospective loss is given by:

$$\partial'' PL/\partial^2\theta = P_{\epsilon\pi}\epsilon^2\phi(\phi-1)(\theta\epsilon)^{\phi-2} - P_{\pi\epsilon}\epsilon^2\phi(\phi-1)((1-\theta)\epsilon)^{\phi-2} \quad (17)$$

Which is greater than zero if and only if:

$$\Phi^{\beta}_{\pi\epsilon}/(\Phi^{\beta}_{\pi\epsilon} + \Phi^{\beta}_{\epsilon\pi}) < \Phi^{\alpha}_{\pi\epsilon}/(\Phi^{\alpha}_{\pi\epsilon} + \Phi^{\alpha}_{\epsilon\pi}) \quad (18)$$

Where $\beta = 1/(\phi - 2)$.

This condition, if $\Phi^{\beta}_{\pi\epsilon}$ and $\Phi^{\beta}_{\pi\epsilon}$ are strictly positive, leads immediately to $\phi > 1$. Moreover, again as in the previous section, the second derivative is zero at $\phi = 1$, and it is negative (indicating a maximum prospective loss) with $\phi < 1$. Furthermore, the minimum is found at the extremes of the interval ($\theta = 0$ or $\theta = 1$) in the latter case.

Therefore, the following proposition is obtained:

Proposition 3 *Given two stochastic processes as X and Y, identically distributed, with possible outcomes either the expected value π with probability P, or the expected value plus or minus some shock ϵ with probability $1-P$, with any joint probability distribution $P(X,Y)$, with $P(X = \pi, Y = \epsilon)$ and $P(X = \epsilon, Y = \pi)$ both strictly positive, and a prospective loss function as in (4), then the optimal allocation is given by:*

$\theta^* = \Phi^{\alpha}_{\pi\epsilon}/(\Phi^{\alpha}_{\pi\epsilon} + \Phi^{\alpha}_{\epsilon\pi})$ *iff* $\phi > 1$;

Any θ *if* $\phi = 1$;

$\theta^* = 0$ *or* $\theta^* = 1$ *iff* $\phi < 1$.

A visual inspection of the prospective loss in the interval $[0, 1]$ reveals that it is still a parabolic if $\phi > 1$,with a minimum in $\Phi^{\alpha}_{\pi\epsilon}/(\Phi^{\alpha}_{\pi\epsilon} + \Phi^{\alpha}_{\epsilon\pi})$ and maxima in 0 and 1. It is therefore skewed in the direction of the minimum. Conversely, when $\phi < 1$, the function is an inverse parabolic with a maximum in $\Phi^{\alpha}_{\pi\epsilon}/(\Phi^{\alpha}_{\pi\epsilon} + \Phi^{\alpha}_{\epsilon\pi})$ and a minimum in 0 and 1. The interesting feature of this result, which is very general as it applies to any function F used to calculate the weights of the prospective loss, is that the joint probability distribution of (X, Y) does matter only in the case when $\phi > 1$. When $\phi \leq 1$, the joint distribution does not matter

instead (only risk concentration is optimal, irrespective of $P_{\pi\epsilon}$ and $P_{\pi\epsilon}$); if $\phi = 1$ any allocation is equally worth.

In both cases, whether the function is a parabolic or an inverse parabolic (excluding the case with $\phi = 1$), the slope of the curve is steeper, the more ϕ goes toward infinite or zero. We have therefore:

Corollary 4 *The incentives to risk diversification grow for $\phi \to \infty$; those for risk concentration rise if $\phi \to 0$. Such incentives reach a maximum at $\phi = 0$.*

This is the same result obtained in the previous section. Therefore, the only change implied by the probability distribution being more general than in Proposition 1 is where the optimum is located with $\phi > 1$. An intriguing consequence of this is that, *ceteris paribus*, preferences with no resignation effect ($\phi > 1$) will give rise to higher computational costs, as the optimum will require the knowledge of $P(X, Y)$. In the opposite case, when a resignation effect prevails ($\phi < 1$), the allocation problem is very simple: choose $\theta = 0$ or $\theta = 1$. This latter consideration may strengthen the likelihood that agents indeed have preferences with resignation effect, which allow them save in computation and information gathering costs.

Finally, it is rather straightforward to extend the above framework to a more general n-variables setting. Let $X_1, ..., X_n$ a vector of stochastic processes, again with two possible outcomes: the expected or "reference" value π and a shock (positive or negative) ϵ. Let $P_{1,...,n}$ be the joint probability distribution of the 2^n possible outcomes, with obviously $\sum P_{1,...,n} = 1$. Furthermore, let our representative agent have a prospective loss function defined as:

$$PL = \int \lambda F(P_{1,...,n})(\| \sum \theta_i X_i - \pi \|^{\phi} \partial X_1 ... \partial X_n$$

And assuming that the agents wants to minimise the prospective loss so as to achieve the optimal allocation θ_i, for $i = 1, ..., n$. In this case, the calculation of the first and second derivatives with respect to θ_i would be rather problematic, as it would involve the sum of $2^n - 2$ possible outcomes (that is, all possible outcomes, but the case of all processes being equal to either π or ϵ). The analytical solution to this problem is not available in this general setting, because it involves the sum of a number of polynomials greater than two. However, results may be easily derived with numerical methods, once hypotheses on the structure of the joint probability distribution are set down.

4 Is Prospect Theory relevant in consumption and portfolio decisions?

The analysis in previous sections showed, in essence, that the more agents about the mere appearance of a shock (low ϕ), the less incentive they have to undertake risk diversification. In particular, if $\phi < 1$, risk sharing turns out to be counterproductive. Instead, when ϕ grows to infinite, the importance of the size of the shock in addition to its appearance starts to grow, and as a consequence risk diversification becomes increasingly appealing. This finding is independent on the system agents have to weigh probabilities (e.g. it is independent on the expected utility theory to hold or not), provided that weights and values may be separated – as in every utility function studied so far in the literature.

Whilst clear in analytical terms, the above finding must be cast in agents' actual preferences to have sense in explaining real world behaviors. An intriguing feature of the results of this paper is that they hold the potential to be able to explain why in certain circumstances risk diversification is undertaken, and in other circumstances it is not (so that risk concentration is preferable). The explanation put forward is very simple: when the appearance of a shock counts more than its size, diversification is not undertaken; when the size of the shock matters more, incentives to diversification arise. Therefore, the further step of this analysis is to explain why in those settings, in which diversification is normally not undertaken, the sole occurrence may matter more than the magnitude of a shock in agents' preferences.

We restrict our analysis to consumption and investment allocation choices where, as already mentioned in the introduction, there is a lot of evidence in the literature of lack of risk diversification (van Wincoop, 1994; Lewis, 1999). A cornerstone of loss aversion and of Prospect Theory, as for instance in the works of Kahneman and Tversky, is the existence of "reference values" which, if missed because of a shock, cause strain to agents *per se* (that is, independent of the size of the deviation). The presence of reference values, or focal points, seems fully consistent with the argument made above, that agents may care of the occurrence of a shock in addition to, and perhaps even more than, the magnitude of the shock itself. This concept, in turn, may be quite directly applied to consumption plans, for which there is already a literature on "keeping up with the Joneses" and habit formation (Constantinides, 1990; Campbell and Cochrane, 1991). However, unlike in the literature on habit formation, the presence of reference values, which are costly to deviate from, are modelled in this paper as diminishing sensitivity to gains and losses (or resignation effect), rather than typically as increased risk aversion.

The conclusion is that consumption choices seem a very good candidate for a specification of preferences, which is against risk diversification. This, *prima facie*, may represent an interesting and sensible solution to the lack of consumption risk sharing often found in the literature, from the point of view of Prospect Theory. Agents endowed with preferences as in equations (3) and (4) with $\phi < 1$ over consumption will not put in place, and make recourse to, (consumption) risk sharing mechanisms.

Things seem to be less clear-cut for financial investment decisions, where the presence of strong reference values appears more difficult to justify. To a certain extent, since financial investment is a delayed consumption decision, the same arguments applied for consumption could also be of relevance here. However, the reasons put forward for current consumption seem much less compelling for portfolio choice, both because habit formation and "keeping up with the Joneses" may have less relevance if shifted in the far future, and because a reference value for the short-term outcome of investment decisions may not be related to the long-term implications in terms of future consumption over the distant future. For instance, investing in the stock market may carry out the best rate of return at low risk in a long (consumption) horizon, but agents do not appear to be such far-sighted (Benartzi and Thaler, 1994). Thus, it appears that consumption and investments decisions should be treated separately, and the arguments put forward for the former may not be easily translated to the latter.

Nevertheless, it is argued here that another factor may contribute to justify a specification of preferences with $\phi < 1$ in the case of financial investment: the presence of information incompleteness and asymmetries. The existence of the former implies that agents do not have full knowledge of the stochastic process driving returns; the latter that it exists an agency problem between a principal, with limited information, and an agent, endowed with full information on the data generating process of asset returns (DGP).

Assume that the investor is unaware of the DGP of the n stochastic processes driving the returns on the financial assets available on the market. Furthermore, this investor is not aware of the n -dimensional joint probability distribution of returns. Therefore, this investor ("principal") gives mandate to a portfolio manager ("agent") to manage some amount of money. Clearly, this leads to a standard agency problem with asymmetric information.

We also assume that the mandate given to the agent is subject to an assessment after one period and may be removed if the result is not "satisfactory". The possibility of annulling the contract stems from the hypothesis that the principal is also uncertain over the quality (e.g. the

professional skills and the moral integrity) of the agent. Therefore, the principal uses the outcome of the investment, the details of which are obviously chosen by the portfolio manager, to infer about the quality of the agent and therefore take a decision about the continuation or the rescission of the portfolio management contract. The situation described in the model appears to be highly plausible for many real world situations, in which investors are uncertain both about returns and about the quality of their intermediaries.

In formal terms, it is assumed that in the initial period the principal has an *a priori* probability distribution about the quality of the intermediary. For simplicity, we assume that such probability distribution is a Normal centered on zero and with a finite variance σ_Q^2:

$$P(Q) = N(0, \sigma_Q^2) \tag{19}$$

At this stage, the investor is "neutral": the expected value of the quality is zero. We may interpret this value as a "benchmark" on the average quality of portfolio managers: if the true value of Q exceeds zero, then the portfolio manager chosen by our investor is superior to the average, and the opposite is true when Q is negative.

In a second stage, the principal looks at the result of the investment to infer about the quality of the agent. The outcome Z of the experiment, which is the excess return over the "benchmark" obtained on the delegated portfolio, is perceived to be governed by the (stochastic) law:

$$Z = \mu Q + \epsilon \tag{20}$$

Where ϵ is a zero mean, distributed Normal, with variance (as perceived by the principal) σ_ϵ^2 white noise process, and μ a strictly positive real scalar. This return may be risk-adjusted: in this case the μ perceived by the agent (for instance on the basis of historical evidence) will be dependent on the required riskiness of the portfolio, which may even be the variance as in the standard specification. Intuitively, the principal perceives that *ex post* returns will be an indication of the (unobservable *ex ante*) quality of the agent, although the signal she receives from actual returns has some noise (the subjective importance of which is given by the variance of ϵ).

Let us turn now to the agent, that is, the portfolio manager. We assume that a menu of n stochastic processes driving returns is available to him: a set $X_1, ..., X_n$ with the same features of the processes in the previous sections: assuming that the manager will be at the benchmark

$Q = 0$ (which will happen on average), the returns of every X_i will be either zero or a (negative) shock ϵ. Then, the portfolio manager chooses a linear combination of the X_i such as $\Gamma = \sum \theta_i X_i$, with θ_i representing the chosen allocation.

The *ex post* probability distribution is perceived by the principal as:

$$P(Q/Z) = N((Z/\sigma_\epsilon^2)/(\sigma_Q^{-2} + \sigma_\epsilon^{-2}), 1/(\sigma_Q^{-2} + \sigma_\epsilon^{-2})) \tag{21}$$

The logic of this latter passage is as follows. After having observed Z, the principal switches to the *ex post* probability distribution, using the Bayes formula. She therefore learns in a Bayesian fashion about the quality of the agent and updates her belief (hence her participation to the contract) accordingly. Intuitively, the weight attributed to the actual return Z will be an inverse function of the variance of the observational error ϵ, as well as a positive function of the variance of the *a priori* probability distribution on the quality of the portfolio manager.

We can assume that the principal may quit the contract, for example, if the value $V = (\ Z/\sigma_\epsilon^2)/(\sigma_Q^{-2} + \sigma_\epsilon^{-2})$, which is the mode and the mean of the above *ex post* probability distribution, overcomes a multiple of the variance of the same distribution, a value $S = -a/(\sigma_Q^{-2} + \sigma_\epsilon^{-2})$, with a a positive real scalar. Clearly, the problem could be more complicated in a multi-period setting: in this case, the advantage for the agent to quit the contract, provided that the manager is of poor quality $Q < 0$, should be weighted by the advantages to gain more information as new observations become available. Since the core of the argument is not changed with the introduction of this complication, this issue is left aside, albeit being a plausible feature of real world situations.

In the point of view of the portfolio manager, *ex ante*, the condition of the principal quitting the contract (which in turn he seeks to avoid) may be expressed as:

$$Z < Z^* = -a\sigma_\epsilon^2 \tag{22}$$

Assuming that the manager has full knowledge of this condition, which is exogenous to him (a and σ_ϵ^2 being both chosen or perceived by the principal), and that he is only interested to the continuation of the delegated portfolio management contract (e.g. because of the trading fees we draws from it) it is straightforward that his loss function will have the following specification:

$$L = \rho P(\Gamma < Z^*) \tag{23}$$

Where ρ is a strictly positive real scalar. Intuitively, the portfolio manager cares about the probability of the contract being quitted by the principal. As a consequence, he wishes to minimise the probability that the outcome of the investment is poorer than a certain threshold Z^*, on which he has no control. Thus, his loss function very much resembles that of the scientists in section 2. He only cares of the actual return not overcoming a certain threshold, which is given to him and acts as salient or "reference" value as in the literature on Prospect Theory. Therefore, he is indifferent to the size of the shock, as long as the overcoming of the threshold is not affected. It is on these grounds, that he chooses the actual portfolio allocation, which – as discussed at length previously – will not in general lead to risk diversification, but rather to risk concentration. In fact, in his loss function, for shocks overcoming the reference point, ϕ will be equal to zero.

Clearly, this model is very stylized and it is evidently not able to capture the complexities of actual financial investment with asymmetric and limited information. However, it appears *prima facie* robust to some complications. For instance, in the case of a multi-period setting, when the contract between the principal and the agent is updated after $n > 1$ periods, the threshold will be calculated on the basis of the n observations, making it less demanding for the agent to deviate in a single case. On the other hand, however, the threshold will be more demanding as the uncertainty in ϵ wanes along with the increasing number of available observations. Furthermore, although the portfolio manager will be less constrained not to overcome a certain threshold, if he does not want to lose the trust of the principal, he will still be forced to do so on average, and this may again lead to – given the presence of transaction costs which impede a continuous reshuffling of the portfolio weights – risk concentration.

Another criticism to the above argument of the presence of a "resignation effect" in financial investment might be that such severe information asymmetries and complete delegation not always take place in real world situations. The principal might always exert some degree of control on the portfolio allocation strategies of the manager, and even manage savings herself. Whilst the relevance of this counter argument is left to the speculation of the reader, it may be noted that it is not *per se* able to rule out the presence of a resignation effect altogether. For instance, even if the portfolio manager (the agent) is the principal herself, she may act as if the "self" managing the portfolio allocation were a different person or entity, again judging results and the quality of the

investment strategy on the basis of some quantitative threshold. Thus, it cannot be ruled out that, even in this case, strategies characterized by risk concentration will tend to prevail over time.

5 Conclusions

A central tenet of economic theory is that people dislike risk, so that they are willing to turn down some expected return in order to be insured against it. But what measure of risk? Clearly, risk is a multifaceted phenomenon that it is highly unlikely to be captured by a single, simple indicator. This paper, rather than defining risk in general terms, has focused on the features of the notion of risk (in agents' prospective utility) which may or may not lead to risk diversification. By risk diversification and lack thereof (risk concentration), we are clearly not referring to hedging: in that case, no ambiguity exists as, for instance, one process may be used to *offset* shocks in another. Indeed, the core of the literature on risk diversification does not refer to hedging, but rather to a "statistical averaging" effect; this is in part determined by the empirical observation that many relevant stochastic processes, as for instance the returns on different financial assets, are positively, albeit not perfectly, correlated. This study has focused on the choice of a representative agent, to whom a set of independent, or positively but not perfectly correlated, identically distributed stochastic processes is made available. Which allocation will she select? Economic theory, from Markowitz (1952), predicts that she will choose an allocation so as to minimise the variance of the possible outcomes, and this in turn leads to risk sharing: do not put all eggs in the same basket. Whilst clear in theory, this prediction is regularly rejected by the empirical evidence, which shows that in many important allocation choices, agents refrain from diversification. For instance, this has been observed in financial investment (Tesar and Werner, 1992) and in consumption choice (van Wincoop, 1994).

This paper sets out to provide an explanation of the observed deviation from the behavior predicted by theory by relying on Prospect Theory and of loss aversion, which have received considerable experimental support in recent years (Kahneman and Tversky, 1991). In short, the argument of this paper is that whenever agents fear the mere occurrence of a shock, in addition to or instead of, the size of the shock itself, they find risk diversification counterproductive, and prefer to stick to risk concentration: put all the eggs in a basket, so that you will minimise the probability that *anything* unforeseen will happen. It has also shown that the more agents care about the occurrence of the shock vis-à-vis the

size, the greater are the incentives to abstain from risk diversification, and vice versa. Therefore, if only the occurrence of a shock matters, the incentives to risk concentration are the largest.

The existence of a tendency to put value to the occurrence of a shock in itself (which this paper referred to as the "resignation effect") is clearly not obvious, and the limited experimental evidence reported for instance in Kahneman and Tversky (1991) does not seem sufficient to safely warrant it. Therefore, this paper has also discussed at some length the conditions, under which the existence of a resignation effect may be considered plausible, and therefore explain the emergence of rational risk concentration phenomena. Resignation effects may mainly stem from the existence of focal points or "reference values" on which agents coordinate expectations. Thus, a deviation from these reference values may be costly *per se*, if any in emotional terms, independent on the quantitative amount of the deviation itself. In some problems, the emergence of reference values is a fundamental element of the problem. In an example proposed here, some scientists want to send a rocket on the moon. Clearly, the simple fact of missing the moon is costly for them, independent of the entity or direction of the miss. In other contexts, the emergence of resignation effects is not immediately obvious and should be qualified. This paper has sought to demonstrate that in two important settings, namely financial investment and consumption, the emergence of a resignation effect is likely. In the latter case, the existence of reference values is a direct consequence of habit formation and "keeping up with the Joneses" popular in the literature (Campbell and Cochrane, 1991). In the case of financial investment, the issue is apparently more complicated because the presence of reference values seems less straightforward. However, to a closer look, reference values may also arise in this context as agents have limited information on the stochastic processes generating returns so that they use "rules of thumb" to infer about the quality of their own investment strategies or of those of the portfolio managers who they have delegated to.

In other fields, like health and life insurance, agents seem to undertake risk diversification, evidently because the size of the shock dominates over the occurrence of the shock itself. Therefore, the theoretical existence of resignation effects does not imply that they are necessarily a permanent trait of human preferences. Instead, such effects depend on specific circumstances, that should be specified. In some important respects, the presence of resignation effects may be plausible and the prevalence of mean/variance–type utility functions may be maintained. To quote just a salient example, models of banking and financial supervision (e.g. the Value at Risk) are explicitly based on the minimization

of the probability of the heaviest possible loss, which for instance occurs in some "catastrophic" circumstance (like a financial crisis). In this particular setting, clearly the loss function of the supervisor is characterized by a very high parameter ϕ, possibly much higher than the value of 2, popularized in the mean-variance literature.

An intriguing consequence of the analytical results of this paper is that it provides testable conditions which is possible to verify on financial market data. For instance, in principle it should be possible to test the value of the parameter ϕ and therefore of the structure of preferences to the extent that it matters for the incentives to risk diversification. It is important to notice that the intuitions of Prospect Theory have been founded on psychological insight and on experimental evidence, but there is – to the author's knowledge – no published study on real world data, that is outside the laboratory. However, there is at least theoretically the possibility, for instance, to regress the size of shocks in several financial markets (e.g. the daily changes in a number of stock markets) on the rates of return, to see what is the value of ϕ which maximizes the fit (or put it alternatively, for which value of ϕ the possible emergence of shocks is priced *ex ante* by agents).

Therefore, this paper may have provided some insights useful to explain apparently irrational risk concentration phenomena and also possibly spur empirical research on the "right" specification of agents' preferences. At a minimum, this paper has aimed to demonstrate that the incentives to risk diversification are not necessarily a datum of "human nature", so that it depends on preferences, which in turn are a function of the circumstances, and that it needs to be further investigated.

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