Are SMEs sensitive to systematic risk ?

a study of probabilities of default and assets correlations

in French and German SMEs

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Summary and implications of the study

Recently, the Basel Committee asked for an appropriate treatment of exposures to small and medium-sized enterprises (SMEs) in the new risk-weight formulas. The main concern was that capital charge could be too high for SMEs. Indeed, many comments on the New Basel Accord (BIS, 2001a) pointed out a calibration problem for SMEs credit risk. Most of the criticisms argued that the risk-weight curve was to steep and to high, what induced too-high risk-weights for most of the SMEs, due to the fact that these firms are generally characterized by relatively high probabilities of default, as compared to large businesses. There were two options to reduce the risk-weight on SMEs exposures. The first one was to propose different risk-weight functions for SMEs and large businesses. The other was to assume that asset correlation declines with the probabilities of default (PDs). This is the way that the Committee chooses (BIS, 2001b, annex). The objective of the modified risk weights formulas recently proposed by the Basel Committee (BIS, 2001b) was to provide an appropriate treatment of exposures to small and medium-sized enterprises (SMEs). The impact of this change is a risk-weight curve that is generally lower and flatter than the formula proposed in January.

In this study, we show that the best way to adjust risk weights formulas for SMEs would be to propose a different treatment of these firms. In the paper, we use an internal ratings based approach of credit risk and we run a one factor model – the same model that served to calibrate the Basel risk weights formulas – to provide estimates of stationary default probabilities and assets correlations in two large populations of around 440.000 SMEs in France and 280.000 in Germany. The study retain the legal bankruptcy as definition of default in the two countries. Main results are :

- Firstly, the PDs are decreasing with size, what verifies that, on average, SMEs are riskier than large businesses.
- Secondly, the estimated SMEs correlations are very weak (in the magnitude of 1% to 3%), far from the 10% to 20% levels assumed by the Basel Committee. But they are on average higher in large firms.

- Thirdly, results do not show a negative relationship between correlations and PDs : on average, the relationship is U shaped in France, while it is positive in Germany; moreover, the correlation decreases with size in the 2 countries.
- Fourthly, a bootstrap-like simulation which was run in order to test the robustness of the previous results, confirms the relatively low level of correlation in SMEs portfolios. However, it shows that portfolios of large businesses have a greater probability to exhibit high assets correlations.
- Fifthly, the positive effect of diversification is verified in SMEs portfolios, what reinforces the previous results.
- Sixthly, results concerning PDs and assets correlation seem demonstrate that it is possible to distinguish different segments inside the SMEs population. In particular, very small SMEs are less risky on average than the medium-sized SMEs in Germany (and the same result could be observed in France when considering very small personal businesses) and these two size classes of SMEs are riskier than large SMEs. At least, it is possible to distinguish between very small and small SMEs, on the one hand, and large SMEs, on the other hand.

The implications of this study are twofold. First, the weak sensitivity to systematic risk of SMEs and the positive effects of diversification in the European SMEs portfolios advocate for a reducing of the SMEs risk weights. This could be done by introducing lower values of the correlations in the corporate risk weights formulas. Too large capital charges for SMEs could favor a credit rationing process, especially in periods of economic recession, what could potentially intensify the business cycles. In addition, the assumed negative relationship between PDs and correlations in the Basel risk weights formulas could also induce two high capital charges for the less risky European SMEs, so that the less risky firms should “pay” for the more risky firms.

Secondly, it seem necessary to distinguish into the SMEs population at least between small SMEs and larger SMEs. Large SMEs should probably received a more favorable treatment than large firms, because they seem to be less sensitive to systematic risk than the latter. On the other hand, even if smaller SMEs are on an individual basis riskier than the large SMEs, their very weak sensitivity to systematic risk and the positive effects of large portfolios diversification invite to integrate these firms into the retail segment.

Abstract

Recently, the Basel Committee asked for an appropriate treatment of exposures to small and medium-sized enterprises (SMEs) in the new risk-weight formulas. Assets correlations are a major determinant of the distribution of losses in a portfolio credit risk model and a central element of these formulas. This paper uses the same one factor model used by the Basel Committee to compute the risk weights functions and provides estimates of correlations in two large populations of SMEs containing around 450.000 French firms and 280.000 German firms. Results show that the correlations are on average in the magnitude of 1 to 2%. So, the estimated correlations are much lower than the 10% to 20% levels assumed by the Basel Committee. In addition results do not show a negative relationship between correlations and PDs. Moreover, the paper presents also a bootstrap-like simulation that permits to test the robustness of the previous results. This test confirms the relatively low level of assets correlations in SMEs portfolios. However, it shows that portfolios of large businesses have a greater likelihood to exhibit high values of the assets correlation.

1. Introduction

The structure of default rate correlations is an important determinant of the distribution of losses in a portfolio credit risk model. Capturing the correlations between individual exposures is crucial in order to assess the risk at the portfolio level. In most of the credit risk models, the correlations measure the degree of sensitivity of the PDs to the systematic risk factors which represent the influence of the "state of the economy". Portfolio risk will be greater the more the individuals credits in the portfolio tend to vary simultaneously in reaction to the realization of these risk factors. So, a crucial element in the estimation of loans losses distribution is a good calibration of parameters – the probabilities of default and their variance - which determine assets correlations.

One objective of the modified risk weights formulas recently proposed by the Basel Committee (BIS, 2001b) was to provide an appropriate treatment of exposures to small and medium-sized enterprises (SMEs). Indeed, many comments of "The New Basel Accord" (BIS, 2001a) pointed out a calibration problem for SMEs credit risk. Most of the criticisms argued that the risk-weight curve was to steep and to high, what induced too-high risk-weights for most of the SMEs, due to the fact that these firms are generally characterized by relatively high probabilities of default, as compared to the large businesses. There were two options to reduce the risk-weight on SMEs exposures. The first one was to propose different risk-weight functions for SMEs and large businesses. The other was to assume that asset correlation declines with the probabilities of default (PDs). This is the way the Basel Committee chooses (BIS, 2001b, annex). The impact of this change is a risk-weight curve that is generally lower and flatter than the formula proposed in January.

This paper uses the same one factor model that the Basel Committee uses to compute estimates of the default probabilities and correlations into two large populations of around 400.000 French SMEs and 300.000 German SMEs. Results show a positive relationship between correlations and PDs, contrary to the negative one that was assumed by the Basel Committee. In addition, results show on average a negative relationship between correlations and firms size and, finally, they show that the correlations are much lower than the 10% to 20% levels introduced in the Basel risk-weights formulas. Using a bootstrap test, results show also that portfolios of large businesses have a greater likelihood to exhibit a high correlation.

The second section of the paper presents the standard one factor model we used to compute the assets correlations. This is the same as the one used by the Basel Committee (2001a and 2001b) to calibrate the risk weights. Section 3 presents the data, and section 4 the results. Section 5 proposes a bootstrap-like test that permits to measure the volatility of the default correlations and to test the robustness of the results. Section 6 concludes.

2. The one factor model and the computation of the correlations

We computed the correlations by using a one systematic factor probit ordered model (Gordy, 2000, Dietsch and Petey, 2002). This model was used by the Basel Committee to calibrate the risk weight functions of the IRB approach. In this model, each borrower's financial position at the end of a planning horizon (default or not) is determined by one systematic risk factor and one idiosyncratic risk factor. Indeed, the end of period borrower i state is driven by an unobserved latent random variable U, which is defined as a linear function of a single systematic factor x and a specific idiosyncratic factor e_i

$$U = wx + \sqrt{1 - w^2} e_i \qquad (1)$$

where x et e_i are standard normal and independent variables ($E\left[xe_i\right] = 0$). The systematic factor represents the state of the economy. It measures the effect of the business-cycle on the default rate. The state of the borrower at the end of the planning horizon depends on the location of the latent variable relative to a "cut-off" value, which defines default. If the latent variable is a standard normal variable, the default cut-off value is simply equal to $\Phi^{-1}(\overline{p})$, where $\Phi^{-1}(.)$ is the inverse standard normal CDF and $\overline{p}$ is the unconditional PD for a borrower belonging to a given risk class. Let us define Z_i as the standardized latent variable of borrower i. Therefore, a borrower makes default when:

$$\frac{w\ x + e_i}{\sqrt{1 - w^2}} < \Phi^{-1}(\overline{p}) \qquad (2)$$

or, alternatively, for a given value of the systematic factor x, when :

$$e_i < \sqrt{1-w^2}\,\Phi^{-1}(\overline{p}) - wx \qquad (3)$$

This condition allows to compute *p(x)*, the individual PD conditional to the realization of the systematic factor x. This probability is simply derived from (3), as follows:

$$p(x) = \Pr\left[e_i < \sqrt{1-w^2}\,\Phi^{-1}(\overline{p}) - wx\right] = \Phi\left[\sqrt{1-w^2}\,\Phi^{-1}(\overline{p}) - wx\right] \text{with } e_i \sim N(0;1) \quad (4)$$

This is the conditional default probability of a borrower with stationary PD equal to $\overline{p}$. If the realization of the systematic factor is good (if the state of the economy is good), the firm will default only if the realization of the specific idiosyncratic factor is worse. Otherwise, the (standardized) latent variable Z_i will not cross the default cut-off value $\Phi^{-1}(\overline{p})$. The value of $p(x)$ fluctuates around the stationary probability depending of the values of the systematic risk factor and of w. Moreover, the degree of correlation between defaults is determined by the sensibility of the latent variables to the systematic factor, that is by w. For two borrowers i and j with the same rating grade, the (non conditional) covariance between latent variables is given by:

$$Cov[Z_i;Z_j] = E[Z_iZ_j] - E[Z_i]E[Z_j] = \frac{w^2}{1-w^2} \qquad (5)$$

Therefore, correlations between latent variables are due to the existence of aggregate shocks in the economy. In addition, in the probit model, it is the existence of correlation between defaults that determines the shape of the end of period value distribution of the portfolio[1].

In order to compute *w,* it is necessary to compute the variance of $p(x)$. We adopted the non-parametric method proposed by Gordy (2000). Thus, the variance of $p(x)$ is determined as follows : assuming serial independence for the realizations of the systematic factor and conditional independence between defaults, the probability that two borrowers jointly default is:

[1] We verified that the shape of this distribution is skewed to the right, and that its degree of asymmetry directly depends on the weight *w* of the systematic risk factor.

$$\Pr\left[Z_i < \Phi^{-1}(\overline{p}) \& Z_j < \Phi^{-1}(\overline{p}) \middle| x\right] = \Pr\left[Z_i < \Phi^{-1}(\overline{p}) \middle| x\right] \Pr\left[Z_j < \Phi^{-1}(\overline{p}) \middle| x\right] = p(x)^2 \quad (6)$$

with variance:

$$Var[p(x)] = E\left[p(x)^2\right] - E[p(x)]^2 = E\left[\Pr\left[Z_i < \Phi^{-1}(\overline{p}) \& Z_i < \Phi^{-1}(\overline{p}) \middle| x\right]\right] - E[p(x)]^2 \quad (7)$$

The latent variables are standard normal variables, with correlation equal to $w^2 / \left(1 + w^2\right)$. Therefore, the (non-conditional) expected value $E\left[\Pr\left[Z_i < \Phi^{-1}(\overline{p}) \& Z_i < \Phi^{-1}(\overline{p}) \middle| x\right]\right]$ is given by the bivariate normal distribution. So, the variance of the conditional default probability is:

$$Var[p(x)] = Bivnor\left(\Phi^{-1}(\overline{p}), \Phi^{-1}(\overline{p}), \frac{w^2}{1 - w^2}\right) - \overline{p}^2 \qquad (8)$$

Knowing the values of the stationary default probability and the variance of the conditional default probability, the weight w of the systematic factor is derived as solution of the non-linear equation (8).

3. The data

The data come from the internal ratings systems of two large credit insurers : Coface in France and Creditreform in Germany. They give records of all rating grades changes over the same 1997-2001 period in the two countries. In addition, the two files contain accounting information for all firms we retained in the estimations. Following a quite conventional definition, SMEs are defined as firms with turnover lower than 40 millions €. Very small firms with annual turnover lower than 150.000 € were excluded. The reason is that these firms (non incorporated firms, like small shops) belong more to the retail segment of the banks portfolios than to the corporate one. The final database contains around 450.000 French and 280.000 German incorporated firms.

By using a quite conventional SMEs size classification, we distinguished three size classes of SMEs. Class 1 puts together the small SMEs with turnover between 0,15 millions € and 1 million €, class 2 groups together the medium-sized SMEs with turnover between 1 to 7 millions €, and class 3 the large SMEs with turnover between 7 to 40 millions €. Table 1 presents the distribution by size of the SMEs in the two samples. We also consider in this study large firms (with turnover higher than 40 M€).

Table 1 : The size distribution of French and German firms

Size	**France**		**Germany**	
(turnover in M€)	Number of firms	%	Number of firms	%
1 (0,15 to 1)	287.586	64.04	140.660	49.7
2 (1 to 7)	131.977	29.39	116.175	41.1
3 (7 to 40)	29.538	6.58	26.112	9.2
Total SMEs	449.101	100	282.947	100
Large firms	6.213		12.081	

Sources : Coface and Creditreform

Notice that all sub-samples are very representative of the incorporated French and German companies population of each size.

From the original ratings system, we have built a 9 positions ratings system[2]. Rating grade 1 corresponds to the lowest degree of credit risk and rating grade 9 corresponds to default. In the two countries, the legal bankruptcy is used as the definition of default. Table 2 shows the risk distribution in the three size classes in the two countries.

Table 2 : The risk distribution of French and German SMEs year 2001

a) France

	Risk classes (from low risk - class 1 - to high risk – class 8)								
Size	1	2	3	4	5	6	7	8	Default
1 : <1M€	0.4	39.5	10.9	13.6	13.9	14.6	2.7	2.1	*2.4*
2 : 1 – 7M€	5.7	42.8	13.1	13.3	12.7	8	1.2	1.1	*1.6*
3 : 7 – 40 M€	17.8	32.4	14.5	13.8	12.5	5.7	0.7	0.3	*0.9*
Total	*3.34*	*40.02*	*11.86*	*13.65*	*13.41*	*11.85*	*2.18*	*1.65*	*2.0*

b) Germany

	Risk classes (from low risk - class 1 - to high risk – class 8)								
Size	1	2	3	4	5	6	7	8	Default
1 : <1M€	0.1	21.7	31.3	9.4	7.1	14.6	5.5	6.2	*4.0*
2 : 1 – 7M€	1.0	24.6	32.4	9.0	4.2	11.2	3.7	4.8	*4.1*
3 : 7 – 40 M€	6.3	47.6	25.8	2.2	4.0	6.1	3.3	2.1	*2.6*
Total	*1.0*	*27.2*	*31.3*	*8.6*	*5.7*	*12.5*	*4.6*	*5.2*	*3.9*

Sources : Coface and Creditreform

[2] Notice that we do not try here to build completely consistent risk classifications. Therefore, as shown later, the PDs are not necessarily the same in each country for a given risk class.

Because the risk classification come from two different ratings systems, the two risk scales are not directly comparable. That explains partially that the distribution of the SMEs in the 9 risk classes is different from one country to the other one. Here, the objective of the study was not to build a unique ratings system, but only to analyze the buildings blocks of SMEs credit risk, and more particularly the correlations, in the two countries. Notice also that the higher percentage of firms in default in Germany is due to the fact that the state of the economy was characterized by a increase in the number of bankruptcies in that country in 2001.

4. The results

The ratings grades were used to compute PDs and assets correlations. We took the rating grades at the end of each year. Only one transition within a year was retained and the other transitions within the same year were neglected. Then, we computed annual PDs and stationary default probabilities - the weighted mean of the annual PDs over the period – for each size and risk class. Before to present the correlations results, it is interesting to observe how the stationary PDs vary with the firms size.

4.1 The stationary probabilities of default decrease on average with firm's size

Table 3 presents the stationary PDs in the various sub-portfolios of French and German SMEs and large firms. It shows that, on average, the PDs tend to decrease in a monotonic way with the firm size in France. In Germany, the relationship is not monotonic in the SMEs sample. The PDs are lower in the first size class than in the second one, whatever the risk class. However, the PDs differences are not so high. The PDs are also higher in these two first classes, as compared to the last size class. In other words, the smaller SMEs seem to be less risky than the medium-sized SMEs and these two classes of SMEs seem to be riskier than the largest SMEs. In France, additional results (not shown here) show a similar result, in that sense that the PDs are also lower in the very small (personal) businesses (with turnover lower to 0,15 M€). So, these results tend to show that we can likely distinguish three categories of SMEs : the small or very small ones, in which the default risk is lower than in the medium-sized SMEs, the medium-sized SMEs that are more the riskier SMEs on average and the largest SMEs where the credit risk is lower. Table 3 allows also to compare the SMEs and the large corporate firms. Results show that in France as well as in Germany, on average, the average stationary PDs are much lower in the large businesses. These results confirm that, in the two countries, the default risk tend to decrease on average with the size of the firms[3].

Table 3 : The stationary default probabilities in French and German firms (in %)

a) France

	Size classes				
Risk classes	SMEs size 1 <1M€	SMEs size 2 1 – 7 M€	SMEs Size3 7 – 40 M€	Large firms >40 M€	Total SMEs
1 (low)	0.33	0.24	0.15	0.03	0.19
2	0.41	0.27	0.19	0.07	0.32
3	0.9	0.68	0.48	0.12	0.72
4	1.64	1.35	0.84	0.37	1.33
5	2.79	2.61	1.53	1.05	2.39
6	4.94	4.51	2.44	1.11	4.23
7	9.99	9.44	5.49	2.29	8.61
8 (high)	14.89	16.24	13.28	(*)	13.78
Total	2.63	1.74	0.79	0.28	2.21

(*) no default in this class

b) Germany

	Size classes				
Risk classes	SMEs Size 1 <1M€	SMEs Size 2 1 – 7 M€	SMEs Size3 7 – 40 M€	Large firms >40 M€	Total SMEs
1 (low)	0.17	0.43	0.56	0.21	0.49
2	0.77	0.98	1.05	0.38	0.91
3	1.16	1.68	1.43	0.56	1.41
4	1.95	2.56	2.34	1.32	2.21
5	2.38	3.04	2.35	1.24	2.55
6	2.37	3.55	3.10	1.90	2.87
7	5.45	7.63	9.44	3.39	6.42
8 (high)	16.22	18.66	18.61	11.43	17.28
Total	2.73	3.0	2.05	0.53	2.78

4.2 The very weak values of the assets correlations and the existence of a positive relationship between the correlations and the PDs in French and German SMEs populations

Table 4 presents the assets correlations for the same sub-portfolios of French and German firms. Three main results come to light. First, the values of the default correlations are very weak. The average value is around 1% in the French and German SMEs. These values are far

[3] Notice that this could be due in part to the criterion of default that was chosen. In fact, the legal bankruptcy is a mean of financial difficulties resolution which use appears to be less frequent in the large businesses sector than in the SMEs sector.

from the 10% to 20% values retained by the Basel Committee in the last risk-weight formulas (BIS, 2001b). The maximum value per risk class climbs to 10,72 % in France and to 6,52 % in Germany.

Secondly, the assets correlations decrease significantly on average with the SMEs size. They are quite low in the large SMEs, compared to the small and medium-sized ones. This result tends to show that the SMEs credit risk is less sensitive to systematic risk factor as the size of the firms increase. However, results also show that the average correlation is higher in the larger businesses (with turnover higher than 40 M€) in the two countries. The average values of the correlations are 1,45 % in Germany and 2,21 % in France during the same period.

Thirdly, results do not show a negative relationship between the correlations and the PDs. In the French SMEs population, the correlation increases with the risk of default in the class of smaller firms and it is U-shaped in the other two classes, what explains also the U-shaped form of the relationship in the total sample. In Germany, results show a positive relationship between correlations and PDs on average and in the two first size classes of SMEs. In the largest SMEs and in the large firms, no clear relationship appears between the two variables.

Table 4 : The assets correlations in French and German firms (in %)

a) France

	Size classes				
Risk classes	SMEs Size 1 <1M€	SMEs Size 2 1 – 7 M€	SMEs Size3 7 – 40 M€	Large firms >40 M€	Total SMEs
1 (low)	0.79	2.95	2.79	1.5	2.19
2	0.12	1.95	1.56	0	2.29
3	1.55	0.61	0.71	4.39	2.31
4	1.34	0.95	0.57	2.79	2.67
5	1.53	0.98	0.37	2.77	1.51
6	1.78	1.47	0.82	0	1.99
7	2.67	2.08	2.07	0	2.98
8 (high)	2.71	2.79	10.72	0	3.07
Total	1.54	0.97	0.49	2.21	1.28

b) Germany

	Size classes				
Risk classes	SMEs Size 1 <1M€	SMEs Size 2 1 – 7 M€	SMEs Size3 7 – 40 M€	Large firms >40 M€	Total SMEs
1 (low)	0	0	0	1.21	0.11
2	1.86	1.33	0.57	2.51	1.29
3	1.52	1.29	0.24	0	1.19
4	2.21	1.42	6.52	1.61	2.01
5	3.18	2.02	0.25	0.75	2.59
6	1.21	0.62	0.25	0.49	0.79
7	3.97	1.97	0.57	1.69	2.75
8 (high)	2.71	2.62	2.03	0	2.59
Total	1.23	0.79	0.14	1.45	0.93

The differences between the observed correlations between latent variables and the Basel Committee correlations could received two kind of explanations. First, we used a legal bankruptcy definition of default. Second, the 1997-2001 period was mainly characterized by economic growth, with only one “bad” year in Germany and a noticeable decrease of the number of defaults in the SMEs population at the end of the period in France, and in the middle of the period in Germany. However, additional simulations we performed showed that even if the introduction of additional “bad” years contributes logically to increase the correlations, the latter stay well below the 10 % value in all sub-samples.

5. Are SMEs less sensitive to systematic risk than large firms : a bootstrap-type experiment to assess the stability of the correlations

In this section, we address the issue of the robustness of our results. In this perspective, we propose a bootstrap-like methodology which permits to assess the possible variation of the assets correlation inside each size class.

Two main reasons lead to the fact an estimated correlation might be too low (or too high). The first reason come from the length of the ratings time series. The period of observation might be too short to cover at least an entire business cycle, leaving a misleading apparent stability of default rates (low or high). This could induce a simultaneous bias in the measurement of PDs and PDs volatility. To avoid this shortcoming, the direct solution is to accumulate new data through time. The second reason for a possible underestimation of correlation stems from the fact that the estimated correlations were computed on a very large sample (quasi-exhaustive) of businesses. In general, the size of the banks SMEs portfolio is lower. Consequently, a bank could observe as well a higher or a lower assets correlation in its book. This last point is of particular importance if the regulator wishes to enforce a uniform value of default correlation across banks in the computation of risk weights.

The correlations presented in the previous section are average correlations into risk-size classes. However, the regulator problem would be to assess the likelihood of potential very high correlations in some risk classes, that is to know the "confidence interval" for such average computed correlations. Recall that, by using equation (8), we can compute the conditional variance of $p(x)$ for given values of the correlation and of the PDs. Consequently, we can compare the conditional variance of PDs computed by taking "assumed by the regulator" values of the correlation with the value of the conditional variance computed by taking the "true" correlation values. For a given value of each risk class stationary PD, there is a monotonic relationship between the variance of the conditional default probability $p(x)$ and the correlation of latent factors r. By repeating this procedure a large number of times, it is possible to build a non parametric distribution of the conditional variance, which could be translated in a distribution of the correlation.

We built five different portfolios, splitting the entire database into the same five size classes as before. So, the simulated portfolios are homogeneous in terms of firms size. These portfolios are voluntarily small, the likelihood to find high correlation being higher in small portfolios. So, the simulated portfolios concerning the first four SMEs size classes contain 5.000 borrowers. The simulated portfolio of large businesses contains only 2.000 borrowers, because our sample entails only 4.377 firms (again, our sample is very representative of the French large businesses population). The size of each portfolio is maintained constant over the seven years period by replacing firms in default a given year by firms which did not default and are still present in the database at the beginning of the next year.

Table 5 presents the ratio a of the "regulator induced" conditional variance to the estimated conditional variance. This ratio measures the number of times the observed conditional variance should be multiplied in order to reach the correlation assumed by the regulator. Figure 1 puts the two conditional variances on a graph. Two different values of the "regulator" correlation were chosen : 8% and 20%. As in the previous results, we observe that the (median) correlation is decreasing with size, even after controlling for the differences in PDs. This result is not completely surprising, the simulated portfolios being drawn from the same population.

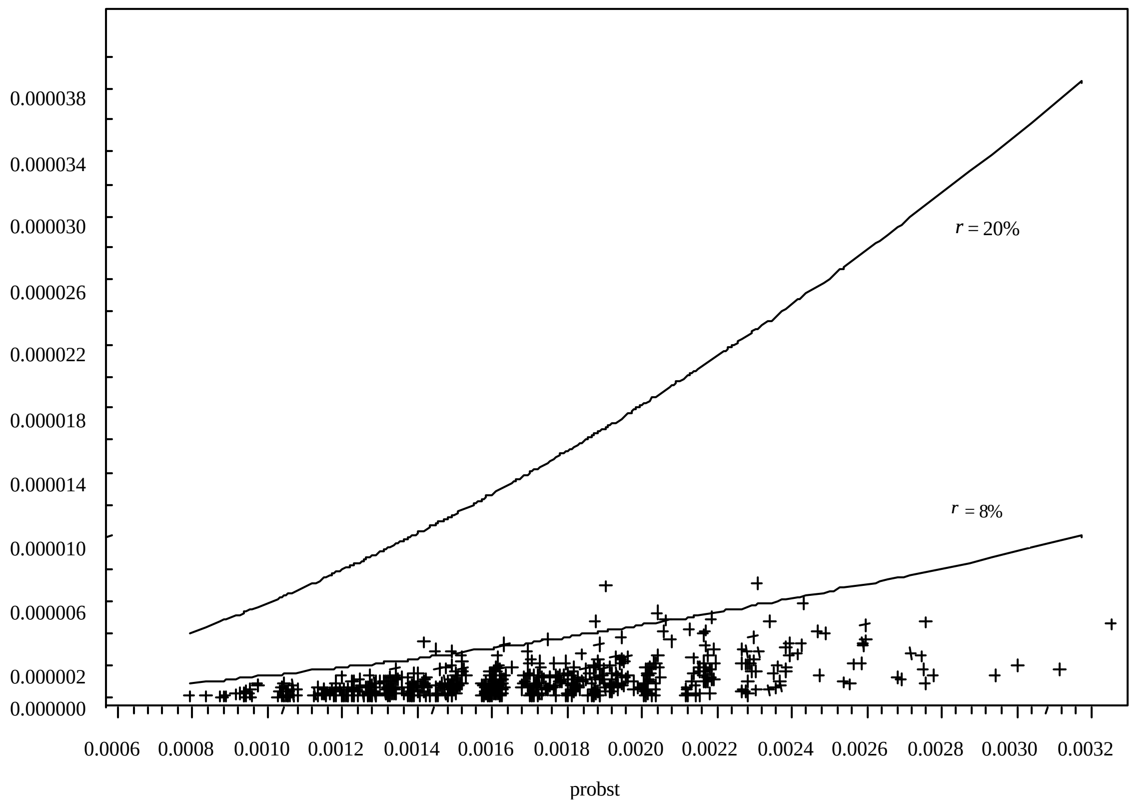
Table 5 shows two different results. First, concerning the SMEs portfolios (size classes 1 to 4), we observe that the ratio α remains higher than 1 even for the lower correlation of 8%. That means that, in the French SMEs borrowers population over the studied period, the 8% correlation level over-estimates the effective correlation and acts as a very conservative value. Indeed, our result confirms that the correlations in the SMEs portfolios are quite low. Second, this result does not hold for large businesses (size class 5), where the ratio α takes less than 1 values (min=0.61). Thus, portfolios of large borrowers have a greater likelihood to exhibit a higher correlation. However, the correlation of 20% is never reached, as figure 1 shows.

Table 5 : The ratio a of regulator conditional variance to observed conditional variance

	r = 8%			r = 20%		
Size Turnover in Millions of euros	Median (std dev)	Min	Max	Median (std dev)	Min	Max
Less than 0,75	1,74 (0,22)	1,22	2,70	5,52 (0,69)	3,84	8,63
From 0,75 to 3	2,77 (0,97)	1,21	9,93	9,86 (3,48)	4,28	35,2
From 3 to 7,5	2,45 (0,83)	1,21	8,36	8,93 (3,04)	4,44	30,6
From 7,5 to 40	2,73 (1,17)	1,05	11,62	10,37 (4,24)	4,02	44,6
More than 40	4,79 (16,8)	0,61	2841	20,19 (489)	3,9	10792

Figure 1 : Conditional variances and stationary default probabilities in large businesses (size class 5)

The upper curve represents the conditional variance implied by a correlation of 20%, given the stationary PD of the simulated portfolio. The lower curve shows the variance given a correlation of 8%. Any point located over a curve implies that the simulated portfolio has a correlation greater than these assumed correlation.



The main result of our simulation is the difference of the volatility of conditional variances across size classes. Table 5 shows that if the median conditional variance is decreasing with size, its standard deviation increases with size. This is especially true for the large businesses (turnover over 40 millions of euros). For large corporate firms, the likelihood to observe a correlation higher than a given value is higher than for other size classes, despite the fact that the median (and consequently the correlation) is much lower than in other size classes. Moreover, in the large businesses portfolios, the empirical conditional variance exceeds 8 times out of 629 the value induced by the choice of a 8% correlation[4]. This remains low (1,3%) but significant[5].

The stability of the conditional variance in the small SMEs class (turnover less than 0.75 millions of euros) is quite surprising despite the relatively small size of simulated portfolios (5.000 exposures to be compared to the over 150.000 businesses of that size class). An explanation of this result is that the sensitivity of smaller businesses to economic conditions is relatively uniform across sectors and regions, at least in the French case. The same seems also true for medium-sized SMEs. On the contrary, larger businesses appear to be potentially more sensitive to the degradation of economic conditions. In other words, if the large business PD is much lower than that of SMEs, a degradation on business conditions might result in a relatively greater increase of the number of defaults in this population.

Moreover, the volatility of the correlation in the large businesses population shows that the diversification of risk might be more difficult for portfolios of large corporate firms than for SMEs portfolios. Indeed, even if the SMEs loans portfolio of a given bank contains a small number of borrowers, the value of the correlation into this portfolio can converge rapidly to the average value for this class of borrowers. On the contrary, this appears to be more difficult in large corporate portfolios, because an even large portfolio of large businesses can show an high default correlation. Our simulated portfolios of large borrowers could be considered as diversified because they represent nearly half of the complete population of the large potential borrowers. The credit risk of large businesses comes less from the likelihood of individual

[4] For low values of the PD, the estimated conditional variance can become negative. We excluded portfolios showing this result, which is equivalent to set the variance equal to zero. As we are only interested in the higher values of the variance, this has no real effect on the results. However, this is a limitation of our non-parametric estimator. It was in fact impossible to proceed to simulations by using smaller portfolios of large firms, because the estimated variance became negative in most cases, (legal) defaults being very rare events in this population.

[5] Additional simulation shows that in SMEs population this result never occurs over the range of one thousand simulations.

defaults than from the likelihood of possible multiple defaults in case of an economic downturn due to the relative concentration of the portfolio (Notice that this result is independent of the effective size of the credit exposures). If the objective of the credit risk regulation is to avoid an underestimation of the credit risk in portfolios of large corporate exposures, in order to avoid corresponding very severe losses for the bank that holds this portfolio, we can notice that this objective could induce quite large economic capital requirements on average. Simulation results show, for instance, that the highest simulated correlation amounts to 12% in this size class, while the average correlation for the entire size class is only of 1,1% (figure 1).

To resume, the results of our bootstrap-like test show that we can distinguish three types of borrowers in the corporate sector : the very small SMEs, which are less sensible to systematic risk, an intermediate category of medium-sized SMEs, where the likelihood to get a loans portfolio characterized by high default correlation remains relatively low and, finally, the large businesses, which exhibit an small correlation on average, but where the risk to get a loans portfolio with an high correlation is high. This result could reinforce the choice of higher correlation values made by the regulator. However, this choice could induce severe costs for SMEs, if we consider the very low values of the correlation in this population. An alternative rule could be to impose stringent concentration rules for large corporate portfolios, however with the risk of becoming rapidly obsolete in the context of large consolidation and M&A waves.

6. Conclusion

In this paper, we show that the best way to adjust risk weights formulas for SMEs would be to propose a different treatment of these firms. We use an internal ratings based approach of credit risk and we run a one factor model – the same model that served to calibrate the Basel risk weights formulas – to provide estimates of stationary default probabilities and assets correlations in two large populations of around 440.000 SMEs in France and 280.000 in Germany. The study retain the legal bankruptcy as definition of default in the two countries.

The results show that, in the two samples of French and German SMEs, the sensitivity to a one systematic risk factor ("the state of the economy") is quite low and that the estimated defaults correlations are well under the assumed 10% to 20% levels in the new formulas of risk-weights proposed by the Basel Committee. A bootstrap-like test confirms that these values over-estimates the effective correlation in the French SMEs borrowers population. However, the same result does not hold for large businesses and results show that portfolios of large borrowers have a greater likelihood to exhibit a higher correlation. Moreover, results do not show a negative relationship between assets correlations and PDs.

Our results suggest also two ways of future research. First, the results could be determined by the definition of default. So, it will be interesting to see if the results will stay stable when choosing another criterion of default, such as a too large delay of reimbursement of the bank or non banking default of payment. Second, because the SMEs sector is a central part of the productive system in most of the European countries, it would be interesting to see if the relationships between correlations and PDs and between correlations and firms size are the same in other European countries than the two countries considered in this study.

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