Value at Risk with Informed Traders, Herding, and the Optimal Structure of Trading Divisions

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Actual version: February 2002

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Abstract

We scrutinize the use of value at risk as traders' limit in banks. Thereby, we compare a bank with uninformed traders dealing on a perfect capital market, with a bank in which traders receive a noisy signal about the future price of the stock they are dealing in. Additionally, they are able to deduce some information about the market trend from the observation of the behavior of other traders. In the imperfect market setting, informed traders tend to herd in informational cascades, which increases the probability of extreme results and value at risk. Thus, banks should either avoid or optimize information flow between traders. We discuss different optimization approaches to maximize a value at risk-based RORAC through an efficient information policy. Likewise, we compare our results with „neoclassical" value at risk both from an ex ante and ex post-perspective and identify systemic risks from neoclassical negligence of informational herding.

JEL Classifications: D7, D8, G21, G31

1. Introduction

Value at risk is used both by banks and bank regulators as a device to control risk. Although it has been argued that it does not measure risk in a theoretically sound way,[1] it is a useful device to limit the probability of high losses that might endanger the existence of a bank or even the stability of a financial system. Therefore, science and practice take great efforts to correctly calculate value at risk for a portfolio or even a whole bank, either through a variance-covariance approach or through historical or other simulation methods.[2] A key factor in such calculations is the correlation between the different risky positions.

However, the use of value at risk as limit in risk management must be seen under two different perspectives: Ex post, managers or regulators want to know about the riskiness of a given portfolio with respect to large losses. If the probability of such a loss exceeds the limit, they demand adjustment measures: E.g., bank management will compel the treasurer to take a position in opposition to the traders of a trading department to reduce overall value at risk. Bank regulators will ask the bank to look for additional equity to comply with the capital adequacy rules. Because diversification is a key factor in banking, the necessity of such actions relies heavily on using the correct correlation between the different risky positions when calculating value at risk. These correlations are calculated in models using historical data.[3] However, the traders themselves are not restricted to historical data when taking the respective positions.

From an ex ante perspective, value at risk is used as a limit for each risk taking decision unit, e.g., for each trader in a trading department. The bank's management should be able to correctly aggregate these individual limits to an overall value at risk for good reasons: It should not set limits arbitrary but in conformity with its overall risk policy. Otherwise, either ex post adjustments would be large and costly because the individual limits are too loose, or the overall risk policy would be obsolete because individual limits were too narrow. However, even if large ex post adjustments were not too costly, it is rather risky to rely on them only. Particularly under turbulent and therefore risky market conditions it might prove difficult to detect and hedge concentrations of risk as fast and easily as needed.

A crucial insight of our paper is that, from an ex ante perspective, it is not the correlation between share prices that determines value at risk but the correlation between traders making the right or wrong decision. In a neoclassical setting, traders have no other knowledge than the other market participants. Thus, their behavior should be uncorrelated and the correlation be-

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[1] See Artzner et al. (1997, 1999).
[2] See, e.g., Duffie/Pan (1997) or Jorion (1997).
[3] See, among others, J.P.Morgan/Reuters (1996) presenting RiskMetrics™.

tween share prices has no influence on overall value at risk at all. The same holds if traders receive a noisy signal about the share they are trading in, but, apart from that, act in isolation. However, if traders are able to observe the trading decisions of other traders, the correlation between share prices gets relevant again, because it influences the correlation between the traders' decisions.

The last case allows for herding behavior due to informational cascades. Through learning from the behavior of others about the general market trend traders can found their decision on a signal of greater precision. However, we will argue that this signal is not much more precise, because learning stops once a trader follows the market signal and therefore ignores his private information. Also, traders induce greater risk when following the behavior of their forerunners. Therefore, organizational measure should be taken to either avoid herding or to control information flow to achieve optimal learning processes.

Although our paper mainly deals with the ex ante perspective, there are also implications for the ex post calculation of value at risk. If traders receive a noisy signal and act in isolation, value at risk decreases sharply compared to its neoclassical calculation. As a consequence, equity capital is not used efficiently, but at least overall risk is below what is perceived by the respective bank or its state supervisors. However, if traders communicate with each other, we might observe a strong increase of value at risk if the decisions of many traders were not done in response to their idiosyncratic signal but in accordance with the market signal, at least compared with the isolation case. We do also observe that, if the precision of the traders' signal is small, neoclassical calculation might sometimes underestimate value at risk and cause insufficient reserves. If bank regulation relies on capital adequacy, this result might have negative consequences for the stability of the financial system.

In the following second section, we review the literature on value at risk with respect to its use as limit in risk management. We also take a short look at the literature on rational herding, a concept used in the fourth section to analyze the behavior of informed traders. Before doing so, we outline the basic model and the benchmark case of perfect capital markets with uninformed traders in the third section. In the fourth section, we assume that traders receive a noisy but informative private signal about the share they are trading in, and that they are able to observe the trading decision of other traders in their trading department. We deduce the conditions for the appearance of rational herding in informational cascades and calculate overall value at risk for trading departments with informed traders. In the fifth section, we look at the efficiency effects of herding and draw some consequences for the organization of trading departments. The sixth section contains some additional insights from the model, in particular a look at the ex ante consequences of informed traders. The seventh section concludes.

2. Related Literature on Value at Risk and Rational Herding

Value at risk of a position is the amount of loss that will be exceeded only with a given (small) probability within a certain (short) holding period.[4] Thereby, value at risk is the amount of equity capital needed to reduce downside risk of a risky position, here defined as probability to suffer losses greater than reserves, to a certain predefined level. As such, it is used to obtain the economical or regulatory capital requirements for banks and other financial intermediaries. If equity capital is below value at risk, the respective firms should either reduce risk or look for additional equity.

Originally, value at risk was proposed by the Group of Thirty to measure the risk of derivatives.[5] However, it is applicable on any risk for which data are available to construct a distinct distribution of returns. As a risk measurement tool, value at risk has the advantage of being a very simple conception that can easily be understood, in particular because it measures risk in currency units. Also, for deciders in business, it is intriguing to compare different portfolios by one single risk measure. However, from a theoretical point of view, value at risk as a measurement tool has some obvious drawbacks.[6] A basic shortcoming is its violation of subadditivity. Therefore, value at risk is no coherent risk measure as defined in Artzner et al. (1997, 1999).[7] Furthermore, value at risk for non-normal distributed returns is not consistent with the expected utility theory. Nevertheless, within banks, the use of value at risk is widespread, which is to some degree caused by the approval of value at risk in internal models for the calculation of regulatory capital.[8]

According to the Basle Capital Accord, banks that use internal models for measuring market risk are also obliged to set up value at risk limits to control risk exposure on a bank-wide basis. For this reason, but also due to developments concerning the internal risk budgeting of banks, value at risk is also used in risk management today. Within this scope, each business

[4] See e.g. Jorion (1997). The probabilities applied in value at risk models are usually 1% or 5%, the holding period 1 to 10 days. For the different techniques to calculate value at risk see, e.g., Chung (1999), Hendricks (1996), Pritsker (1997) or more technically Ridder (1998).
[5] See Global Derivatives Study Group (1993).
[6] For evidence on this criticism see Artzner et al. (1997, 1999), Guthoff/Pfingsten/Wolf (1997) and Johanning (1998), using arguments developed by Rothschild/Stieglitz (1970).
[7] Value at risk is only a coherent risk measure if returns are multivariate normally distributed.
[8] See Basle Committee on Banking Supervision (1996), Section B.2.

unit gets assigned a certain amount of risk capital in terms of a value at risk limit. Limit setting is understood as a process of internal capital allocation in a top down process. However, due to correlation effects the total limit of the bank should be significantly smaller than the sum of the individual limits.[9] Risk management in banks is not sufficiently advanced to disaggregate the overall limit into individual limits correctly. Also, value at risk lacks time-consistency, i.e., it leads to decisions at a later date that, from an ex ante perspective, are not optimal and might, if rightly anticipated, violate the initial value at risk limit.[10]

Whereas general portfolio theory identifies the investor's utility-maximizing portfolio choice given a trade off between risk and return, the aim of banks using value at risk as controlling device is somewhat different. Banks restrict themselves with respect to downside risk as perceived by value at risk, and maximize expected return under this limitation. Below, we will use a value at risk-based RORAC as adequate instrument to measure the success of such a policy.[11]

Value at risk is usually calculated in a neoclassical context. Implicitly, risk controllers and regulators thereby assume that portfolios are randomly composed. In contrast to this assumption, banks hire traders to invest in some stocks and neglect others, depending on the traders' assumptions concerning the future development of the stock prices. If such a conception is meaningful, portfolios are in reality not randomly but deliberately composed. Dresel/Härtl/Johanning (2002) discuss the allocation of risk capital in a setting with informed traders.[12] As a conclusion, they allow for higher individual risk limits for each trader, in particular if risk concentrations can be hedged through an active treasurer. In their model each trader acts in isolation and does not react to the decisions of other traders, as is the case in our model presented below.

To describe traders' behavior we exploit arguments from the literature on rational herding. Although there are numerous applications of herding arguments in financial economics,[13] the actual paper is, to our best knowledge, the first on the calculation of value at risk. The concept of herding in informational cascades itself was formalized by Banerjee (1992), Bikhchandani/Hirshleifer/Welch (1992) and Welch (1992). The basic logic of our paper is along the

[9] For a discussion of the effect of diversification within the scope of risk management by value at risk limits and possible solutions see Dresel/Härtl/Johanning (2002).
[10] See Franke (2000) and appendix 2 below.
[11] Note that the literature does not clearly distinguish between the different risk adjusted performance measures, like RORAC, RAROC or RARORAC. For a discussion of risk-adjusted performance measures see e.g. Crouhy/Turnbull/Wakeman (1999) and James (1996). For approaches based on risk capital in terms of value at risk see Stoughton/Zechner (1999) or Wilson (1992).
[12] See also Beeck/Johanning/Rudolph (1999), who discuss the disaggregating of value at risk limits over time.
[13] For an overview see Devenow/Welch (1996) or Hirshleifer/Teoh (2001).

lines of Banerjee (1992). He studies the asset choice of *n* investors out of *i* assets, including one unknown optimal asset *i**. Some investors get a noisy signal to support their decision. Every person deciding can observe the preceding decisions of other investors. All investors follow Bayesian rationality, which might make them ignore their own signal and follow the behavior of the preceding deciders only. Bikhchandani/Hirshleifer/Welch (1992) find that such an informational cascade occurs with a probability approaching one if the number of individuals is very large. However, because in a cascade the deciders ignore their own signal, cascades develop on the basis of very little information. Therefore, Banerjee (1992) proposes to hide the decisions of some early deciders to raise the ex ante welfare of the economy. Cao/Hirshleifer (2000) show that delaying the observation of former actions can improve the precision of decisions of each individual decider in a cascade as well as the average welfare.

An early application of rational herding on financial economics is Welch (1992), who explains investor behavior in IPOs. Welch (2000) finds empirical proofs for herding in the buy or sell recommendations of security analysts. Further empirical evidence on herding in financial economics be found in, e.g., Grinblatt/Titman/Wermers (1995), Oehler (1998) and Graham (1999), the first two papers dealing with the investment strategies of mutual funds and the last-mentioned with the recommendations of investment newsletter.

The first to detect informational cascades in the laboratory experiments were Anderson/Holt (1997). Nöth/Weber (1999) and Kremer/Nöth (2000) confirm their results. However, the laboratory experiments reveal that deciders use simple heuristics rather than Bayesian updating. However, non-Bayesian mechanisms to update expectations effecting informational cascades would not change the fundamental insights of this paper, thus below we use Bayesian updates.

Given this state of the literature, the contribution of our paper is to link rational herding due to informational cascades[14] with banks' risk management based on value at risk. In doing so, our paper is one of very few in the literature on rational herding that discusses not only the occurrence but also the consequences of herding in a relevant application.[15] In our simple setting, we are able to quantify the efficiency losses caused by rational herding and the effectiveness of different remedies.

[14] For the distinction between informational cascades and herding see Hirshleifer/Teoh (2001), p. 4.

[15] Another example for a study on the effects of herding is Lee (1998), who explains market crashes through the existence of transaction costs.

3. The Benchmark Case: Value at Risk with Uninformed Traders

In the following, we construct a model of a bank consisting of a trading department only. In such a trading department, numerous traders deal in different stocks. These traders are restricted each by an individual value at risk limit. Thus, the „true" value at risk of the bank does not depend only on the risk the individual trader is allowed to take, but presumably also on the correlation between the share prices of the different stocks they are trading in. To capture this element, we assume that the stocks returns depend not only on the private influences, but also on a systematic factor we call the market trend. The market trend is not observable.

To be precise: Assume that the market trend is positive with probability t_g and negative with probability $t_b = (1 - t_g)$. The relevant market for the trading department consists of N shares, each traded by a separate trader. If market trend is positive, the price of the individual share will go up with probability q and down with probability $(1 - q)$, with $½ < q < 1$. To keep the model symmetric, a negative market trend leads to the adverse result, i.e., the share price goes up with probability $(1 - q)$ and down with probability q. Let r_i be the return of share i and r_j the return of share j, both being either positive or negative and with the normalization $|r_i| = |r_j| = r$,[16] and let s represent the different states of nature representing the four permutations of the return of the shares. We assume that the market is risk neutral and that the riskless interest rate is 0. Thus, $t_g = t_b = 0.5$. Under these assumptions, we can write the correlation between the price developments of any shares i and j as:[17]

$$(1)\ \rho_{ij} = \frac{\sum_s p(s) r_i r_j}{\sqrt{r_i^2 r_j^2}} = (2q-1)^2.$$

Traders inform themselves about the outlook of the share they are responsible for. Thus, they receive a costless noisy signal θ with precision p. I.e., whatever expectations the trader has about the share price going up or down from the signal, she is right with probability p and wrong with probability $(1 - p)$, with $p \geq 0.5$. If the trader assumes that the price will rise, she will take a long position, otherwise she will go short. The position of each trader is measured by its return л, which is symmetric. Thus, if she is right, she earns л, otherwise she incurs a loss of -л. Allowing the trader to take a position л is equivalent to setting an individual value

[16] Note that because markets are risk neutral and the riskless interest rate is 0, $E(r_i) = E(r_j) = 0$.

[17] In the symmetric model with uniform correlation for all shares presented in this paper negative correlation are not well defined. If a share price were negatively correlated with the market, the respective short position would serve as long position in the model. To allow for different degrees of correlation would greatly contribute to the complexity of the model without changing the basic insights.

at risk limit of л.[18] We assume that such limits are fixed and independent of the decision of other traders.[19] The signal θ is unobservable for everybody else but the respective trader and cannot be communicated. All other parameters are common knowledge.

However, as long as we assume that markets are perfect, traders do not know more about the future share prices than markets, i.e., $p = 0.5$. Knowing this, traders are indifferent between going long and short and will decide randomly to go long or short with the same probability 0.5. The assumption that specialized traders don’t know more about the development of share prices than other market participants does not seem to be very realistic. However, it should be noted that it is an implicit element of almost all calculations of value at risk as found in the literature, which usually work in a neoclassical setting and use market information without taking any special ability or knowledge of traders into account.[20]

In such a setting, value at risk depends on the degree of diversification. Because the standard deviation of the individual share is normalized to 1, we would expect value at risk to be a function of the number of traders and the correlation between the shares. In opposition to this assumption, the correlation between traders making the right or wrong decision determines the distribution of the overall result of the bank from an ex ante perspective. I.e., if many traders make a mistake, the bank will have to bear a great loss, and if many are right, the bank will earn a lot. The distribution of gains and losses is independent of how many traders went long or short, or which correlation exists between stock returns. However, in perfect markets with uninformed traders making their decisions randomly, the correlation between the traders making the right or wrong decision is 0 and risk is perfectly diversified.

Thus, for N traders, the probability $p(m)$ that m traders make the right decision (and that, therefore, $N - m$ are wrong), is

$$(2) \qquad p(m) = p^m (1-p)^{N-m} \binom{N}{m}.$$

If m traders are right, overall return is mл - (N - m)л = (2m - N)л. Given $p(m)$ for any $m = 1, \ldots, N$, one can write the distribution function of overall return and deduce value at risk,

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[18] We thereby assume that p is below the confidence level of value at risk.

[19] To assume that the limits of traders are fixed and independent of the decisions of other traders represents the actual practice. Seemingly, such a strategy disregards the findings of portfolio theory. However, on the one hand, many practical problems impede simultaneous decision making of all traders, thus an optimal allocation of value at risk with regard to diversification effects of the actual portfolio positions is not possible. On the other hand, the compensation of traders usually contains limited liability and is thus asymmetric. Therefore, traders have positive risk preferences and might collude to achieve a wider limit and greater risk, e.g. through taking opposite positions. Thereby, they trade their private knowledge against their gains from higher risk. To avoid such risks and get unbiased decisions, bank managements should (and do) keep the limits fixed.

[20] Exceptions from this rule are Dresel/Härtl/Johanning (2002) and Beeck/Johanning/Rudolph (1999).

e.g., on a 99% or 95% confidence level. The following graph shows the diversification effect with respect to overall value at risk per trader, given that, independently of the number of traders N, each trader gets the same individual value at risk limit:

insert graph 1 around here

This diversification effect would allow a bank with fixed economic capital to increase the size of its activities significantly if the trading department consists of many traders. This result does not depend on the stocks the traders are active in. It also holds if traders all deal in the same stock. However, at perfect markets there are no gains from trading, and thus there is no motivation for up- or downsizing the bank.

As a general result, we can conclude that in the neoclassical setting, from an ex ante perspective, the correlation of stocks is irrelevant.[21] This result is not robust with respect to the assumption that uninformed traders randomize their decision fairly. Traders could, e.g., have a certain bias towards going long, because in the real world short markets are often less liquid and going short is more costly.[22] As can easily be seen, in this case correlations between stock returns are no longer irrelevant, but will increase the standard deviation of the overall return. In our model, this would mean inducing more risk without earning a premia. Thus, it would be in the interest of the bank to make it equally costly for traders to go long or short.

4. Learning, Informed Traders, and Value at Risk

The discussion above does not explain why there is a bank at all. An investor could likewise randomize the investment decisions (or let his computer do it) and will not have to hire any traders. Also, if nobody has an informational advantage, there is no need to change a decision once made, so if he hires some traders, these should trade only once and stay at home afterwards. Seemingly, the delegation of decision-making needs justification. The easiest way to do this is to assume that traders know better than their principal about the potential development of the shares they are dealing in. However, because perfect markets aggregate all information in prices, which the principal can easily observe, traders must know more than markets to make better decisions than the principal.[23] In the symmetric model presented above the probability for traders to get the right signal p must be greater than 0.5.

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[21] Of course, this does not hold for a given portfolio of long and short positions in different shares. However, see the discussion of value at risk from an ex post perspective in the herding case in chapter 5.

[22] As a consequence, hedge funds specialized in short portfolios are known to be not very successful.

[23] Thus, if we don't count traders as insiders, we now assume that there is only weak information efficiency. See Fama (1970).

The setting allows for a second way to get informed: If traders know that other market participants do also receive an informative signal, they would like to observe their behavior and update expectations respectively. These other market participants might be traders from other banks dealing in the same stock. However, they could trade on their own account or to serve uninformed liquidity traders, and obviously will not tell for which of the motives they bought or sold. It is reasonable that traders in the same trading division will not hide their trading motives. Although they usually trade in different stocks, their behavior is at least informative with respect to the market trend. Thus, placing numerous traders together into one room, as it is usually done in the trading divisions of banks, should provide synergetic effects in the production of information.

Letting traders receive an informative signal about their stock without observing the behavior of others will allow them to make a profit even on the average, but it does not change the fundamental insight gained above. The fact that a trader makes a wrong or a right decision is not correlated with other traders being wrong or right. Therefore, the density function is defined by equation (2) as above, but now puts greater weights on higher results due to p being strictly greater than 0.5. Consequently, overall value at risk per trader decreases with the informativeness of the signal, as can be seen below.[24] Likewise, due to a better diversification value at risk decreases with the number of traders. Thus, equity capital is most efficiently used in trading departments with a large number of traders N. Because we assume that neither traders nor their signals are costly, trading departments should be as large as possible.

insert graph 2 around here

Thus, individual learning is an unmixed blessing. Learning from the behavior of others should likewise increase expected return, but might have adverse effects on risk if it induces many traders to make the wrong decision. In the following, we assume that traders decide sequentially in the order of indexation, and that trader n can observe the decision of traders 1 to n - 1. This information might be valuable, if stock returns are correlated (thus if $q > ½$ and therefore $\rho_{ij} > 0$), and if it induces some traders to make a different decision than without.

Whereas the first condition is fulfilled by assumption, we can imagine many parameter settings where the second is never fulfilled: Even if a trader knew for sure which market trend is relevant, she would not react to it, because her own signal is stronger. Assume, e.g., that she

[24] In the following, we uniformly use a confidence level of 1% to calculate value at risk.

knows that there is a good market trend and receives signal to go short, θ_S.[25] The conditional probability, given a good market trend, that the share price will go up is

(3) $$q(u|\theta_S) = \frac{q(1-p)}{q(1-p)+(1-q)p},$$

the probability for the share price going down is

(4) $$q(d|\theta_S) = \frac{(1-q)p}{q(1-p)+(1-q)p}.$$

Therefore, the trader will follow her own signal if $p > q$, and follow the market trend otherwise. With respect to correlation, the condition can be written as

(5) $$p > \frac{\sqrt{\rho}+1}{2},$$

and we get the following parameter sets:

insert graph 3 around here

In the area above the critical value for p, market trend and correlation between the shares are always irrelevant. Each trader decides on her own account and will not look at what other traders do, because the correlation between the different shares is too small in relation to the precision of his signal. Below, she will form her expectations conditioned on her own signal and the trading decisions of the traders that had to decide before her.

Obviously, the first trader will follow her signal, because she has no other information. The second trader will update her expectations about the market trend, taking into account the decision of the first trader. E.g., if the first trader went long (L_1), according to the Bayesian rules the second trader expects the market trend to be good with probability

(6) $$t_g(L_1) = \frac{t_g\, p(L_1|g)}{t_g\, p(L_1|g)+t_b\, p(L_1|b)},$$

i.e. the probability that the first trader decides to take a long positioning in a good market, which is $t_g(pq + (1 - p)(1 - q))$, divided by the overall probability that the first trader takes a long position, i.e. $t_g(pq + (1 - p)(1 -q)) + t_b((1 - p)q + p(1 - q))$. The probability $t_g(S_1)$ that the first trader goes short is formulated analogously. To generalize our notation, we call

[25] Because our model is symmetric, we don't have to present the calculation for the complementary case where market trend is bad and the signal urges to take a long position.

$h(1,\ldots, n - 1)$ the history of decisions taken by traders 1 to $n - 1$. We can write the general algorithm to update expectations of trader n, with $n > 1$, as

$$t_g(h(1,\ldots, n - 1)) = t_g(t_{n-1}, h(n - 1)), \quad (7)$$

with $h(n - 1)$ consisting only of the decision of trader $n - 1$. According to the law of large numbers, $t_g(h(1,\ldots, n - 1))$ would approach 1 or 0 if trader n could observe a very large number of independent decisions of other traders before deciding herself.

In our model, the information about the market trend is valuable for the trader only if it helps her to better estimate the probability for the stock she is trading in to go up or down. The probability that the share price of stock n will rise, given the history $h(1,\ldots, n - 1)$ and without taking into account the signal of trader n, is

$$p_u(h(1,\ldots, n - 1)) = t_{ng}q + (1 - t_{ng})(1 - q), \quad (8)$$

the complementary probability for the price to go down

$$p_d(h(1,\ldots, n - 1)) = 1 - p_u(h(1,\ldots, n - 1)). \quad (9)$$

Thus, the trader receives two signals about the future development of her stock with different precision and sometime opposite direction. The first is from her private observations of stock n with precision p, and the other from the history of earlier decisions with precision $p_u(h(1,\ldots, n - 1))$, if history advises to go long, or $p_d(h(1,\ldots, n - 1))$ if its advice is to go short. The trader will follow the signal with the greater precision.[26] Thus, if she receives a signal θ_{nL} to go long, he will go long if $p > p_d(h(1,\ldots, n - 1))$, and if the signal is θ_{nS}, she will likewise follow the individual signal if $p > p_u(h(1,\ldots, n - 1))$. As general condition, the trader will act in accordance with θ as long as

$$(1 - p) < p_u(h(1,\ldots, n - 1)) < p. \quad (10)$$

What if not? The first trader for whom this condition is violated, let us call her trader n_h, will follow the assumed market trend to maximize the probability to make the right decision. However, if, e.g., $p_u(h(1,\ldots, n_h - 1))$ was large enough to induce her to take a long position despite θ_{nS}, this will also hold for all following traders. They will compute $p_u(h(1,\ldots, n_h - 1))$ and discover thereby that the decision of trader n_h was not based on her private information. Consequently, they will not take into account the behavior of trader n_h to update $p_u(h)$. Thus, the said conditions are likewise violated for trader $n_h + 1$, who maintains $p_u(h(1,\ldots, n_h - 1))$ as precision of the market signal. Because $p_u(h(1,\ldots, n_h - 1)) > p$, all further private information is lost because all traders from n_h onwards herd. I.e., they follow trader n_h without taking into

[26] For a proof see appendix 1.

account their private knowledge. We call such an event an upward cascade, if it induces all traders after n_h to take a long position, and a downward cascade if they all go short.

Learning about the market trend is therefore restricted to what can be found out from the behavior of traders from 1 to n_h - 1. As can be deduced from equation (8) and (9) in combination with equation (1), the maximum achievable value for t_{ng} is

$$(11) \qquad t_{ng}^{\max} = \min\left(1, \frac{1}{2} + \frac{2p-1}{2\sqrt{\rho}}\right),$$

and the same condition holds for 1 - t_{ng}. Note that $t_{bg}^{\max} = 1$ exactly if, according to condition (5), the traders never react to the market signal. Thus, they achieve the best knowledge about the market trend in a world in which they will never use this information only.

To calculate value at risk for informed traders with potential herding, we construct a grid of long and short decisions, given that the market trend is good, but traders don't know.[27] The last assumption is without loss of generality because the model is symmetric. We characterize each knot of the grid $\Delta L(n)$ by the surplus of long over short decisions $\Delta L = \#L - \#S$, and the number of the trader n who has to make a decision in $\Delta L(n)$. In the grid, traders will go long with the probability of a positive signal θ_L given a good market,

$$(12) \qquad p(\theta_L \mid g) = qp + (1 - q)(1 - p),$$

or with probability 1 if there is an upward cascade, or probability 0 for a downward cascade. Otherwise they go short. Condition (10) for the occurrence of a cascade is equivalent to having a certain surplus of long over short decisions ΔL_h, which will lead to an upward cascade, or short over long decisions ΔS_h, in which case there will be a downward cascade. The actual numbers for ΔL_h and ΔS_h are calculated using condition (10). Note that, as a closer inspection of (6) and (7) reveals, these critical values are path independent and equal for upward and downward cascades.

Using the transition probabilities according to the rules stated above, forward induction generates the probability to reach knot $\Delta L(n)$, $p(\Delta L(n))$, which is also the probability of n traders to make $(n + \Delta L(n))/2$ long and $(n - \Delta L(n))/2$ short decisions. For each $\Delta L(n)$, and in particular for all $\Delta L(N)$, we can calculate a specific discrete distribution function over the number of right decisions of the traders, $f(m \mid \Delta L(N))$, with m being the number of traders out of N who

[27] The structure resembles the modelling of a double barrier option with a binomial tree in discrete time. See, e.g., Clewlow/Strickland (1998).

made no mistake. To achieve the overall distribution, we can aggregate the distributions of each knot $\Delta L(N)$ with weight $p(\Delta L(N))$, i.e.

(13) $$f(m) = \sum_{\Delta L(N)} f(m|\Delta L(N))\, p(\Delta L(N)).$$

However, the straightforward application of the binomial density function (2) to calculate $f(m \,|\, \Delta L(N))$ is not possible, because the probability to make the right decision is not always the same. As long as traders follow their private signal, they are right with probability p. In an upward cascade, they are right with probability q, in a downward cascade this probability is $(1 - q)$. Because m right decisions out of N can be due to any feasible combinations of m_θ traders being right when acting in accordance with their signal θ, and m_C traders being right when following the assumed market trend and ignoring θ. To complete notation, let $\#n_\theta$ be the number of decisions following the private signal, and $\#n_C$ the number of decisions taken when in a cascade, these numbers being unequivocally defined by $\Delta L(N)$, ΔL_h, and ΔS_h. The probability for each m at $\Delta L(N)$ is

(14) $$p(m|\Delta L(N)) = \begin{cases} p(m_\theta, \#n_\theta) & \left| -\Delta L_h \le \Delta L(N) \le \Delta L_h \right. \\ \sum_{m_\theta, m_C} p(m_\theta, \#n_\theta)\, p(m_C, \#n_C) & \left| \begin{array}{l} \text{else, with } m_\theta, m_C \ge 0, m_\theta \le \#n_\theta, \\ m_C \le \#n_C, m_\theta + m_C = m \end{array} \right. \end{cases}$$

In the second case, $\Delta L(N)$ is reached by a cascade, and we add up the probabilities of all combinations of m_θ and m_C that are feasible to achieve m right decisions in $\Delta L(N)$. The partial probabilities of these calculations are binomial according to

$$p(m_\theta, \#n_\theta) = p^{m_\theta}(1-p)^{\#n_\theta - m_\theta}\binom{\#n_\theta}{m_\theta}, \text{ and}$$

(15) $$p(m_C, \#n_C) = q^{m_C}(1-q)^{\#n_C - m_C}\binom{\#n_C}{m_C} \text{ or}$$

$$p(m_C, \#n_C) = (1-q)^{m_C} q^{\#n_C - m_C}\binom{\#n_C}{m_C},$$

the last distinction depending on $\Delta L(N)$ being reached by an upward or downward cascade.

Overall return is mл - (N - m)л = (2m - n)л, thus using $f(m)$ we are able to calculate the distribution function of overall return. In the following graph, we compare the distribution function for fifty informed traders with and without herding potential:

insert graph 4 around here

The distribution function for traders with herding potential clearly reveals its origin from three overlapping distribution functions: One from traders deciding without regard to the market, the second from an upward cascade and the third from a downward cascade. The two last-mentioned distributions are responsible for the remarkably fat tails of the overall distribution compared with the distribution for traders without herding potential. This effect depends on correlation between the share prices, which is also a measure for the relevance of the market trend. Thus, value at risk increases with correlation, converging to a maximum value that is equivalent to the value at risk without any diversification effect.

In graph 5 below we compare value at risk per trader for fifty traders given a herding possibility, with value at risk per trader for the same fifty traders under different assumptions. E.g., if all traders receive a signal but do not take the behavior of other traders into account, value at risk is at a minimum of 0.238 and independent of correlation. In the herding case, this minimum is reached only if the correlation is rather small and therefore nobody follows the market. Value at risk is also independent of correlation between share prices if traders receive no signal and decide randomly. However, in this case traders on the average don't earn money. Nonetheless, value at risk for informed traders with herding option might exceed this value if correlation is high enough.[28]

To link our observations with the conventional way to calculate value at risk from an ex post perspective, we assume that all traders, for what reason so ever, take a long position and that therefore correlation between share prices is relevant again.[29] With respect to the ex ante perspective, we cover value at risk for the worst case of traders' decisions this way. The resulting value at risk shows a structure similar to value at risk with herding, rising from a minimum value resulting from perfect diversification to a maximum equivalent to the non-diversification case. However, its values are always higher, and the relative increase compared to the herding case changes much with correlation. Thus, the conventional calculation will not only achieve the wrong level but also a wrong structure of value at risk as a function of correlation between share prices. We will come back to this issue at the end of in section 6 when dealing with the ex post-calculation of value at risk.

insert graph 5 around here

[28] In our example, this is the case for $\rho > 0.12$.
[29] In this case we dropped the assumption that the market trend is good to maintain the symmetric structure of our analysis.

5. Optimal Structure of Trading Divisions

For optimization we have to introduce an efficiency measure. As such, we apply a value at risk based RORAC, i.e., the expected return per unit of capital which is needed to limit the probability of failure to a certain level, in our case to 1%. However, with respect to the maximization of RORAC, the discreteness stipulation for value at risk stemming from the over-simplifying two-point distribution of returns introduced above poses severe problems. For 50 traders, it allows only for returns (and thus value at risk) on equal numbers on the interval [-50, 50]. Particularly if value at risk is low, minor policy changes can lead to jumps of the RORAC function if, e.g., value at risk thereby increases from 2 to 4. To a great degree, an optimal policy would consist of the exploitation of this effect.

Therefore, we further randomise the return distribution to (almost) normality. Let the return for each trader be determined by the following simple stochastic process: The trader draws independently 20 times and with equal probability 0.5 from the returns 0.1 and 0, if her expectation about future stock prices proved to be correct, and from -0.1 and 0 if she is wrong. Thus, she still earns 1 on the average if she is right and -1 if not, but returns are binomially distributed on the interval [0, 2] if she is right, and [-2, 0] otherwise. Note that, as a consequence of this modification, a position earning л on the average if the trader makes the right decision is no longer equivalent to setting an individual value at risk limit of л.[30]

If we assume that the number of traders is *N*, these draw independently 20*N* times. Let л(*N*) be the overall return of *N* traders and assume for a moment that all traders are correct in their anticipation of the share price development. Thus,

$$(16) \quad p(\pi(N) | m = N) = 0.5^{20N} \binom{20N}{10\pi(N)},$$

where л(*N*) is defined, and 0 otherwise. The respective distribution over л(*N*) approximates a normal distribution around the mean *N*. The return distributions for other values of *m* are the respective (almost) normal distributions around the mean 2*m* - *N*. Thus, for any *m* we can generate a distribution *f*(л(*N*)|*m*) that is almost normal and discrete on the rather small interval 0.1 over [*m* - *N*, *m* + *N*] and therefore less prone to distortions from the discreteness stipulation. Overall distributions are attained according to (13) with *m* substituted by л(*N*). We use

[30] Value at risk of a position earning л on the average if the trader makes the right decision is 1.5 for $p = 0.5$, and somewhat smaller if the trader calculates value at risk in a non-neoclassical manner and attributes a higher precision to his decision.

this overall distribution to calculate value at risk and expected return μ to achieve our efficiency measure RORAC = μ/value at risk.

What are the efficiency effects of herding? At first sight, it should increase the value of a bank because traders follow a signal of greater precision. Consequentially, the expected return is strictly greater in the herding case. However, compared to the precision of the private signal, the market signal in a cascade has an only somewhat greater precision, which will not increase beyond what the first trader in a cascade receives. On the other hand, risk will increase with every trader in a (potentially wrong) cascade. Thus, if a bank wants to maintain a certain level of default probability with a given amount of capital, traders in a bank where herding is possible should have much tighter limits and consequently will earn less money.[31]

We demonstrate this effect for a bank with one unit of capital, using the same parameters as in the foregoing section. As can be seen in the chart below, for low levels of correlation, RORAC is high, because traders don't react to the market signal and the portfolio of 50 traders is well diversified. As soon as correlation is strong enough to generate informational cascades destroying this diversification effect, RORAC decreases dramatically.[32] Thus, herding can be very costly.

insert graph 6 around here

A potential response to this is to keep trading departments small to avoid herding, and the smaller, the greater the correlation and the higher the precision of the traders' signal (see the arguments for graph 3). However, thereby the original diversification effect is also destroyed. The trade off between these two effects allows to determine an optimal size of a trading department with unlimited communication. However, if, for any reason, trading departments must be large, they should be as large as possible, because the expected return per trader is constant in the number of traders in a cascade, the relative changes of the proportion of traders in a cascade in N converges to 0 when N is large, and value at risk per trader decreases in the number of traders through diversification.

Larger trading departments can also be justified if the bank can control information flow. Of course, in doing so the bank management could always force the traders to decide in isolation.[33] It is more interesting to look for communication rules that allow and optimize learning.

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[31] We could argue alike using any utility function in which expected return of a cash flow has positive and risk negative value.

[32] The jumps in the RORAC-function are due to the necessarily discrete changes of the critical level ΔL_h in ρ. If ΔL_h does not change, RORAC increases because with a higher value for ρ traders in a cascade act due to a signal of greater precision.

[33] See the discussion in Banerjee (1992), p. 811.

Assume, e.g., that, to avoid a premature cascade, the trading decisions of the first $n_l > \Delta L_h$ traders could be hidden from these and full information will be given to traders $n_l + 1$ to N only.[34] Thus, market signals inducing trader $n_l + 1$ to begin a cascade will have at least the precision of the equivalent market signal in a cascade with unlimited communication. Obviously, given an unlimited number of traders, n_l could be set that high that traders learn about the true market trend with almost certainty, and, if according to (5) this information is valuable, the overall surplus converges to first best, i.e., what could be achieved with full knowledge about the signals of all traders, as N approaches infinity.[35]

In a more realistic setting with a limited number of traders, when deciding about n_l the bank management must look at the trade off between the precision of signals obtained from traders 1 to $n_l - 1$ and the exploitation of this additional information through traders n_l to N. Of course, potential gains depend on the number of traders, and for a small number of traders it remains second best not to allow any herding. In the following, we calculate RORAC for $N = 50$ traders for different levels of correlation:

Insert graph 7 around here

Note that n_l is rather large and the efficiency gains are not very impressive for low correlations compared to traders' isolation, because the learning mechanism is not very precise: The paths of traders' decisions which are on the interval between ΔL_h and ΔS_h in n_l might enter a cascade later on, and if they do so, this cascade will have the minimum precision for a cascade. Also, some paths will lead into a cascade in n_l with the minimum or a slightly higher precision. In both cases, a cascade should be prevented to enhance efficiency.

Thus, given that the bank management can distinguish between long and short decisions,[36] the simple communication rule described above is only third best. Optimal learning can be achieved if the bank management makes the information transfer contingent on both the surplus of long over short decisions and on the number of remaining traders that could exploit the market signal. However, to implement such optimal information policy is rather difficult because value at risk is not time consistent. I.e., a policy to maintain a certain level of value at risk for all 50 traders from an ex ante perspective is not equivalent to a policy which main-

[34] See Cao/Hirshleifer (2000) for a similar proposal.
[35] The logic is equivalent to the application of the folk theorem in infinitely repeated principal-agent games without discounting, e.g., in Radner (1981), except that in this case observation concerns the market trend and not the behavior of an agent.
[36] This assumption is not trivial but reasonable: In the model, long positions are positions whose payoff is positively correlated with the market. Bank management might not know about the actual correlation of each paper with the market. However, traders have to know about the correlation of other papers with the market if we allow them to learn about the market trend from the trading decision of the respective traders. Thus, we might expect this information to be common knowledge at least inside the bank.

tains the same level in every knot of the decision grid.[37] As a consequence, maximization of value at risk-based RORAC is also not time-consistent. Thus, we cannot use methods of flexible planning through backwards induction to identify the optimal information policy.[38]

At first sight, the optimal information policy could be derived through the comparison of any combination of disclosure and non-disclosure in any knot of the decision grid. However, such methods of brute force will not avail. For 50 traders, and therefore 1250 knots, the number of permutations is 2^{625}, which is not tractable. Even a substantial reduction of the relevant area of disclosure on the decision grid or a reduction of the number of traders will not reduce the number of information policies sufficiently.[39] Thus, we have to entertain some heuristic algorithms to stepwise approach the optimal information policy, which consists of functions of optimal trigger levels for cascades, $\Delta L^*(n) > 0$ and $\Delta S^*(n) = -\Delta L(n) > 0$, with $n = 1,\ldots, N$ traders. Thereby, we follow some intuitive axioms:

1. Because bank management does not know if the market trend is good or bad, the optimal information policy is symmetric with respect to a surplus of long over short and short over long decision, i.e., $\Delta S^*(n) = -\Delta L^*(n)$.

2. Obviously, $\Delta L^*(n)$ is for all n greater or equal to ΔL_h.

3. To hide information in a certain knot of the grid can be understood as an investment into a better precision of the market signal, which might trigger off a cascade later on. The potential gains from such an investment decrease with the number of traders who already made their decision, and with the precision already achieved. Thus, we assume $\Delta L^*(n)$ to (weakly) decrease in n.

In general, the respective algorithms start at an extreme solution, e.g., no information is revealed at all (or no information hidden at any time), and stepwise reveal (or hide) information as long as doing so increases RORAC.[40] As can be seen below, the resulting information policies depend on correlation. Because a high correlation makes herding more attractive, there is tendency to reveal more information when correlation is high.

Insert graph 8 around here

The table below demonstrates that the respective gains from an (almost) optimal information policy are much higher if correlation is high. Thus, although a policy in accordance with

[37] See Franke (2000).
[38] Appendix 2 presents a simple numerical example for the time inconsistency of value at risk.
[39] Optimization over 20 relevant knots only would make it necessary to calculate the RORAC for more than one million permutations already.
[40] See appendix 3 for a description of the algorithm.

$\Delta L^*(n)$ dominates any other information policy, in managerial practice it might not be worth to implement such a policy if this is costly and correlations are modest. In these cases, bank management should rely on simple communication rules like information release in trader n_l, as discussed above, or even overall isolation. In markets with generally higher correlations between stock prices and in situations where markets tend to show strong correlations (like in a market crashes or booming market), a more efficient information policy might promise substantial gains.

Correlation	Isolation	Fixed n_l (n_l)	$\Delta L^*(n)$
0.2	0.4202	0.4327 (47)	0.4440
0.4	0.4202	0.4624 (46)	0.4946
0.6	0.4202	0.4957 (45)	0.5744

Table 1: RORAC under different information policies

We conclude that the optimal structure of trading departments is at first hand a structure of information flow between traders. The most unfavorable way to organize this information flow is to allow unrestricted communication. By comparison, the isolation of traders achieves far better results, although it is not optimal. To accomplish the needed degree of isolation, it is sufficient to let traders act in isolated groups that are sufficiently small to avoid herding. An active use of market information is made if the bank's management informs the traders after a fixed number of decisions n_l, or in accordance with a function $\Delta L^*(n)$ defining the trigger level of disclosure. Both strategies are very demanding with respect to the knowledge of the bank management, and an implementation seems not realistic at the actual state of the art. Nonetheless, banks with large trading departments should be aware that free flow of information between the traders can be very costly and therefore should rethink their organizational structure accordingly.

6. Discussion and extensions

Although we present a rather rudimentary model of stock markets and a trading department, a number of rather realistic features can be identified and discussed: Thus, we can describe a bearish or bullish mood of a trading department as rational behavior due to imperfect information and learning. However, because such rational herding leads to an alignment of behavior, it substantially increases risk. Meanwhile, expected return increases less, because information aggregation stops when the first trader just follows the market signal (i.e. her colleagues). Thus, rational herding is rather costly for banks that value risk negatively. Our paper presents some determinants for the extent of such costs, which are particularly high if correlation between shares is high.

The efficiency effects of herding in our model are particularly severe because every trader is immediately informed about earlier trading decisions. Extensions of our model could include retarded or stochastic disclosure. Both would not change the precision of traders' decisions in cascades. However, it is less probable that traders follow the market trend, because cascades might break due to additional information stemming from traders who did not follow the market trend. The resulting portfolios are better diversified, we expect RORAC to increase. However, in such an extension distributions would be path dependent, so we would need Monte-Carlo-simulation for solving.

In the following we consider the efficiency effects of „gurus" or opinion leaders in a bank, meaning traders who are said to be particularly well informed and therefore have great influence on the trading decisions of other traders. Although at first sight it seems favorable to hire traders whose judgment has greater precision, these might trigger off cascades based on insufficient precision of the resulting market signal. If the precision of their private signal is not very high, the overall effect is negative and banks employing such gurus have a markedly higher risk or a lower RORAC than banks without, although they might not be aware of it.

Assume, e.g., that the first trader's signal has precision $p + \Delta p$ with $\Delta p > 0$. All other traders receive signals with precision p. As we can see in the graph below, RORAC increases continuously in Δp, but jumps downwards whenever the information surplus reduces ΔL_h by one step. If bank management wants to be certain that the ability of the guru increases bank's RORAC, for 50 traders, $p = 0.55$ and a correlation $\rho = 0.2$, the surplus of the guru over the non-information case $p_0 = 0.5$, $p + \Delta p - p_0$, must be about 550% larger than the surplus of a „normal" trader, $p - p_0$. We would not expect even the best expert to have such an advantage over her colleagues. For higher correlations, this value decreases, for $\rho = 0.4$ to about 200% and for $\rho = 0.6$ to 100%. Also, for higher correlation the result in between p and $p + \Delta p$ is very much ambiguous. Thus, banks should be very careful about the doings of their opinion leaders, particularly if correlation is low. Or to put it another way: Trendsetters are valuable only if the trend is strong. Otherwise, they are at first hand risky.[41]

insert graph 9 around here

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[41] Note that we regard the case of unrestricted information flow only. It is obvious that in any of three information policies described above, RORAC increases with increased knowledge of any traders.

So far, we did not discuss traders' incentives or the costs of traders' salary and transactions. In our setting, it can be easily shown that traders always decide to their best knowledge if they earn more if their bet goes right. Obviously, if traders were risk averse, the respective second-best incentive function would be almost flat. Under risk neutrality any salary allowing them to earn more in the case of success would be second best to induce the assumed behavior.[42] Thus, we do not have to minimize the costs of the incentive scheme like in the canonical principal agent models, but pay every trader a salary representing her outside option. Therefore, we do not expect any substantial new insights from the inclusion of the traders' incentive scheme into the model. However, a potential extension of the model might deal with the optimal number of traders for a given outside option value and a given amount of equity capital.

The agency-perspective offers some additional insight into the co-ordination problem. Seen from this perspective, the adverse effects of herding are a consequence of the imperfect implementation of the bank's notion of risk into traders' behavior through the traders' individual value at risk limits and earnings maximization. This imperfection is no surprise, because traders' decisions are interrelated in their consequences, whereas limits are fixed and independent of other traders' decisions. However, it is interesting to note that the optimal information policy $\Delta L^*(n)$ discussed above is a perfect substitute for such an optimal incentive function because it uses any available information efficiently.[43]

Finally, the model gives new (and not very favorable) insight into the logic of „neoclassical" risk limits in risk management. A crucial assumption in the respective calculations is symmetric information, i.e., prices fully reflect all available information. Thus, traders have no additional information and do hold risky positions quasi accidentally. To give traders additional information should at first sight enhance the situation of the bank, and this both from the ex ante and ex post perspective. Assumed that is true, the perceived value at risk would be wrong, but at least above its true value. Banks and regulators enjoyed an additional safety cushion due to neoclassical ignorance. However, the exact confidence level that is used to calculate value at risk is arbitrary, at best experience based. Thus, the safety cushion might be included already, and it might be desirable that every bank has roughly the same safety cushion.

In section 4 we compare value at risk with herding to value at risk in the worst case in a neoclassical setting, i.e. if all traders take a long position, and find that the relative size of the

[42] Thus, other reasons might be responsible for the high performance sensitivity of traders' salaries observed in reality, e.g., the possibility to justify exorbitant salaries.
[43] Note that in our contract setting it is not possible to make traders reveal their signal truthfully except through their trading decisions. We might think of signaling games that can.

safety cushion changes much with correlation. The same holds from the ex post perspective of, e.g., a treasurer who intends to control risk for a given portfolio. As we demonstrate below, its size also depends a great deal on the actual composition of the portfolio.

To calculate ex post value at risk, we construct the return distribution $f(\pi(N)|\Delta L(N))$ for any portfolio defined by the number of short and long decision for N traders, which is described by $\Delta L(N)$. Following the procedure above, in a first step we find out about the distribution of traders making the right decision, $f(m \mid \Delta L(N))$, before modifying the return distribution according to (16). Because bank management does not know about the true market trend, we have to take both the good and the bad market trend into account. However, due to the symmetry of our model,

$$\begin{aligned} & f\left(m|\Delta L(N)\right) \\ (17)\quad & = \frac{p_g\left(\Delta L(N)\right)}{p_g\left(\Delta L(N)\right)+p_b\left(\Delta L(N)\right)} f_g\left(m|\Delta L(N)\right)+\frac{p_b\left(\Delta L(N)\right)}{p_g\left(\Delta L(N)\right)+p_b\left(\Delta L(N)\right)} f_b\left(m|\Delta L(N)\right) \\ & = \frac{p_g\left(\Delta L(N)\right)}{p_g\left(\Delta L(N)\right)+p_g\left(-\Delta L(N)\right)} f_g\left(m|\Delta L(N)\right)+\frac{p_g\left(-\Delta L(N)\right)}{p_g\left(\Delta L(N)\right)+p_g\left(-\Delta L(N)\right)} f_g\left(m|-\Delta L(N)\right), \end{aligned}$$

thus it is sufficient to calculate the distributions for the good market only.

To calculate value at risk along neoclassical lines, we have to ignore traders' decisions and take portfolios as arbitrary. Therefore, in a good market, long positions earn money with probability q and loose with $(1 - q)$, the probabilities for short positions to earn money vice versa. Thus the probability that m_L out of a total of $(N - \Delta L)/2$ long positions earn money in a good market is

$$(18)\qquad p\left(m_L|g,\Delta L\right)=q^{m_L}\left(1-q\right)^{\frac{N-\Delta L}{2}-m_L}\begin{pmatrix}\frac{N-\Delta L}{2}\\ m_L\end{pmatrix},$$

the respective probability for short positions

$$(19)\qquad p\left(m_S|g,\Delta L\right)=\left(1-q\right)^{m_S} q^{\frac{N-\Delta L}{2}-m_S}\begin{pmatrix}\frac{N-\Delta L}{2}\\ m_S\end{pmatrix}.$$

Using the symmetry of our model we get

(20)
$$p(m|\Delta L(N)) = p_g \left(\sum_{\substack{m_L, m_S, \\ m_L + m_S = m}} p(m_L|g, \Delta L) p(m_S|g, \Delta L) \right) + (1 - p_g) \left(\sum_{\substack{m_L, m_S, \\ m_L + m_S = m}} p(m_L|g, -\Delta L) p(m_S|g, -\Delta L) \right).$$

This way, we construe the distribution $f(л(N)|\Delta L(N))$ for both the herding and neoclassical case. For $p = 0.55$, we get the value at risk numbers as follows:

insert graph 10 around here

For most portfolios and correlations, the safety cushion exists. However, its relative size differs a lot for different correlations and portfolios, as can be seen below:

insert graph 11 around here

For the given parameters, the safety cushion is particularly small for well diversified portfolios and for badly diversified portfolios, and it also decreases in size if correlation gets high, i.e., when „extreme" portfolios with a great surplus of long or short positions are more probable. Thus, „neoclassical" value at risk will not dampen booms nor be able to provide a large safety cushion when it is most urgently needed, i.e., when we observe strong market trends due to extreme developments on the markets.

Neoclassical value at risk is below its herding equivalent if correlation is high and portfolios are well diversified. With high correlation, the well-diversified portfolios occur with a very low probability only, because in this case an informational cascade is almost certain. Thus we might like to ignore the resulting systemic risk. However, the relationship between value at risk in the neoclassical and herding case is also defined by the precision of traders' signal. If precision is low, banks need more capital, and we might expect that neoclassical value at risk is not sufficiently high for a wider range of parameter constellations. We demonstrate this effect for $p = 0.51$:

insert graph 12 around here

Now, even with modest correlations, well-diversified portfolios are riskier than perceived. For high correlations, the gap reaches about 40% of „neoclassical equity", and there is no safety cushion for many portfolios. Again, this result raises questions about the validity of value at risk, particularly in states of the world in which sufficient reserves are vital for the stability of the financial system.

Whereas our model could be applied also to herding of traders across different banks, there is no strategic interaction across the market, because all traders are price takers. In the real world, herding behavior of banks with significant trading volume would lead to price adjustments on the markets. It is reasonable to assume that these price reactions would make it less favorable to follow the trend and might even break informational cascades. The respective extension of our model is beyond the scope of our paper.

Note finally that many of our results are not due to the use of value at risk in risk management. Any risk measures calculated under the assumptions of perfect markets suffer the same fallacy. However, with respect to practical consequences we must admit that it is much easier to demonstrate the failures of such measures under imperfect market conditions than to calculate risk for informed traders correctly. Therefore, a general conclusion of our results with respect to prudential regulation is that it should not rely too much on quantitative rules as long as the underlying incentive aspects are not sufficiently well understood and the resulting effects cannot be included in the respective models.

7. Conclusion

In our paper, we demonstrate the effects of rational herding of informed traders on risk management with value at risk. Taking a neoclassical calculation of value at risk as benchmark, we achieve lower numbers for value at risk because risk is diversified through the individual traders' decisions. However, the size of the respective safety cushion differs a lot with correlation between share prices and the respective degree of herding activities. It is particularly small if correlation is high, like, in a boom or market crash.

From an optimization perspective, banks should control information flow in their trading departments, particularly if correlations are high. To simply isolate traders from each other already doubles RORAC. A yet better, third-best strategy is to hide the first n_l decisions of traders. Second best could be achieved through an information policy $\Delta L^*(n)$ that makes the disclosure of traders' decisions contingent both on the number of traders n who already made their trading decision and the surplus ΔL of long over short decisions of these n traders. Due to time-inconsistency of value at risk and value at risk-based RORAC, we are not able to identify this optimal policy and have to approach it through imperfect heuristic procedures. The result is a further increase in RORAC, which is particularly strong if correlation is high. Thus, if the correlation between the risky position of the different deciders in a bank is low and sophisticated information policies are costly, banks could rely on rudimentary information policies like, e.g., perfect isolation or if delayed disclosure. They have to invest into information policy if the general market trend is stronger.

In an extension of the model, we discuss the effect of a „guru", i.e. an expert who is very well informed and has great influence on the decision of other traders. Thereby, with unlimited information flow she could effect informational cascades and increase overall risk. The respective efficiency losses are high if correlation is low. Seemingly, gurus are useful if market trends are strong and should be fired if markets are back to normal. Obviously, gurus are always valuable if the bank uses one of three information policies described in section 5.

Finally, we compare value at risk and RORAC for the neoclassical and herding case. We observe that neoclassical calculation often provides a safety cushion if the precision of the traders signal is high. However, in this case the relative size of this cushion is particularly small if correlations are high and traders more often choose „extreme" portfolios, i.e., portfolios with a great surplus of long or short decisions, or if they are particularly well diversified, i.e. $\Delta L(N)$ is close to 0. If the precision is low, neoclassical calculation underestimates risk for many portfolios. As long as value at risk is calculated in a neoclassical setting, these results diminish the worth of value at risk to limit systemic risk in banking.

Appendix 1

To prove that traders follow the signal with the greater precision, we regard the case when trader n observes θ_{nS}. According to the rule stated in the text, she should go long if
$p_u(h(1, n - 1)) = t_{ng}q + (1 - t_{ng})(1 - q) > p$,

and short otherwise.

The decision to go long is right if the probability that the share price goes up, given $h(1, n - 1)$ and θ_{nS}, is greater than ½. This condition can be written as

$$\frac{p(\theta_{nS}|u)}{p(\theta_{nS})} = \frac{p(\theta_{nS}|u)}{p(\theta_{nS}|u) + p(\theta_{nS}|d)}$$
$$= \frac{t_{ng}(1-p) + (1-t_{ng})(1-q)(1-p)}{t_{ng}(1-p) + (1-t_{ng})(1-q)(1-p) + t_{ng}(1-q)p + (1-t_{ng})qp} > \frac{1}{2},$$

which simplifies to $t_{ng}q + (1 - t_{ng})(1 - q) > p$.

Appendix 2

To prove time-inconsistency, we employ a simple numerical example. We regard the choice between four different distributions, V_{11}, V_{12}, V_{21} and V_{22}. Returns according to V_{11} are -2, -1,…, 7 with equal probabilities of 10%. Likewise, returns in V_{12} are -3, -2,…, 1, 4, 5,…, 8 with the same probability for each realization. Expected return for both is 2.5. Thus, V_{12} results from a mean preserving spread and should always be inferior for risk avers deciders. V_{21} is constructed like V_{11} with returns of -4, -3,…, 5, and therefore a lower mean 0.5. However, through choosing the riskier V_{22} one could earn a risk premia: Returns are -5, -4,…, -1, 2, 3 with equal weights of 10% and 6 with the weight of 30%. The corresponding mean is 0.8.

In t_1, the decider is allowed to make a strategy choice maximizing his value at risk-based RORAC, with value at risk at a confidence level of 10%. In t_2, he receives a signal telling him if he is in a good state of nature and is allowed to choose between V_{11} and V_{12}, or in bad state with only V_{21} and V_{22} available. Thus, in t_1 a strategy defines his choice in these two states. We can calculate RORAC for the four alternative strategies, and observe that, despite the risk premia earned with V_{22}, the optimal strategy is (V_{11}, V_{21}), i.e. to minimize risk in any state of nature.[44]

However, the revelation of the state of nature in t_2 reduces risk. Therefore, the risk premia earned with V_{22} is now sufficiently high to compensate for the additional risk. Consequently, in the bad state the decider would like to deviate from the optimal strategy for t_1 and to choose V_{22} now.[45] Thus, decisions based on a value at risk-based RORAC are not always time consistent.

Appendix 3

To calculate the almost second best information policy, we start with the non-disclosure case. In a first step, we identify the disclosure level $\Delta L_l(50)$ in column $n = 50$ of the decision grid that leads to the largest increase in RORAC, given non-disclosure in all other columns. In the second step, given disclosure in $\Delta L_l(50)$, we test for the largest increase of RORAC through disclosure in column $n = 49$. Thus we achieve a function $\Delta L_l(n)$ defined on ($n(l)_{\min} = 49$, 50). In the following steps, we look for the largest increase of RORAC through (more) disclosure in one of the columns for which $\Delta L_l(n)$ is already defined, or in $n = n(l)_{\min} - 1$, and modify $\Delta L_l(n)$ accordingly. We test for changes of $\Delta L_l(n)$ with $\Delta L_l(n)$ monotonously decreasing only.

[44] The respective RORAC is 0.75 for (V_{11}, V_{21}), 0.55 for (V_{12}, V_{22}) and (V_{11}, V_{22}), and 0.5 for (V_{12}, V_{21}).
[45] RORAC in t_2 is 2.5 for V_{11}, 1.25 for V_{12}, 0.167 for V_{21} and 0.2 for V_{22}.

The algorithm stops when there is no column in which more disclosure would increase RORAC. We use the resulting $\Delta L_l(n)$ as our approximation of the second best information policy $\Delta L^*(n)$. Additionally, we control for stability of our result with respect to less disclosure in any column. This procedure does not increase RORAC in any instance.

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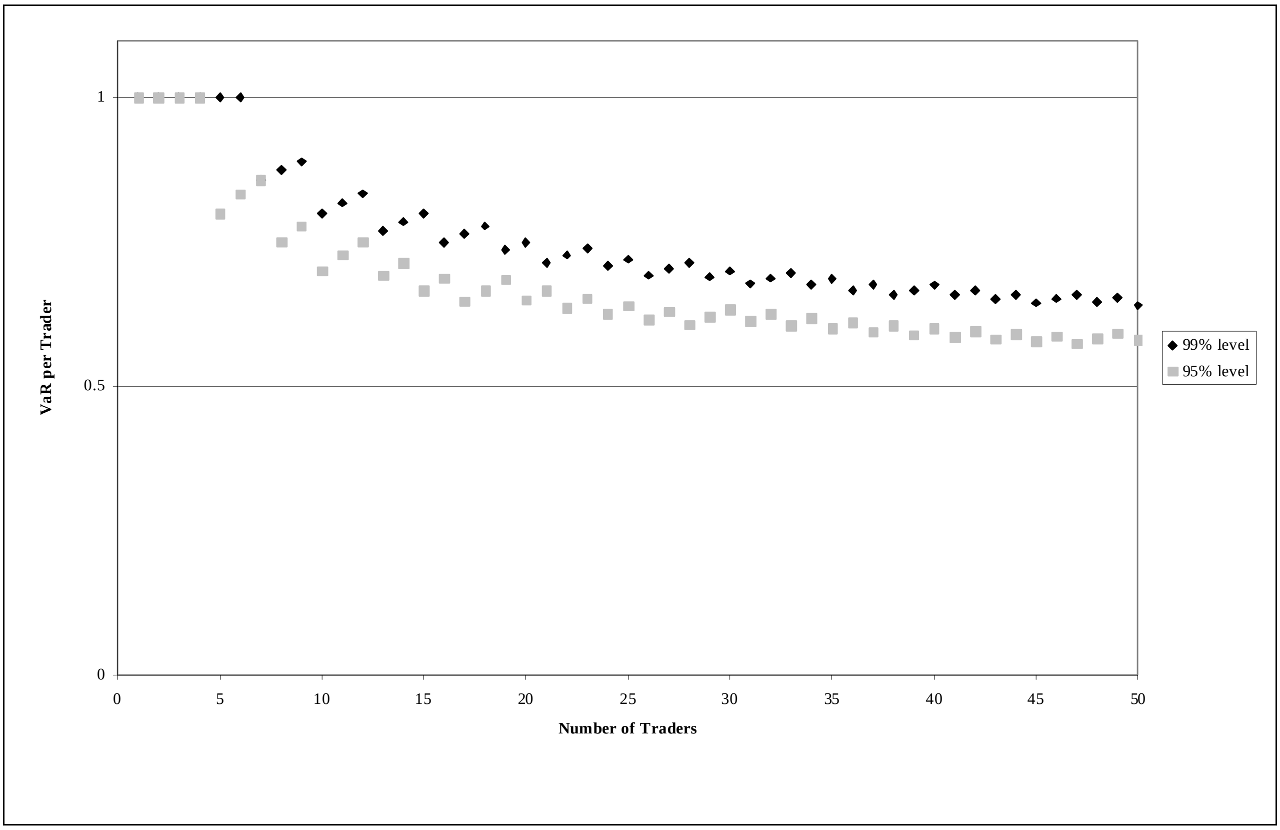
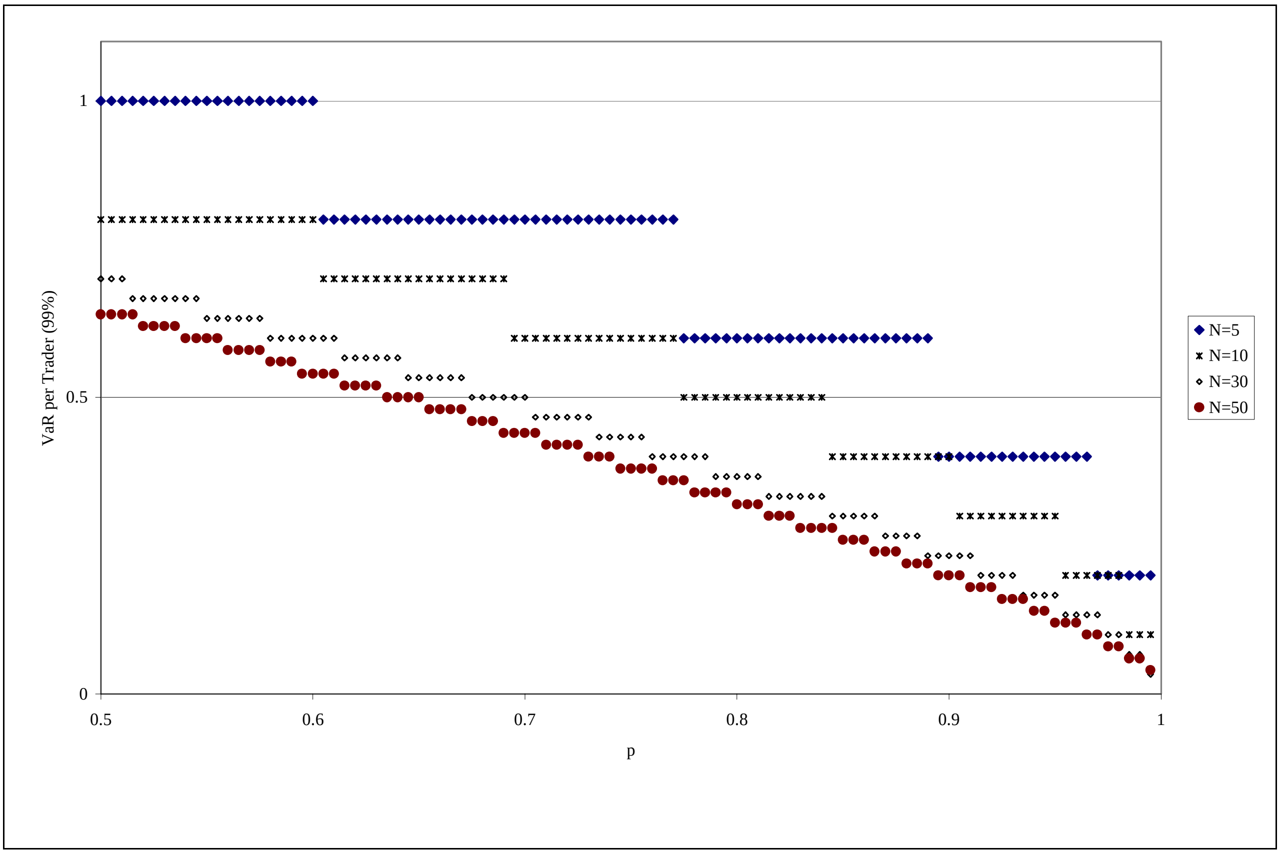
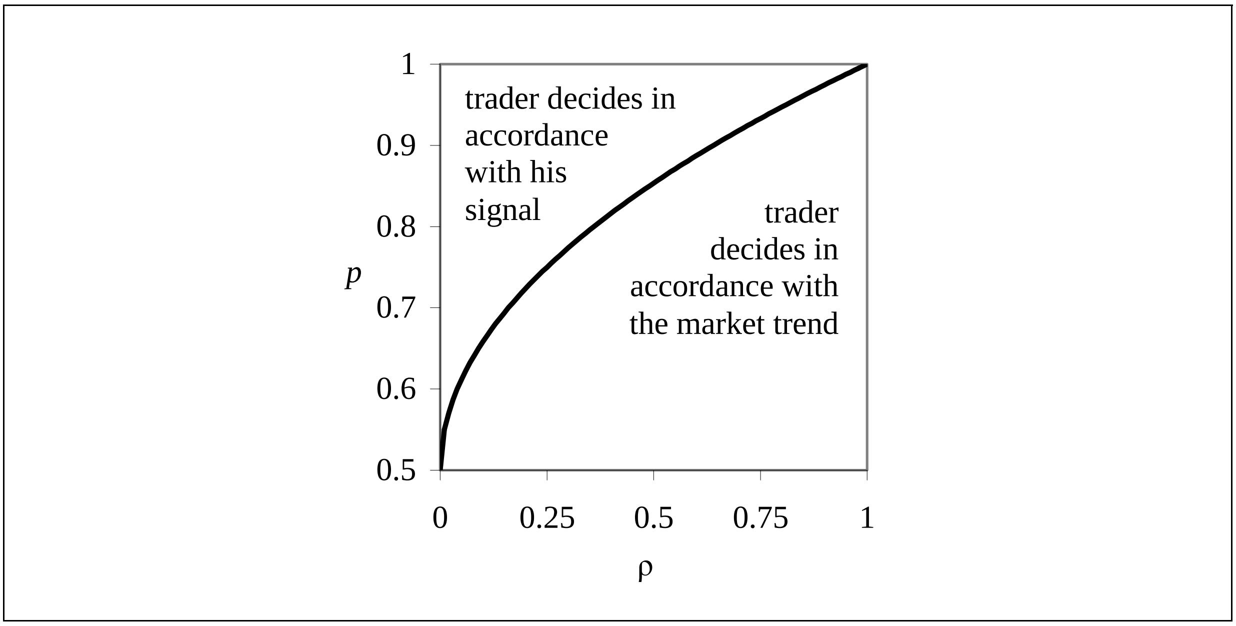
Graph 1: Overall value at risk per uninformed trader subject to the number of traders



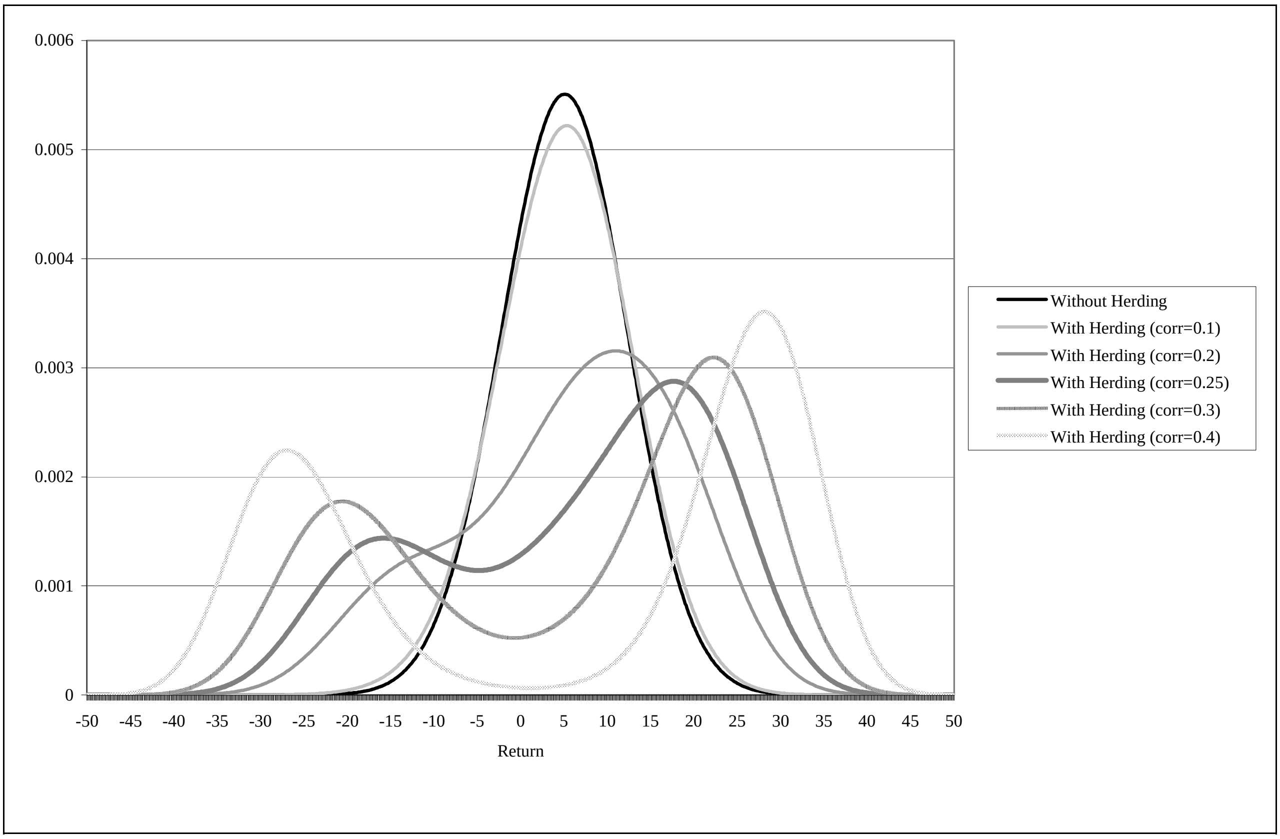
Graph 2: Value at risk per trader's position subject to the precision of the private signal



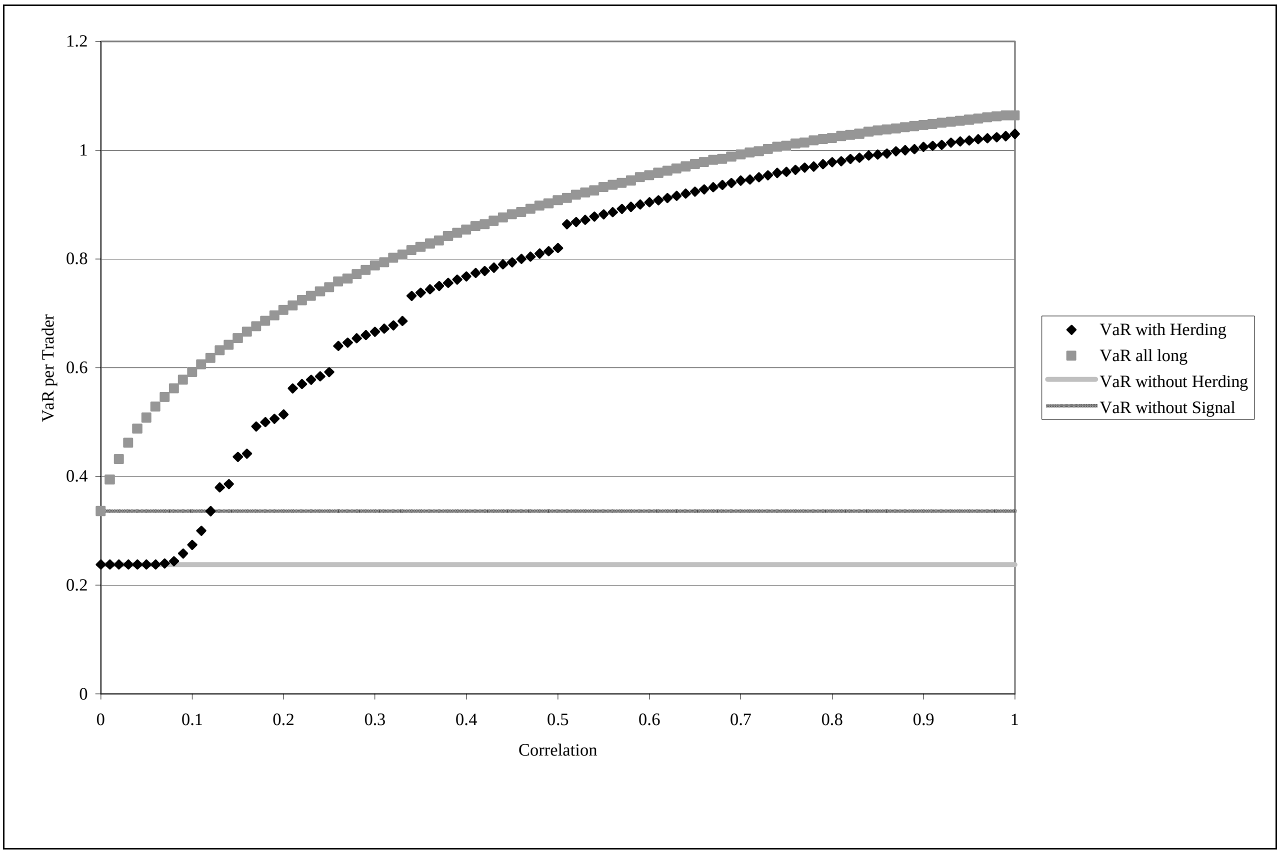
Graph 3: Trader’s decision given full knowledge about the market trend



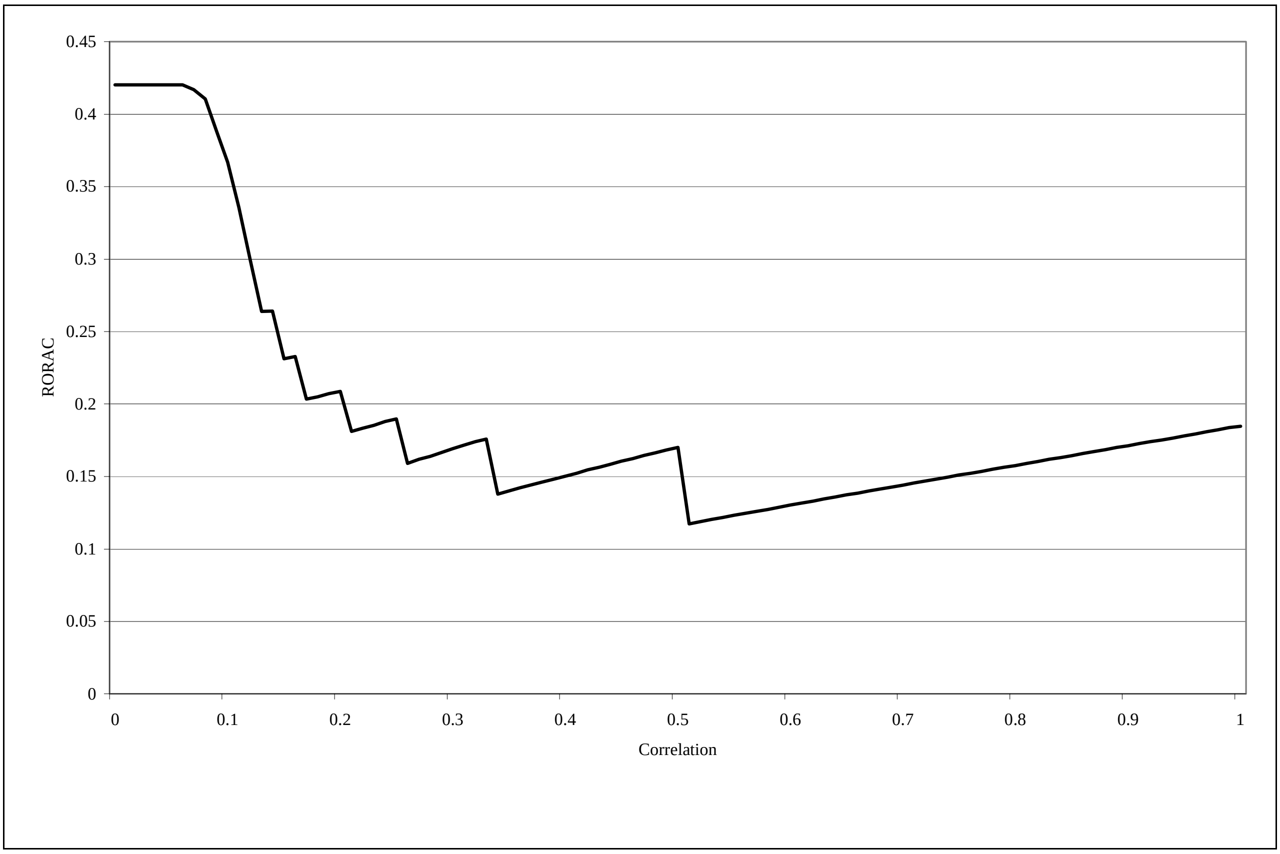
Graph 4: Return distributions with and without herding



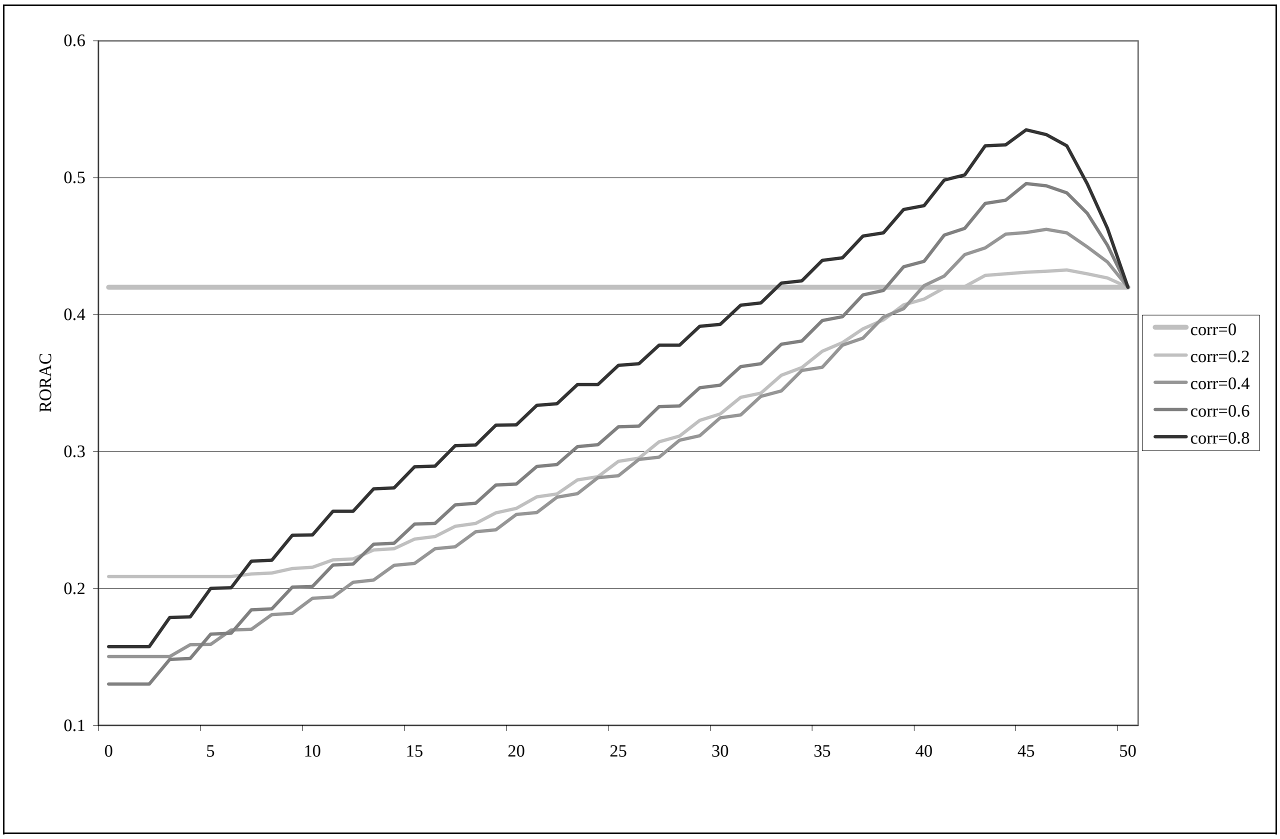
Graph 5:Value at risk per trader subject to correlation for different scenarios



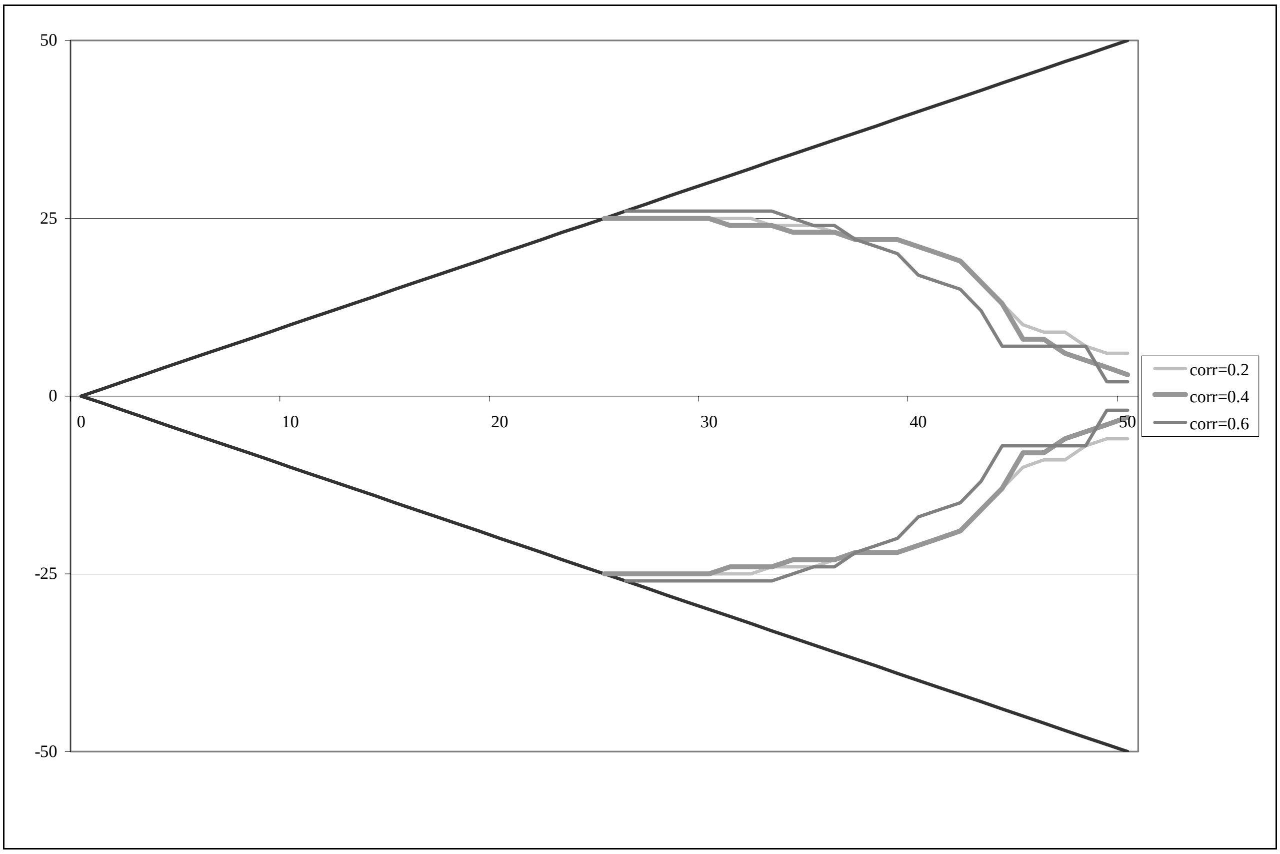
Graph 6: RORAC with herding subject to correlation between the shares



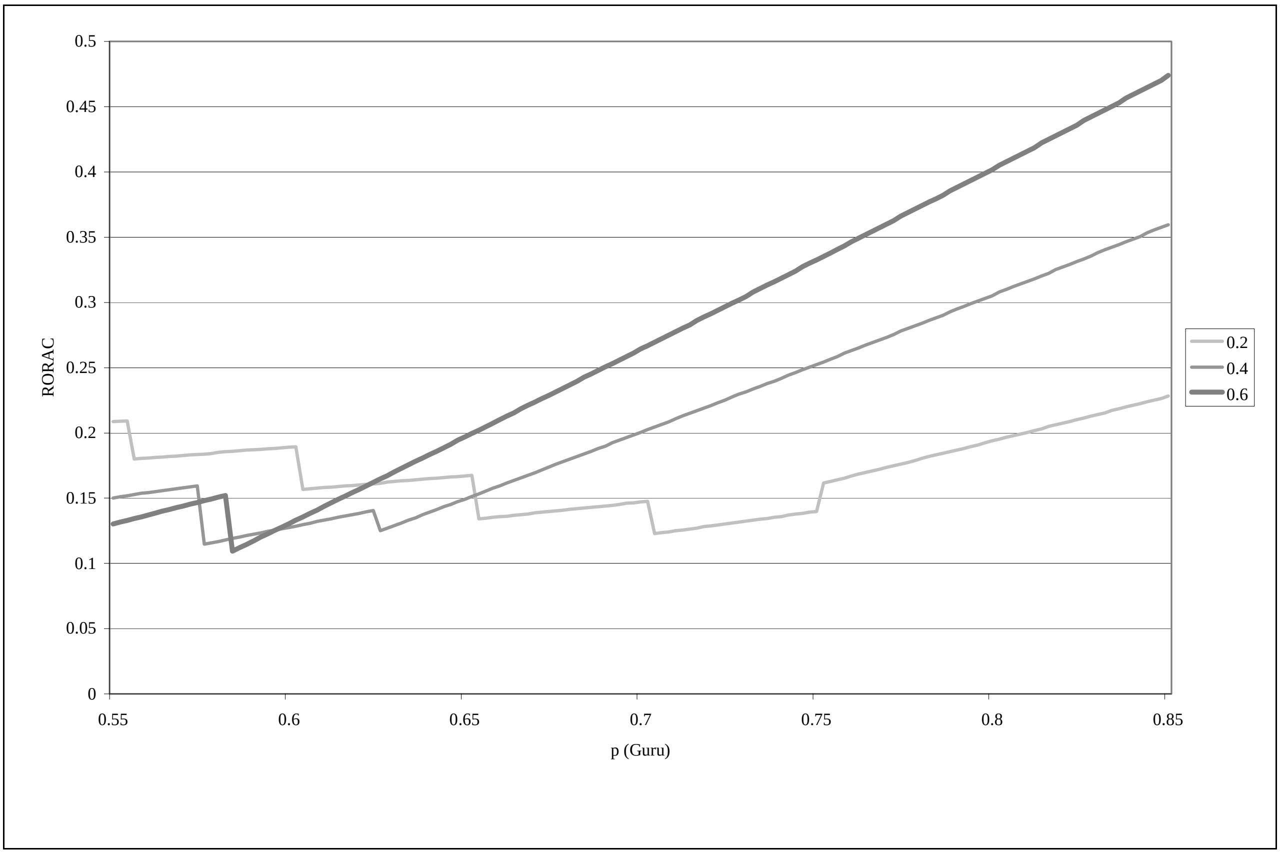
Graph 7: RORAC with herding and third best information policy „n_l”



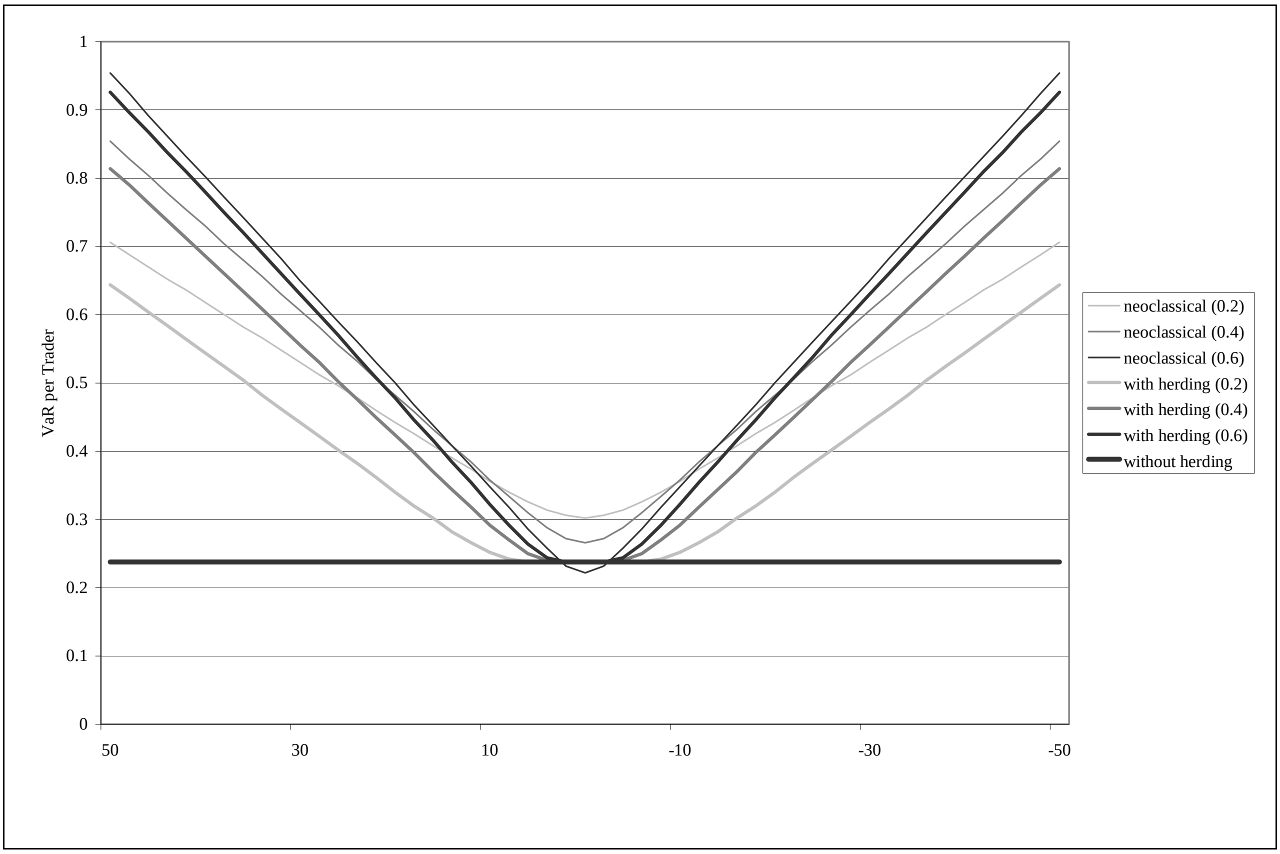
Graph 8: $\Delta L^*(n)$ for different correlations



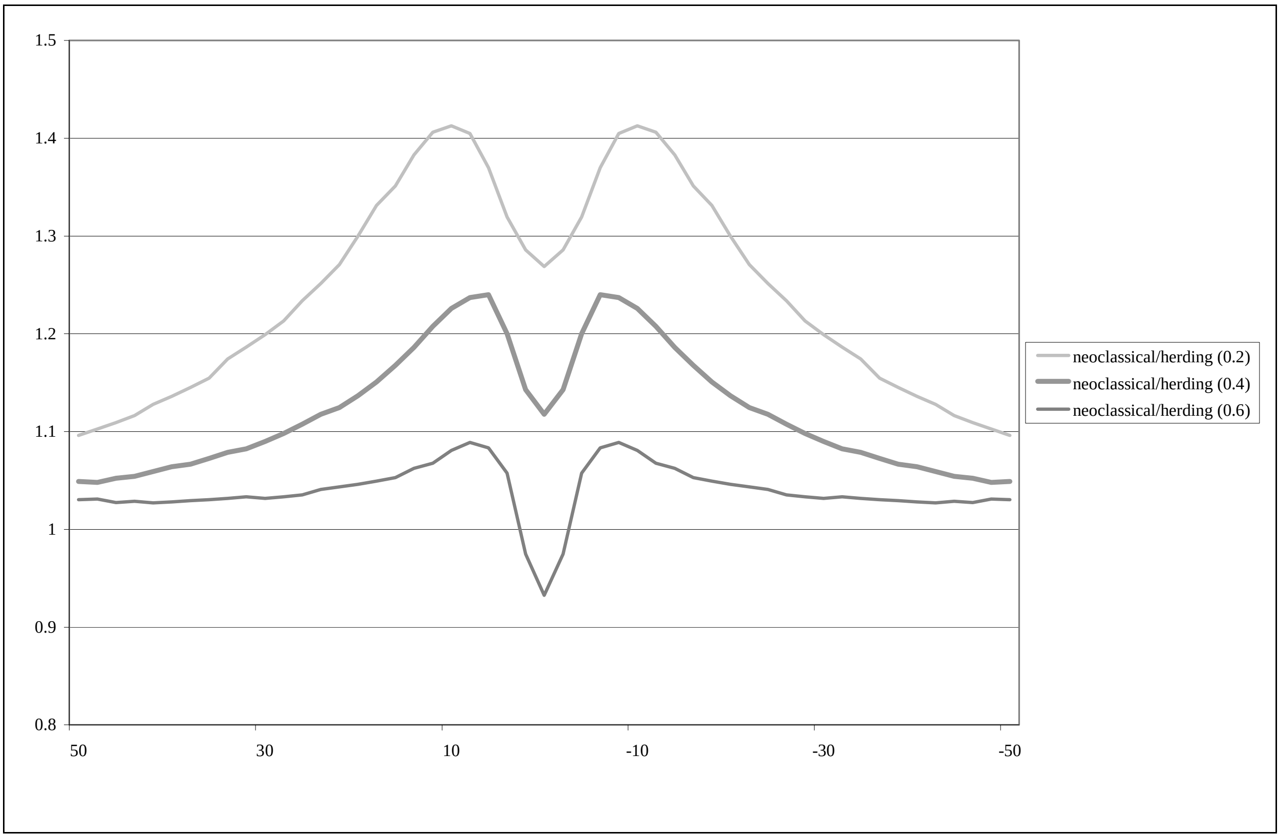
Graph 9: RORAC with herding and superiorly informed first trader („guru")



Graph 10: Ex post-value at risk in the herding and neoclassical case



Graph 11: Relative size of the safety cushion of neoclassical calculation of value at risk (p=0.55)



Graph 12: Relative size of the safety cushion of neoclassical calculation of value at risk (p=0.51)