Parsimonious estimation of credit spreads

Rainer Jankowitsch
Vienna University of Technology and CCEFM
Favoritenstrasse 11, A-1040 Vienna, Austria
rjankowi@pop.tuwien.ac.at

Stefan Pichler
Vienna University of Technology
Favoritenstrasse 11, A-1040 Vienna, Austria
spichler@pop.tuwien.ac.at

March 2002

This work is based on a research project funded by the Austrian National Bank.
(OeNB – Jubiläumsfondprojekt Nr. 8808)

Parsimonious estimation of credit spreads

Abstract

The traditional method of credit spread estimation is based on subtracting independently estimated risk-free and risky term structures of interest rates which in many cases yields unrealistically shaped and often irregular credit spread curves. A parsimonious joint estimation of the risk-free term structure and the credit spread as proposed by Houweling et al. (2001) might serve as a valuable alternative to overcome this drawback but it is hard to decide whether a seemingly irregular shape of the credit spread curve is economically caused by the data or is only an artefact of the functional form of the estimation model. Results of an empirical examination of EMU government bond data show that traditional estimation models with different functional forms yield differing irregularities in the credit spread curves whereas joint estimation procedures result in well-behaving and coinciding curves. Moreover, the explanatory power of the more parsimonious joint estimation procedures is virtually equal to the traditional methods. This is strong evidence for the superiority of a joint estimation procedure of credit spread curves. Finally, we conclude that a simple linear joint cubic splines specification performs surprisingly well compared to a numerically more affording non-linear model.

JEL: G12, G13, G15; C13

1 Introduction

During the last decade the importance of understanding and modelling credit risk has increased rapidly. Growing markets for corporate bonds and credit derivatives stimulated the development of advanced valuation and risk management models in the field of credit risk. Finally, the publication of the consultative papers of the new Basle Capital Accord by the Basle Committee of Banking Supervision additionally fuelled the evolution of theoretical and applied research in this field. Many of the new credit risk models require accurate observations of the term structure of interest rates of different credit risk classes as an input. Prominent examples for such models are the Markov chain framework first introduced by Jarrow and Turnbull (1995) and extended by Jarrow et al. (1997) and the class of reduced form models which exogenously model the default intensity of a Poisson process as a function of stochastic state variables. This approach introduced by Duffie and Singleton (1997) and Lando (1998) enables the use of well established results and techniques from the world of affine risk-free term structure models. Both classes of models crucially rely on the availability of reasonable data for term structures of credit spreads, i.e. the term structure of differences between risky term structures and the risk-free one. Apparently, there is a need for accurate and reliable procedures which estimate the term structure of credit spreads from observable coupon bond prices.

The estimation of the discount function or the zero-coupon yield curve from observable prices of coupon bonds is a well established field of research in the academic literature. The most established procedures are due to the pioneering work by McCulloch (1971, 1975), Schaefer (1981), Vasicek and Fong (1982), Nelson and Siegel (1987), and Langetieg and Smoot (1989). Recently, more advanced estimation methods were suggested, e.g. by Steeley (1991), Svensson (1994), Linton et al. (2001) and Subramaniam (2001). Interestingly, there are only rare contributions to the specific problems of credit spread estimation. Traditionally, credit spreads are estimated by subtracting independently estimated risk-free and risky term structures of interest rates which in many cases yields unrealistically shaped and often irregular credit spread curves. Düllmann and Windfuhr (2000) and Geyer et al. (2001) report twisted credit spread curves for EMU government debt when the German curve is used as the risk-free reference curve.

Figure 1 shows a representative example where the yield curves for two government issuers have a similar shape. This suggests a rather well-behaved or even linear shape of the credit spread curve. Figure 2, however, shows a strong non-linear twisted pattern of the credit spread curve.

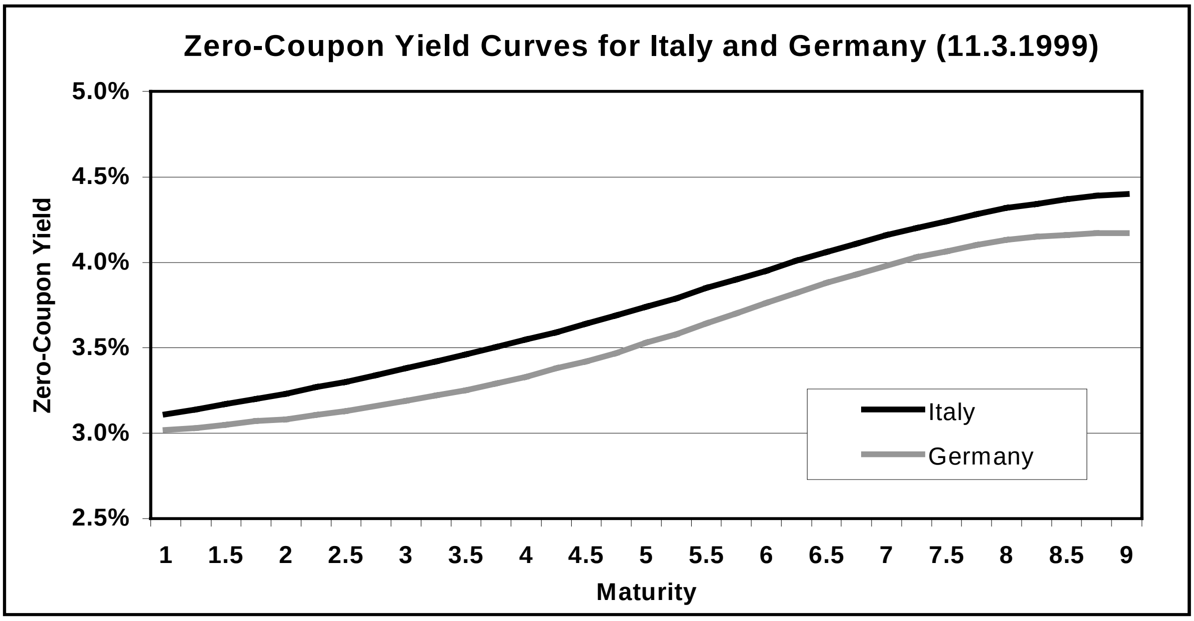

Figure 1: Zero-coupon yield curves obtained using a cubic splines model for Italy and Germany on March 11, 1999

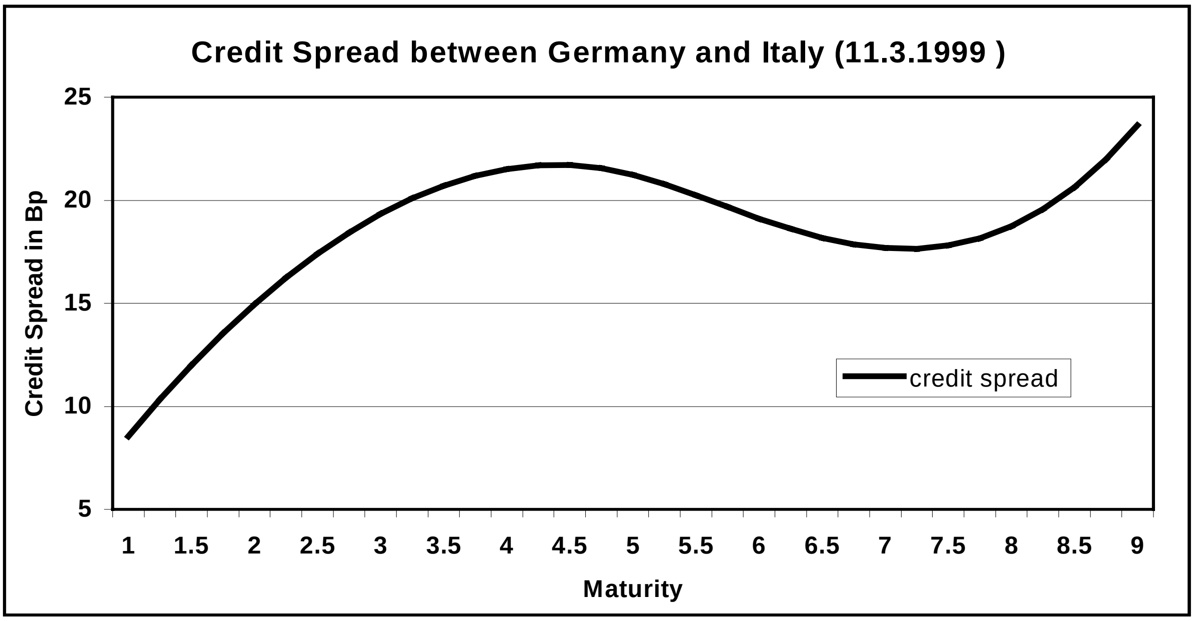

Figure 2: Credit spread curve between Italy and Germany on March 11, 1999 using a cubic splines model

This twisted pattern is regarded as unrealistic or irregular by many researchers since it is seen as a contradiction to the implications of many economic credit risk models. But before jumping to conclusions one should carefully check whether this observation is not an artefact of the estimation procedure rather than an economic fact. In general, traditional models of calculating the credit spread may produce doubtful results which hinder their economic interpretation.

Houweling et al. (2001) were the first to propose a new framework to overcome this possible drawback. They suggested to use a multi-curve approach where the risk-free term structure and the credit spread curve are estimated jointly. This method leads to more realistic and smoother credit spread curves in their empirical example. However, it is hard to decide whether a seemingly irregular shape of the credit spread curve is caused by the data or is only an artefact of the functional form of the estimation model. There is no natural benchmark which can be used to assess the quality of an estimation model. Houweling et al. tried to formalize the trade-off between the gain in smoothness and the decrease in explanatory power of the more parsimonious joint estimation procedure. Based on this rather relative and arbitrary benchmark they find strong evidence in favour of their model.

It is the main objective of this paper to provide empirical evidence that a procedure which jointly estimates the risk-free term structure and the credit spread curve is superior to traditional models by comparing the results of two completely different single-curve and two completely different multi-curve models. This peer comparison provides an additional benchmark and allows for deciding whether a flexible shape of an estimated curve is an artefact of the functional form of the model or not. The empirical study makes use of EMU government bond data already used by other studies but the results should also hold for corporate bond markets as well.

The remainder of the paper is organized as follows. Section 2 gives a short general overview over the basic yield curve estimation problem and describes the procedures used in the subsequent analysis in detail. In section 3 data and methodology are described and the results of the empirical study are presented. Section 4 summarizes and concludes the paper.

2 Estimation models for the credit spread

In this section we briefly describe the basic estimation problem arising when only prices of coupon bonds can be observed in an incomplete bond market. In the following we describe the two different approaches to this estimation problem applied in this paper, the cubic splines model and the Svensson model. The choice of these models may seem arbitrary but these models span a broad class of estimation procedures and due to their practical importance might be seen as representative examples. The main conclusions of this paper, however, are very unlikely to be affected by the choice of the specific models. Thus, we do not refer to more advanced spline interpolation methods, like B-splines or exponential splines.

2.1 Basic estimation problem

First of all we describe the estimation problem in a single market setting. Then we turn to the multi-curve estimation problem when credit spread curves are to be estimated. Consider a bond market with *N* coupon bonds. Each bond *i* is characterised by its market price P_i (quoted price plus accrued interest), its cash flow vector Z_i, and its vector of cash flow dates T_i. A bond market is said to be complete if the total number of distinct cash flow dates in the market is equal to the number of linearly independent cash flow vectors, i.e. if arbitrary cash flow structures can be replicated using portfolios of existing bonds. The absence of arbitrage implies the existence of an unique set of discount factors *D(t)*, where *t* denotes the time to any cash flow date in the market, for which

$$P_i = \sum_j Z_{ij} \cdot D(T_{ij})$$

holds for all *i* = 1, ..., *N*. The zero-coupon yields *r(t)* are related to the discount factors via

$$r(t) = \left(\frac{1}{D(t)} \right)^{\frac{1}{t}} - 1$$

In incomplete markets like the government bond markets in EMU countries the set of arbitrage-free discount factors is not unique. Given an arbitrary set of discount factors

$$P_i = \sum_j Z_{ij} \cdot D(T_{ij}) + \varepsilon_i$$

applies, where ε_i denotes the pricing error of bond *i*. The basic estimation problem is aimed to find a set of discount factors which has optimal explanatory power, i.e. which minimizes the pricing errors with respect to a given norm, and is represented by a continuous function depending on a parsimonious number of free parameters. Let *a* denote the set of parameters and *f* denote the specified function then we have

$$D(t) = f(t;a)$$

and

$$P_i = \sum_j Z_{ij} \cdot f(T_{ij};a) + \varepsilon_i$$

There are numerous models suggested to solve this estimation problem. These models mainly differ in the functional specification, the number of free parameters, and the quantity for which the functional form is specified, i.e. for the discount factors, the zero-coupon yields or the forward rates.

2.2 Single-curve splines model

In the *single-curve splines model* introduced by McCulloch (1975) cubic splines are used to model the discount function. In this approach the maturity spectrum is divided into not necessarily equally spaced intervals. If the maturity spectrum is divided by *k-1* knots there are *k* free parameters to describe the entire discount function which is modelled as a linear combination of *k* prespecified component functions. Let f_k denote the component functions then the estimation problem reads

$$P_i = \sum_j Z_{ij} \cdot \sum_k a_k \cdot f_k(T_{ij}) + \varepsilon_i = \sum_k a_k \cdot \sum_j Z_{ij} \cdot f_k(T_{ij}) + \varepsilon_i$$

Employing the linear structure of this model the optimal parameters can easily be obtained by performing an OLS regression.

Finally, the credit spread curve of a specific country with respect to a reference curve is calculated by subtracting the reference zero-coupon yield curve from the zero-coupon yield curve of the country.

$$s_c(t) = r_c(t) - r_{ref}(t)$$

$s_c(t)$	credit spread between country *c* and the reference country for maturity *t*
$r_c(t)$	zero-coupon yield for country *c* for maturity *t*
$r_{ref}(t)$	zero-coupon yield for the reference country for maturity *t*

2.3 Single-curve Svensson model

In the *single-curve Svensson model* the estimation procedure suggested by Svensson (1994) is used to specify the zero-coupon yield curve directly using the functional form

$$r(t) = \beta_0 + \beta_1 \cdot \left(\frac{1 - \exp(-t/\tau_1)}{(t/\tau_1)} \right) + \beta_2 \cdot \left(\frac{1 - \exp(-t/\tau_1)}{(t/\tau_1)} - \exp(-t/\tau_1) \right) + \\ + \beta_3 \cdot \left(\frac{1 - \exp(-t/\tau_2)}{(t/\tau_2)} - \exp(-t/\tau_2) \right)$$

where $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2$ are parameters.

The interpolation function now becomes

$$f(T_{ij}; a) = \left[1 + r(T_{ij})\right]^{-T_{ij}}$$

where the function *r(.)* is defined as above.

The parameters can be obtained by performing a non-linear optimisation with the parameter restrictions $\beta_0 > 0$, $\tau_1 > 0$, and $\tau_2 > 0$. This procedure which is an extension of the well-known

Nelson and Siegel (1987) approach is widely used by practitioners as well as central banks and other financial institutions. It is regarded to produce more regular or smoother functions than a splines interpolation.

Again the credit spread curves are calculated by subtracting the reference curve from the zero-coupon yield curve of a specific country.

2.4 Multi-curve splines model

In the *multi-curve splines model* the zero-coupon yield curve of the reference country and the credit spread curves of the other countries are estimated jointly. All the curves are estimated with cubic splines as introduced in section 2.2. In order to have a more parsimonious specification the number of parameters for the credit spread curve can be reduced compared to the reference curve. The joint estimation uses the following framework for the discount functions:

$$D_0(t) = \sum_{k=1}^{k_{ref}} f_{ref,k}(t) \cdot a_{0,k}$$ discount function for the reference country

$$D_c(t) = D_0(t) + \sum_{k=1}^{k_{spread}} f_{spread,k}(t) \cdot a_{c,k}$$ discount function for country $c = 1, ..., C$

k_{ref}	number of parameters for the zero-coupon yield curve of the reference country
k_{spread}	number of parameters for the credit spread curves
$f_{ref,k}(t)$	component functions which use the chosen knots of the zero-coupon yield curve of the reference country
$f_{spread,k}(t)$	component functions which use the chosen knots of the spread curves
$a_{0,k}$	parameters of the zero-coupon yield curve of the reference country
$a_{c,k}$	parameters of the credit spread curve of the country c

This multi-curve model uses two features to improve the single-curve model. First it directly estimates the spread curves with a parsimonious splines model and second the reference zero-coupon yield curve and all spread curves are estimated jointly.

Using this framework we obtain the following linear regression model for *C*+1 countries.

$$\mathrm{Y} = \mathrm{X} \cdot \mathrm{a} + \varepsilon \Leftrightarrow \begin{pmatrix} \mathrm{Y}_0 \\ \mathrm{Y}_1 \\ \vdots \\ \mathrm{Y}_C \end{pmatrix} = \begin{pmatrix} \mathbf{X}_0 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{X}_1 & \mathrm{S}_1 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{X}_C & \mathbf{0} & \cdots & \mathrm{S}_C \end{pmatrix} \cdot \begin{pmatrix} \mathbf{a}_0 \\ \mathbf{a}_1 \\ \vdots \\ \mathbf{a}_C \end{pmatrix} + \begin{pmatrix} \boldsymbol{\varepsilon}_0 \\ \boldsymbol{\varepsilon}_1 \\ \vdots \\ \boldsymbol{\varepsilon}_C \end{pmatrix}$$

Y_c is a vector consisting of elements

$$\mathrm{y}_i = \mathrm{P}_i - \sum_j \mathrm{Z}_{ij} \,,$$

X_c is a matrix with elements

$$\mathrm{x}_{ik} = \sum_j \mathrm{Z}_{ij} \cdot \mathrm{f}_{ref,k}(\mathrm{T}_{ij}) \,,$$

S_c is a matrix with elements

$$\mathrm{s}_{ik} = \sum_j \mathrm{Z}_{ij} \cdot \mathrm{f}_{spread,k}(\mathrm{T}_{ij}) \,,$$

a_c is a vector of parameters for country *c*, and

ε_c is a vector of residuals for country *c* assumed to be iid $(0,\sigma_c^2)$.

The residual term is allowed to have different variances for each country because in some countries bond prices a generally more noisy than in other countries. To estimate the parameters a Restricted Feasible GLS procedure (see, e.g. Greene (2000)) is applied. Numerical examples for applications in the given framework show, however, that the relaxation of the variance structure is of minor importance for the results.

2.5 Multi-curve Svensson model

In the *multi-curve Svensson model* the zero-coupon yield curve of the reference country and the credit spread curves of the other countries are estimated jointly. For the zero-coupon yield curve of the reference country we use the original Svensson approach with the functional form presented in section 2.3. To allow for a parsimonious specification the credit spread curves are modelled using the four parameter approach suggested by Nelson and Siegel (1987) which has the functional form

$$s_c(t) = \gamma_{0,c} + \gamma_{1,c} \cdot \left(\frac{1-\exp(-t/\kappa_c)}{(t/\kappa_c)} \right) + \gamma_{2,c} \cdot \exp(-t/\kappa_c)$$

where $\gamma_{0,c}$, $\gamma_{1,c}$, $\gamma_{2,c}$, κ_c are parameters.

Note, that the four-parameter Nelson-Siegel model is nested in the six-parameter Svensson model with $\beta_0 = \gamma_{0,c}$, $\beta_1 + \beta_2 = \gamma_{1,c}$, $\beta_2 = -\gamma_{2,c}$, $\beta_3 = 0$, $\tau_1 = \kappa_c$ and $\tau_2 = 1$. In the following the Nelson-Siegel model is regarded as a four-parameter version of the Svensson model.

The zero-coupon yield curve for the reference country $r_{ref}(t)$ is again specified by a Svensson model with six parameters. The interpolation function is now defined as

$$f_c(T_{ij};a) = \left[1 + r_c(T_{ij})\right]^{-T_{ij}} = \left[1 + r_{ref}(T_{ij}) + s_c(T_{ij})\right]^{-T_{ij}}$$

where $a = \{\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2; \gamma_{0,1}, \gamma_{1,1}, \gamma_{2,1}, \kappa_1; \ldots; \gamma_{0,C}, \gamma_{1,C}, \gamma_{2,C}, \kappa_C\}$.

The reference zero-coupon yield curve and all spread curves are estimated jointly by performing a non-linear GLS procedure analogue to the linear procedure described in section 2.4.

3 Empirical Analysis

3.1 Data

The time series data used in this study include daily prices of EUR denominated EMU government bonds from the period 01/1999 to 03/2001. We make use of this data set for the following reasons: (i) It comprises the largest and most liquid bond market where bonds of different issuers are traded in a homogeneous segment, (ii) in comparison to the corporate bond market the quality and reliability of the price information is much better, and (iii) credit spread curves obtained from EMU government bond prices have already been used by other researchers, e.g. Düllmann and Windfuhr (2000), Geyer et al. (2001). Nevertheless, we are aware of the fact that the observed spreads between different EMU issuers are potentially caused by other effects (e.g. liquidity) than credit risk. Preliminary results from the EMU government bond market, however, show that liquidity effects cannot fully explain the size and the variation of cross-country spreads (see Jankowitsch et al. (2002)). Moreover, the conclusions of this paper are based only on technical properties of the estimation procedures and are expected to hold even if the economic reason for the spread is different.

We restrict our analysis to coupon bonds without any option features with reliable price information and a time to maturity shorter than ten years. The data set includes the basic features of the bonds, e.g. the maturity and the exact cash flow schedule. The daily prices are taken from Bloomberg using the Bloomberg generic price source (BGN). For a detailed description of this data set refer to Jankowitsch and Pichler (2002). In order to provide meaningful results we make use of data collected from the following nine national submarkets: Germany, France, Italy, Austria, The Netherlands, Belgium, Portugal, Spain and Finland.

Table 1 contains the number of government bonds for each country which are used throughout this study:

	Germany	**France**	**Italy**	**Austria**	**Netherlands**	**Belgium**	**Portugal**	**Spain**	**Finland**
Number of Bonds	104	106	104	99	45	36	26	39	56

Table 1: Number of government bonds for each country

Luxembourg is not considered in this data set because there exists no publicly traded government debt. Greece is excluded because it did not take part in the starting phase of the EMU in 1999. Ireland is also excluded because it only has five liquid bonds outstanding.

Though this data set is unique with respect to the countries and variables included offering a great opportunity for research it is important to realise possible weaknesses of this data set. The data set only contains quotation prices and not the actual trade price. The quoted price may only hold for a relatively small quantity, whereas traders demanding higher quantities cannot in advance determine the actual price for the entire quantity they wish to trade. However, practitioners use these quotations in their daily work and the quotes are regarded by market participants to be good for a certain size, typically EUR 10 million, and most of the transactions are taking place within the quoted bid-ask spread.

3.2 Methodology

As the starting point of our empirical analysis we estimate the zero-coupon yield curves for the countries of the EMU government bond market using a standard cubic splines model (*single curve splines model*) as introduced by McCulloch (1975) with equidistant knots and the wide-spread non-linear procedure (*single curve Svensson model*) suggested by Svensson (1994). For the single-curve splines models we use a four parameter specification which turns out to be optimal for most countries throughout the analysis. By taking the German government bonds as risk-free we calculate the credit spreads for the other EMU countries by subtracting the German reference curve from the respective zero-coupon yield curves. We perform a visual inspection of these credit spread curves to check whether their shapes are driven by economic reasons or rather by the functional form of the model specifications.

Given the sparse availability of bonds with longer maturities we restrict our analysis to the maturity spectrum from one to ten years for all countries. The short end of the term structure is eliminated from the analysis mainly because of the lack of liquid price information for very short maturities.

In a second step we extend our analysis to the use of multi-curve models to estimate the credit spread curves. In these models the zero-coupon yield curve of the risk-free country

(Germany) and the credit spread curves of the other countries are estimated jointly by using a parsimonious modelling technique for the credit spreads to filter possible effects which might produce irregular shapes of the credit spread curves. We transform the cubic splines model as proposed by Houweling et al. (2001) into a multi-curve model by using a three parameter credit spread curve. The six parameter Svensson model is transformed by using the nested four parameter Nelson-Siegel model for the credit spread curve, i.e. k_{ref} = 4 and k_{spread} = 3. (for a general description of the models see section 2).

In order to formalise the basis of our conclusions and to incorporate the findings of all countries and of the entire observation period we calculate several statistics which summarise the main properties of the different models. Along the lines of Houweling et al. (2001) we calculate the smoothness of the spread curve function to quantify the effect of eliminating the potential of irregular shapes of the credit spread curves.

Following Poirier (1976) and Powell (1981) the smoothness is derived by computing the integral of the square of the second derivative of the credit spread function *s(t)* over the interval [t_1, t_2]:

$$\text{smoothness} = \int_{t_1}^{t_2} s''(t)^2 \, dt$$

To examine whether a improvement in the smoothness parameter has significant negative influence on the explanatory power models we calculate the average absolute pricing errors for each country.

$$\text{Average absolute pricing error} = \frac{\sum_{i=1}^{N_c} \left| P_i^{market} - P_i^{model} \right|}{N_c}$$

where P_i^{market} market price of bond *i*

P_i^{model} model price of bond *i*

N_c number of bonds in country *c*

Due to its lower degree of flexibility the more parsimonious multi-curve specifications are expected to produce higher pricing errors. The decrease in explanatory power should be comparably small if the flexible shape of the single-curve models is exclusively caused by the functional form of the specification. If the flexible shape of the credit spread curve estimates is at least partly caused by economic reasons, the pricing errors implied by the parsimonious multi-curve models are expected to be much higher than their single-curve counterparts.

3.3 Results

In the first step of our procedure we estimated zero-coupon yield curves for each country on a daily basis using four different estimation procedures, (i) the single-curve splines model, (ii) the multi-curve splines model, (iii) the single-curve Svensson model, and (iv) the multi-curve Svensson model. Before turning to formal comparisons of the different models we perform a visual inspection of the resulting yield and spread curves. The results show that all models produce very similar zero-coupon yield curves (see figures 3 and 4 for a representative example) but the credit spread curves are quite different.

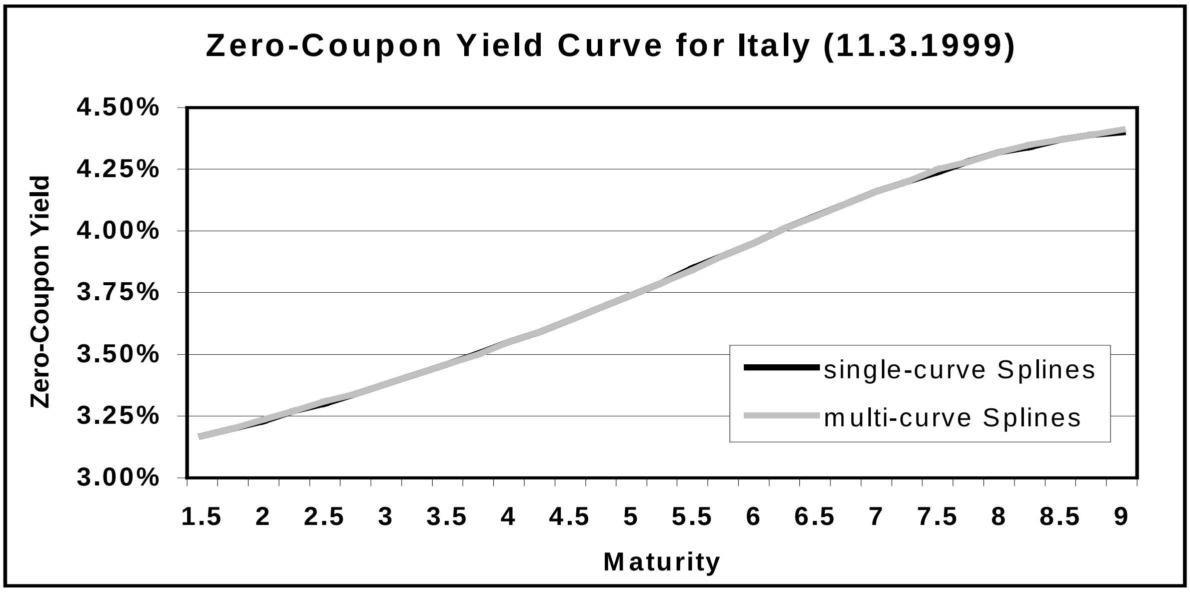

Figure 3: Zero-coupon yield curve for Italy (11. 3. 1999) using the single-curve and the multi-curve splines model

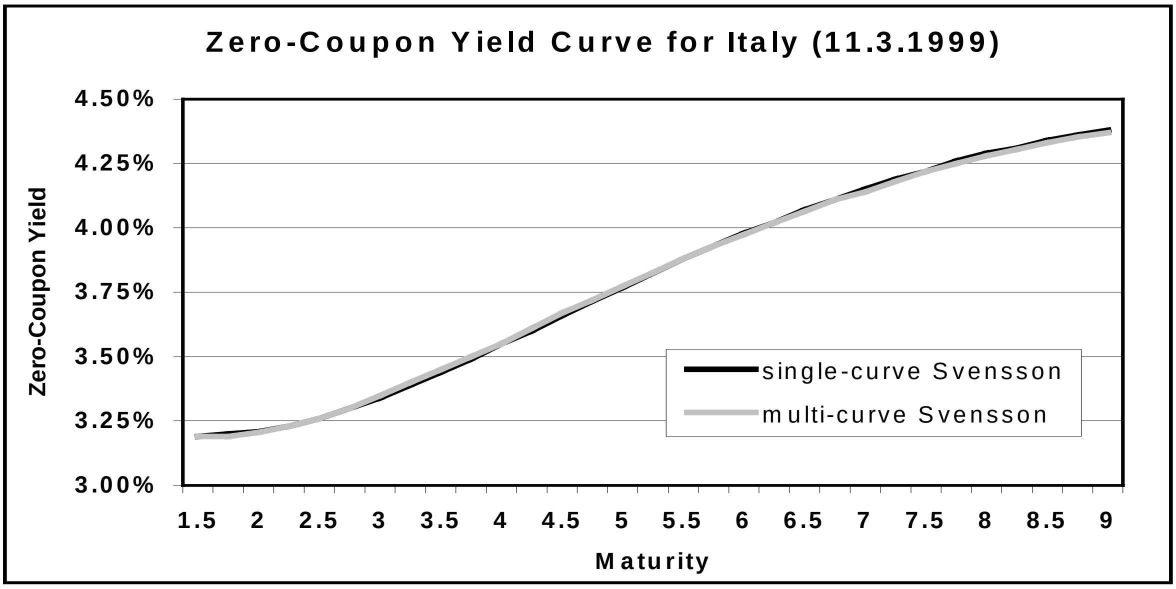

Figure 4: Zero-coupon yield curve for Italy (11. 3. 1999) using the single-curve and the multi-curve Svensson model

In most cases both single-curve models produce twisted curves for the credit spread but the shapes of these curves are very different, however (see figures 5 for a representative example). In general, the minima and maxima of the resulting curves are in different maturity segments and in many cases the Svensson model implies a S-shape whereas the splines model implies an inverse S-shape. Note, that the comparison between the models is based on identical input data drawn from a particular country for one particular day. Based on these observations we conclude that the specific form of the resulting credit spread curves is an artefact of the functional form of the estimation model rather than implied by the data.

The two multi-curve models produce smoother credit spread curves in all observed cases. Moreover, usually the resulting curves are similar to each other, although they are not identical (see figures 6). Typically, the twists and S-shapes are not observed in the multi-curve results. Since the multi-curve models lead to coinciding results and look more 'plausible' from an intuitive perspective we conclude that the shapes of the spread curves resulting from these models are implied only by the data and are not adversely affected by the functional form of a specific model.

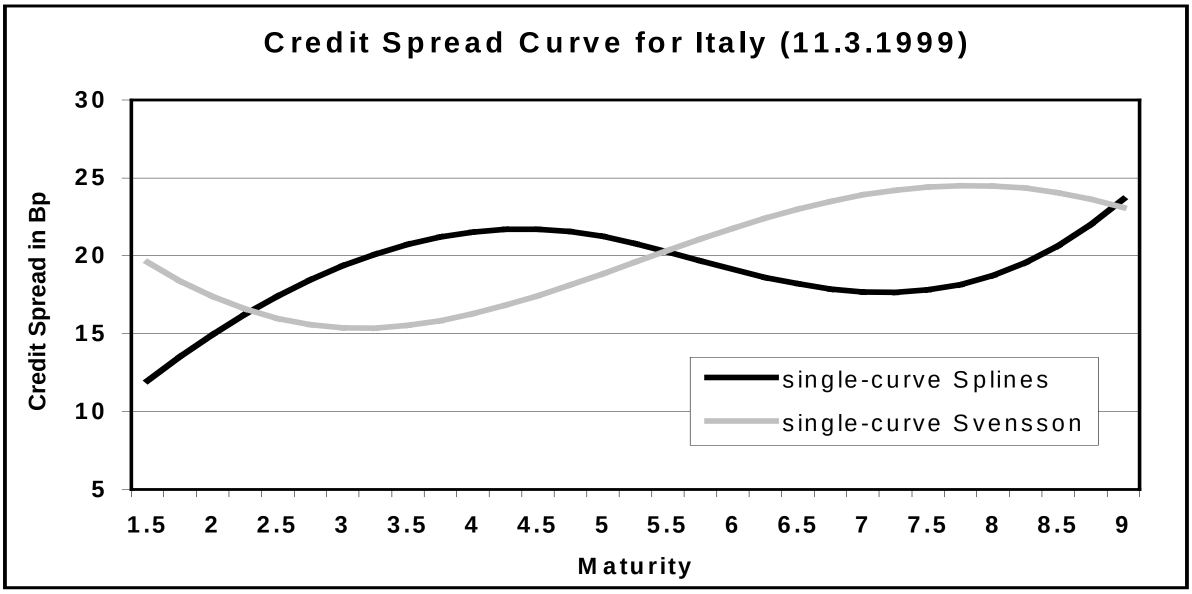

Figure 5: Credit spread curve for Italy (11. 3. 1999) using the single-curve splines and the single-curve Svensson model

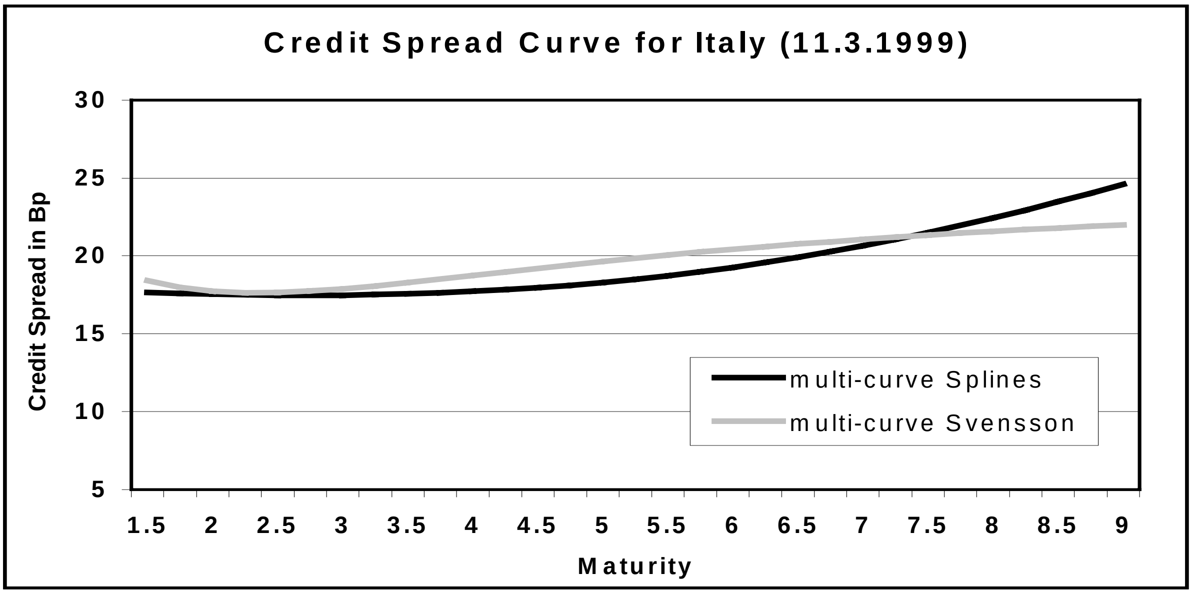

Figure 6: Credit spread curve for Italy (11. 3. 1999) using the multi-curve splines and the multi-curve Svensson model

The difference between the results of the multi-curve models can be explained. Both produce smoother curves compared to their single-curve counterparts by estimating the zero-coupon yield curve of the reference country jointly with the credit spread curves and by reducing the flexibility of the credit spread curves due to their more parsimonious specification. The main difference between the multi-curve models is by which extent they reduce the flexibility of the spread curves. Given the specifications chosen in our study the *multi-curve splines model* reduces the flexibility more than the *multi-curve Svensson model*. Which of these models performs better depends on the quality of the data. If nearly all government bond prices are exactly explained by the zero-coupon yield curve the flexibility need not be reduced as much as if there are some larger pricing errors. So in the first case the *multi-curve Svensson model* is more preferable than the *multi-curve splines model* and vice versa.

Note, however, that these conclusions are only based on a purely informal and subjective visual inspection of the resulting yield and spread curves. In order to formalise the basis of our conclusions and to incorporate the findings of all countries and of the entire observation period we calculate several statistics which summarise the main properties of the different models.

In the following step we examine the improvement of the smoothness of the resulting curves by calculating the smoothness parameter between the maturities of two and ten years for all estimated curves. Table 2 shows the average smoothness of the credit spread curves (multiplied by 10^7) of all four estimation methods:

	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
SC-Splines	N/A	6.04	0.95	5.77	1.81	1.23	4.55	1.27	2.54
SC-Svensson	N/A	6.38	1.37	2.13	0.42	1.77	6.44	1.92	1.63
MC-Splines	N/A	3.49	0.31	0.42	0.52	0.45	0.56	0.26	0.23
MC-Svensson	N/A	3.32	0.76	1.21	0.40	0.78	2.51	1.45	1.25

Table 2: Average smoothness of the credit spread curves (multiplied by 10^7)

Table 3 shows the difference in the smoothness of the credit spread curve between *single-curve splines model* and *multi-curve splines model*. The difference is measured as the smoothness of *multi-curve splines model* minus the smoothness of *single-curve splines model* (divided by the smoothness of *single-curve splines model* for the percentage statistics).

	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
Absolute	N/A	-2.55	-0.64	-5.35	-1.29	-0.78	-3.99	-1.01	-2.31
Percent	N/A	-42.26	-67.32	-92.69	-71.27	-63.55	-87.72	-79.44	-90.86

Table 3: Differences in the average smoothness between single-curve splines model and multi-curve splines model

Table 4 shows the difference in the smoothness of the credit spread curve between *single-curve Svensson model* and *multi-curve Svensson model*. The difference is measured as the smoothness of *multi-curve Svensson model* minus the smoothness of *single-curve Svensson model* (divided by the smoothness of s*ingle curve Svensson model* for the percentage statistics).

	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
Absolute	N/A	-3.06	-0.61	-0.93	-0.02	-0.99	-3.94	-0.47	-0.39
Percent	N/A	-47.96	-44.71	-43.46	-4.71	-56.06	-61.09	-24.64	-23.84

Table 4: Differences in the average smoothness between single-curve Svensson model and multi-curve Svensson model

The results regarding the smoothness parameter clearly support the finding that the artificial twisted form of the credit spread curves in the single-curve models is corrected by the multi-curve models in the overall sample. The improvement (–74% on average) is larger when changing from the *single-curve splines model* to its multi-curve counterpart because of the stronger reduction in the flexibility of the credit spread as explained earlier. But the improvement (–38% on average) for the non-linear Svensson case is also impressive. Note, that in contrast to the spread curves the smoothness of the yield curves resulting from a multi-curve model (not presented here) is virtually equal to the smoothness of single-curve results.

Additional examinations not presented in detail show that the volatility of logarithmic interest rate changes over time remains virtually at the same level when moving from a single-curve to a multi-curve model. Interestingly, the volatility of the multi-curve credit spreads is about 2.3% smaller for the splines model and about 5% smaller for the Svensson model than the single-curve counterpart. This finding supports our conclusion that a multi-curve model explains the variations of the underlying term structures equally well but reduces the effect of artificial volatility caused by the adverse influence of the functional specifications in the single-curve models.

We have shown that the use of a multi-curve model reduces the flexibility of the credit spread function and unrealistic twisted shapes which are artefacts of the estimation procedure can be avoided. The important question is, however, whether this improvement in the smoothness has significant negative influence on other important aspects of model quality, e.g. pricing errors. Obviously, the more parsimonious multi-curve specification is expected to produce higher pricing errors, i.e. is expected to have a lower explanatory power, due to its lower degree of flexibility. But the decrease in explanatory power should be comparably small if the flexible shape of the single-curve models is exclusively caused by the functional form of the specification. On the other hand, if the twisted shape of most of the credit spread curve estimates is at least partly caused by economic reasons, the pricing errors implied by the parsimonious multi-curve models are expected to be much higher than their single-curve counterparts.

In order to check whether the differences in the explanatory power is significant across models we calculate the absolute pricing errors (absolute value of the difference between market price and model price). Table 5 shows the average absolute pricing errors measured in basis points for all countries and for all four models:

	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
SC-Splines	9.91	11.14	8.80	7.88	9.41	9.04	14.97	10.04	6.06
SC-Svensson	10.52	10.54	9.21	7.41	9.29	9.00	15.79	10.48	6.27
MC-Splines	10.06	11.55	9.06	9.52	9.63	9.40	16.58	10.92	6.66
MC-Svensson	10.33	12.41	9.22	7.72	9.35	10.05	16.58	10.68	6.51

Table 5: Absolute average pricing errors for all countries

Table 6 shows the difference in the average absolute pricing error between *single-curve splines model* and *multi-curve splines model*. The difference is measured as the average pricing error of *multi-curve splines model* minus the average pricing error of *single-curve splines model* (divided by the average pricing error of *single-curve splines model* for the percentage statistics).

	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
Basis points	0.15	0.41	0.26	1.64	0.22	0.36	1.61	0.89	0.60
Percent	1.49	3.70	2.94	20.79	2.31	3.96	10.74	8.84	9.87

Table 6: Differences in the average absolute pricing errors between the single-curve splines model and the multi-curve splines model

Table 7 shows the difference in the average absolute pricing error between *single-curve Svensson model* and *multi-curve Svensson model*. The difference is measured as the average pricing error of *multi-curve Svensson model* minus the average pricing error of *single-curve Svensson model* (divided by the average pricing error of *single-curve Svensson model* for the percentage statistics).

	Germany	France	Italy	Austria	Netherlands	Belgium	Portugal	Spain	Finland
Basis points	-0.19	1.84	0.01	0.32	0.05	1.05	0.78	0.20	0.22
Percent	-1.79	17.52	0.11	4.29	0.58	11.68	4.94	1.87	3.55

Table 7: Differences in the average absolute pricing errors between single-curve Svensson model and multi-curve Svensson model

The results clearly indicate that the decrease in explanatory power of the multi-curve models is negligible at least when measured in absolute terms. For most countries the increase of the average pricing errors is very small even when measured in percent (on average 7% for the splines models and 4.7% for the Svensson models). The only outliers are Austria and Portugal for the splines models and France and Belgium for the Svensson models where the percentage increase of average pricing errors exceeds 10%. Note, that these differences are not economically significant given the usual bid-ask spreads of 3-10 basis points to be observed in these markets (see e.g. Jankowitsch et al. (2002)).

Compared to the reduction of the smoothness the other important property of the models does not change significantly supporting the hypothesis that the flexible shape of the single-curve models is exclusively caused by the functional form of the specification. We conclude that for the estimation of credit spread curves the use of multi-curve models is clearly superior to the use of single-curve models.

The differences between the *multi-curve splines model* and the *multi-curve Svensson model* are rather small. As explained above the *multi-curve splines model* is preferable if there are some larger pricing errors. So this model is a better choice except when having very small pricing errors (e.g. when only market data of benchmark bonds are used). Given this observation and the obvious numerical advantages of the linear splines model we suggest to use a *multi-curve splines model* to estimate credit spread curves.

4 Summary

We provide empirical evidence that a parsimonious procedure which jointly estimates the risk-free term structure and the credit spread curve is superior to traditional models where the credit spread curve is obtained by subtracting the risk-free reference curve from the risky term structure. The framework of a multi-curve cubic splines model originally suggested by Houweling et al. (2001) is extended to cover a multi-curve version of the Svensson-Nelson-Siegel approach. This peer comparison provides an additional benchmark and allows for deciding whether a flexible shape of an estimated curve is an artefact of the functional form of the model or not. The empirical study makes use of EMU government bond data already used by other studies but the results should also hold for corporate bond markets as well.

The results regarding the smoothness parameter show impressive evidence that the artificial twisted form of the credit spread curves in the single-curve models is corrected by the multi-curve models in the overall sample. This finding is supported by an examination of the volatility of logarithmic interest rate changes which remains virtually at the same level when moving from a single-curve to a multi-curve model whereas the volatility of the multi-curve credit spreads is remarkably smaller than in the single-curve framework.

The decrease in explanatory power of the multi-curve models is negligible at least when measured in absolute terms. The observed pricing errors of the more parsimonious multi-curve models are approximately equal to the pricing errors of single-curve models. Given the improvement of the smoothness this finding supports the hypothesis that the flexible shape of the single-curve models is exclusively caused by the functional form of the specifications. We conclude that for the estimation of credit spread curves the use of multi-curve models is clearly superior to the use of single-curve models.

The differences between the multi-curve splines model and the multi-curve Svensson model are rather small. Generally, the multi-curve splines model is preferable if there are some larger pricing errors. So this model is a better choice except in a setting with very small pricing errors. Given this observation and the obvious numerical advantages of the linear splines model we suggest to use a multi-curve splines model to estimate credit spread curves.

References

Duffie, D., Singleton, K., 1997, An Econometric Model of the Term Structure of Interest Swap Yields, Journal of Finance 52, 1287-1321.

Düllmann, K., Windfuhr, M., 2000, Credit spreads between German and Italian sovereign bonds – do affine models work?, Canadian Journal of Administrative Sciences 17, 166 - 181

Geyer, A., Kossmeier, S., Pichler, S., 2001, Empirical Analysis of European Government Yield Spreads, Working Paper, University of Economics and Business Administration, Vienna, and Vienna University of Technology

Greene, W.H., 2000, Econometric Analysis, 4th edition, Prentice Hall, New Jersey

Houweling, P., Hoek, J., Kleibergen, F., 2001, The joint estimation of term structures and credit spreads, Journal of Empirical Finance 8, 297 - 323

Jankowitsch, R., Mösenbacher, H., Pichler, S., 2002, Measuring the Liquidity Impact on EMU Government Bond Prices, Working Paper, Vienna University of Technology

Jankowitsch, R., Pichler, S., 2002, Estimating Zero-Coupon Yield Curves for EMU Government Bonds, Working Paper, Vienna University of Technology

Jarrow, R., Lando, D., Turnbull, S., 1997, A Markov Model for the Term Structure of Credit Risk Spreads, Review of Financial Studies 10 (2), 481 - 523

Jarrow, R., Turnbull, S., 1995, Pricing Derivatives on Financial Securities Subject to Credit Risk, Journal of Finance 50 (1), 53 - 85

Lando, D., 1998, On Cox Processes and Credit Risky Securities, Review of Derivatives Research 2 (2/3), 99 - 120

Langetieg, T.C., Smoot, J.S., 1989, Estimation of the Term Structure of Interest Rates, in: Kaufman, G.G. (ed.), Research in Financial Services 1, Greenwich, 181 - 222

Linton, O., Mammen, E., Nielsen, J.P., Tanggaard, C., 2001, Yield curve estimation by kernel smoothing methods, Journal of Econometrics 105 (1), 185 - 223

McCulloch, J.H., 1971, Measuring the term structure of interest rates, Journal of Business 44, 19 – 31

McCulloch, J.H., 1975, The Tax-Adjusted Yield Curve, Journal of Finance 30, 811 – 830

Nelson, C.R., Siegel, A.F., 1987, Parsimonious Modeling of Yield Curves, Journal of Business 60, 473 – 489

Poirier, D.J., 1976, The Econometrics of Structural Change, North Holland Publishing

Powell, M.J.D., 1981, Approximation Theory and Methods, Cambridge Univ. Press, Cambridge

Schaefer, S.M., 1981, Measuring a tax-specific term structure of interest rates in the market for British government securities, Economic Journal 91, 415 - 438

Steeley, J.M., 1991, Estimating the gilt-edged term structure: basis splines and confidence intervals, Journal of Business Finance and Accounting 18 (4), 513 - 529

Subramanian, K.V., 2001, Term Structure Estimation in Illiquid Markets, Journal of Fixed Income 11/1, 77 – 86.

Svensson, L.E.O., 1994, Estimating and Interpreting Forward Interest Rates: Sweden 1992 – 1994, IMF Working Paper, International Monetary Fund

Vasicek, O.A., Fong, H.G., 1982, Term Structure Modeling Using Exponential Splines, Journal of Finance 37 (2), 339 - 348