Comparing risk-neutral probability density functions implied by option prices - market uncertainty and ECB-council meetings *

Martin Mandler

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Keywords: implied probability density functions, risk-neutral expectations, option prices, monetary policy, homogeneity tests

JEL Classification: G14, E58

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1 Introduction

Most central banks of industrialized nations have chosen on the operational level to manipulate a short-term money market rate, in many cases a day-to-day interest rate.¹ The use of short-term interest rates as operating targets is not primarily due to their role in the monetary transmission mechanism but to the fact that they can be controlled by central bank actions to a much larger extent than longer-term money market rates. Much more important for the monetary transmission mechanism are longer-term interest rates that are the result of the interaction of central bank operations on the short end of the maturity spectrum with portfolio decisions of financial intermediaries (Borio (1997)). Since these longer-term rates are heavily dependent on market participants' expectations central banks use signaling strategies. Signaling strategies are intended to influence market expectations with respect to the evolution of monetary policy in order to affect longer term rates. Because of these important interrelations between monetary policy, the monetary transmission process, and market expectations it is of great interest to examine empirically the reaction of market expectations to monetary policy decisions.

Numerous studies have investigated the information on market participants' expectations contained in financial asset prices. Due to their forward-looking nature financial asset prices instantly respond to new information and therefore are a natural indicator for changing market expectations. One important focus of this research has been on prices of financial derivatives. For example, based on the nature of the forward price as the risk-neutral expectation of the underlying asset's price at the forward contract's expiration date there has been a vast amount of studies on the forecasting ability of forward/futures prices with respect to future spot prices.² This approach provides only one piece of information on market expectations: the expected future value of the underlying asset's price. Option prices, however, contain much richer information on market participants' expectations. On a second step therefore researchers have used option prices to compute implied volatilities by inverting an appropriate option pricing formula and interpreted this variable as an approximation of the average volatility

¹For a detailed survey of central bank operating procedures, see Borio (1997).

²See Cuthbertson (1996) for a survey.

of the underlying's price market participants expect to prevail until expiration of the option.³

Over the last years a number of new techniques have been developed that allow for the extraction of considerably more information from option prices than just the expected value or the expected standard deviation (i.e. implied volatility). By using these techniques it is possible to construct implied risk-neutral probability density functions (PDF) from a cross-section of option prices. These risk-neutral densities contain information on the market participants' expectations concerning the underlying assets' price at the options' expiration date and by tracking their behaviour over time it is possible to observe the evolution of market expectations.⁴ Moreover, central banks themselves have taken considerable interest in the development and application of these techniques.⁵ It is thought that an insight into changes in market expectations around events related to monetary policy actions can provide information that might be useful for assessing central bank credibility and the effectiveness of implementation of monetary policy as perceived by financial markets (Bahra (1997)).

As explained in the following section, the interpretation of risk-neutral PDFs is focused on changes in the densities over time, not on their level. However, an important problem is closely linked to this approach because even when using the most robust estimation technique statistics computed from risk-neutral PDFs implied in option prices are subject to substantial day-to-day fluctuations (Bliss and Panigirtzoglou (1999), Cooper

³See, for example, Campa and Chang (1995), Dumas, Fleming and Whaley (1998), Lamoureux and Lastrapes (1993), and Scott (1992). Many papers investigate the forecasting ability of implied volatilities for future standard deviations of the underlying asset's return, e.g. Bank of Japan (1995), Canina and Figlewski (1993), Day and Lewis (1992), Fleming (1998), Jorion (1995), and Neuhaus (1995).

⁴These techniques have been applied to various problems ranging from exchange rate expectations (Campa, Chang, and Reider (1997), Cooper and Talbot (1999)) over the credibility of exchange rate regimes (e.g. Campa and Chang (1996, 1998), Campa, Chang, and Reider (1998), Malz (1996)), to commodity markets (e.g. Melick and Thomas (1997)) and interest rate expectations (e.g. Abken, Madan, and Ramamurtie (1996), Coutant, Jondeau, and Rockinger (1998), Söderlind (2000)).

⁵See for example Bahra (1997), Butler and Davies (1998), Cooper and Talbot (1999), Coutant, Jondeau and Rockinger (1998), Fornari and Violi (1998), Galati and Melick (1999), Levin, McManus and Watt (1998), Malz (1996), McManus (1999), Melick and Thomas (1997), Nakamura and Shiratsuka (1999), Neuhaus (1995), and Bank for International Settlements (1999).

(1999), Melick (1999)). Therefore it is difficult to separate changes in the statistics resulting from important "news" about future monetary policy from those being purely caused by "noise".

In this paper we propose an approach to deal with this identification problem by comparing statistics calculated from risk-neutral densities with a suitable benchmark in order to highlight substantial changes in market expectations. We illustrate our approach with an application in which we estimate risk-neutral probability density functions for short term interest rates (Euribor) on each Wednesday and Friday in 92 weeks in 1999/2000. Then we compute from the risk-neutral PDFs various statistics and compare these for the weeks in which a meeting of the Governing Council of the ECB took place to the weeks in which no meeting was scheduled, thereby using the week without a council meeting as a benchmark. We consider a set of statistics related to uncertainty market participants perceived over future interest rates and statistics that give us information on asymmetries in expectations. Apart from comparing the levels of statistics to those in the benchmark weeks, we investigate changes in these statistics from Wednesday to Friday that might be related to news originating from the results of the council meeting that usually takes place on Thursday, again using the weeks without a council meeting as the benchmark.

The next section gives a brief overview of the theoretical relationships between option prices and risk-neutral probability density functions that form the basis of our application. It also contains a short discussion of issues related to the interpretation of risk-neutral PDFs. The third section provides some information on the estimation technique used to extract risk-neutral probabilities from option prices. After that, section four presents the data set considered in our application. Then, section five explains the methodology used to compare the implied distributions' statistics with the benchmark and presents the results of our application. Finally, section six summarizes and provides some concluding remarks.

2 Option prices and risk-neutral probabilities

As shown by Cox and Ross (1976), the price of an European option on the underlying asset S can be written as the discounted expected value of its payoff at expiration,⁶

$$v_t(S_t, K) = e^{-r(T-t)} \mathbf{E}_t^Q[h(S_T)|S_t],$$
(1)

where K is the exercise price, T is the expiration date, r is the risk-free rate of interest and $h(S_T)$ is the option's payoff function. For an European call option we have $h_c(S_T) = \max[S_T - K, 0]$ and for the put $h_p(S_T) = \max[K - S_T, 0]$. Therefore the call value is,

$$c_t(S_t, K) = e^{-r(T-t)} \mathbf{E}^Q[\max[S_T - K, 0] | S_t]$$
(2)
= $e^{-r(T-t)} \int_K^{\infty} (S_T - K) q(S_T | S_t) dS_T,$

and the put value,

$$p_t(S_t, K) = e^{-r(T-t)} \mathbf{E}^Q[\max[K - S_T, 0] | S_t]$$

$$= e^{-r(T-t)} \int_{0}^{K} (K - S_T) q(S_T | S_t) dS_T.$$
(3)

The expectation is taken with respect to the probability density function $q(S_T|S_t)$. This density is called the *risk-neutral* probability density function and corresponds (in complete markets) to the equivalent martingale measure of Harrison and Kreps (1979) from the asset-pricing literature.

Differentiating equation (2) with respect to the exercise price K yields,

$$\frac{\partial c_t(S_t, K)}{\partial K} = -e^{-r(T-t)} \int_K^\infty q(S_T | S_t) dS_T$$

$$= -e^{-r(T-t)} [1 - Q(K | S_t)],$$
(4)

where $Q(\bullet|S_t)$ is the cumulative distribution function of S_T conditional on S_t . Therefore the first derivative of the European call option value with respect to the strike price

⁶The owner of an European call option has the right but not the obligation to buy the underlying asset at a prespecified price (strike price or exercise price) on a given date (expiration date), whereas a European put option provides him with the right but not the obligation to sell the underlying asset at a prespecified price on the expiration date.

equals the negative of the discounted risk-neutral probability of the option being inthe-money at the expiration date.⁷

The intuition behind this result is that if the option is sufficiently deep in-the-money it will almost surely finish in-the-money at expiration. Since it will be exercised with probability one, an increase in the exercise price by one unit will make the owner of the option pay one additional unit of currency for the underlying, thus reducing the call's value by the present value of one unit of currency. If, on the other hand, the option is far out-of-the-money and will almost surely expire worthless, a small increase in the exercise price should have no effect on the option value because the owner will not exercise it anyway. Finally, if an option will neither almost surely finish in-the-money nor almost surely out-of-the-money, an increase in the strike price by one unit should reduce the call option value by the present value of an amount between one and zero depending on the likelihood of the option being in-the-money at the expiration date.

Differentiating equation (4) once again yields the result of Breeden and Litzenberger (1978),

$$\frac{\partial^2 c_t(S_t, K)}{\partial K^2} = e^{-r(T-t)} q(S_T | S_t) \left|_{S_T = K}\right.$$

$$\tag{5}$$

That implies that the second derivative of the European call option value with respect to the strike price is equal to the discounted risk-neutral probability density of S_T .⁸

The risk-neutral PDF embodies information about the market participants' expectations for the price level of the underlying asset at T. However, the risk-neutral PDF cannot be interpreted straightforwardly as a representation of the market participants' expectations because this equivalence only holds for risk-neutral individuals. The riskneutral PDF is the result of the interaction of market participants' "true" expectations and their risk-preferences which cannot be disentangled without further assumptions (e.g. Chang and Melick (1999)). For example, a high risk-neutral probability for a certain state of the world might be the result of a high "true" probability but can also be associated with a low "true" probability in combination with high risk aversion that makes the individual highly value payouts if this event occurs (e.g. Chang and

⁷A call option is "in-the-money" if the underlying's price exceeds the exercise price, i.e. $S_t > K$, "at-the-money" if $S_t = K$ and "out-of the-money" if $S_t < K$.

⁸The same result can be derived using European put options.

Melick (1999), Galati and Melick (1999)). Without using an economic model specifying marginal utilities under different states of the world the approach most commonly used is to assume that risk-preferences are constant through the observation period and to focus on *changes* in the risk-neutral PDF which are interpreted as providing information on changes in the "true" density (e.g. Bahra (1997), Galati and Melick (1999), and Melick and Thomas (1998)).

3 Estimation procedure

Because of the relationships stated in (2) and (3) it is possible to infer the risk-neutral PDF from observed prices of European options. Many different techniques have been applied to option prices in order to extract the implied risk-neutral PDF. Along broad lines, these techniques can be divided into two categories. One group directly utilizes equations (2) and (3) by making assumptions about the functional form of the PDF and fitting the resulting theoretical option prices to observed option prices in order to estimate the free parameters of the distribution. The other techniques use equation (5) as a starting point.

Using equation (5) to calculate the risk-neutral PDF requires call prices being available for continuous strike prices. However, in reality we are usually limited to just a few discretely spaced observations. Therefore these techniques rely on interpolation methods to construct a continuous option pricing function that can be differentiated numerically to obtain the risk-neutral PDF. Existing comparisons indicate that the interpolation approach might be more robust with respect to errors in option prices than methods based on the first group of techniques (Cooper (1999), Melick (1999) and Bliss and Panigirtzoglou (1999)).⁹

The most straightforward solution would be to interpolate (and extrapolate) or smooth the observed option prices directly. However, the curvature of the option pricing function is difficult to approximate with commonly used methods and small fitted price errors will have a large effect on the risk-neutral PDF, especially in the tails (Bliss

⁹Studies of robustness of the various techniques focus on pricing errors in exchange traded options due to the minimum-tick-size requirement.

and Panigirtzoglou (1999), Cooper (1999), Cooper and Talbot (1999)). Therefore, the approach presented by Shimko (1993) is used who suggests to convert option prices into implied volatilities using the Black-Scholes option valuation formula and then to interpolate or smooth the implied volatilities which are finally converted back into option prices. This procedure does not assume the Black-Scholes formula to be correct but treats it as a convenient mapping from price space into volatility space and back (e.g. Bliss and Panigirtzoglou (1999), Malz (1998), Shimko (1993)). In Shimko (1993) a quadratic polynomial in strike prices is fitted to implied volatilities. Campa, Chang and Reider (1997) suggest using cubic spline functions in order to allow for more flexibility in the shape of the volatility smile. Malz (1998), again, uses a quadratic polynomial but fits implied volatility versus delta, the derivative of the Black-Scholes price with respect to S_t , instead of exercise prices. Bliss and Panigirtzoglou (1999) combine these approaches and estimate natural spline functions in delta/volatility space. Switching from strike prices to delta has the effect of grouping more densely options farer out-ofthe-money or in-the money than options at-the-money, thus allowing for a more flexible shape near the center of the implied density function.

In this paper we follow the approach in Bliss and Panigirtzoglou (1999) in using a spline interpolation approach in delta/volatility space. First, we estimate implied volatilities from daily cross sections of observed option prices. Then, the implied volatility smile is constructed from piecewise cubic polynomials which are separated by the N observed option deltas (the knot points). The polynomials are constrained to be continuous at the knot points and to have continuous first and second derivatives. However, the spline function is not required to fit the observed implied volatilities exactly but to minimize the sum of squared deviations of approximated from actual implied volatilities. The objective function is,

$$\min_{\Theta} \sum_{i=1}^{N} \left(\sigma_i - f_i(\delta_i, \Theta) \right)^2 + \lambda \int_{\infty}^{+\infty} f''(\delta, \Theta)^2 d\delta,$$
(6)

where Θ is the matrix of free polynomial parameters, f_i is the spline function in segment i and δ_i, σ_i are the knot points. λ is a smoothness penalty multiplying a measure of the degree of curvature in the function represented by the integral of its squared second derivative as in Bliss and Panigirtzoglou (1999).¹⁰ Since the spacing of knot points

¹⁰Bliss and Panigirtzoglou (1999) use vega, the derivative of the Black-Scholes-price with respect to

is more dense the farer the observed strike prices are away from-the-money operating in delta space with a constant smoothness penalty allows for greater flexibility in the shape of the approximating function for strike prices at-the-money. The cubic spline is restricted to become linear outside the range of observed option prices.¹¹ The free parameters are estimated via nonlinear optimization.¹²

After estimation of the free polynomial parameters implied volatilities at a large number of equally spaced strike prices are computed including some extrapolation beyond the outer knot points.¹³ In order to calculate the implied volatility for a specific strike price we first need to convert strike prices into option deltas. Unfortunately, delta itself is a function of the implied volatility, so delta and implied volatility have to be calculated simultaneously (Malz (1998)). The interpolated implied volatilities are then transformed back into call option prices by applying the Black-Scholes-formula (or the appropriate variant). Finally, we arrive at the implied PDF by differentiating numerically the call price function twice with respect to the exercise price and multiplying with $e^{r(T-t)}$.

4 Data

In this study we use a set of prices of options written on a three month Euribor futures contract, i.e. a futures contract on a short-term inter-bank interest rate. Both options and futures prices were obtained from London International Financial Futures and volatility, as weight for the squared deviations in (6). This amounts to the assumption of homoscedastic errors in option prices. However, they show that the choice of a weighting scheme does not significantly affect the resulting risk-neutral density functions.

¹¹As Bliss and Panigirtzoglou (1999) point out this restriction can lead to negative tail probabilities if the slope of the polynomial is negative at the extreme knot points. However, this problem does not arise in this study.

 12 After some experimentation we set the smoothness penalty parameter λ equal to 0.01.

¹³Bliss and Panigirtzoglou (1999) compute implied volatilities for equally delta-spaced points. This leads to a more densely grid of implied volatilities at-the-money than away-from-the-money and reduces the accuracy in the tails of the numerically computed implied PDF. Furthermore the very densely spaced strike prices at-the-money can cause problems with the numerical differentiation of the call price function because the effect of small errors in the transformation of delta into price space will be aggravated by the numerical differentiation. Options Exchange (LIFFE). The options are American style and can be exercised at any time up to the expiration date. However, the futures-style margining procedure applied to short-term interest rate options at LIFFE leads to the result that the options are actually priced like European type options. Early exercise, i.e. exercise before the expiration date will not take place because there is no opportunity cost of holding a long position in the option.¹⁴

We use settlement prices of the options and the underlying futures contract which are each day determined by the exchange at the close of trading in the respective contract. One advantage of using settlement prices instead of trading prices is that during trading time transactions based on liquidity considerations might move prices temporarily away from their equilibrium values. This problem is mitigated by using settlement prices (Melick and Thomas (1998)). Since the members of the exchange also act as brokers or hold positions in options themselves there are sufficient incentives for them to determine settlement prices close to their equilibrium values (Söderlind (2000)).

The options and the underlying futures contract expire simultaneously on the third Wednesday of March, June, September, and December.¹⁵ Therefore, our results refer to the expectations of market participants with respect to the Euribor rate at expiry although the underlying asset is the respective futures contract. Exercise prices are spaced by intervals of 0.125, i.e. 12.5 basis points.¹⁶ Since the futures contract is quoted as 100.0 minus the interest rate, our results can be converted into interest rate space by subtracting the respective futures price from 100. The minimum tick size, i.e. the minimum price movement recorded is 0.005.¹⁷

Starting with January, 20th 1999 we select for each week the settlement prices from Wednesday and Friday. Our sample ends on Friday, December, 15th 2000. We always

 16 For seven weeks in 1999 there are only available options with strike prices spaced by 0.250.

¹⁷However, the data on the CD-ROM "Financial Products: End of Day Data" distributed by LIFFE and used for this study is only recorded with an accuracy of 0.01.

¹⁴The option price is not paid upon purchase but option positions are marked-to-market daily giving rise to positive or negative variation margin flows. See LIFFE (2000) for details. Chen and Scott (1993) show that this leads to American options being priced as European Options.

¹⁵This is the regular expiration cycle. There also exist options and futures expiring on the other two of the three nearest calendar months which are, however, not as liquid as the options on the regular cycle. For further details see LIFFE (2000).

select the option and futures contract on the regular cycle with the shortest maturity provided it is not below one month.¹⁸ For two weeks within this period there were no data available for Friday, in four other weeks the ECB Governing Council met on Wednesday as opposed to Thursday so that we had to drop these observations from our sample leaving a total of 92 weeks for our estimation. This data set comprised 8894 observed option prices with maturity between one and four months on Wednesdays and the same number for Fridays. In 43 of these 92 weeks a meeting of the Governing Council of the ECB took place (see European Central Bank (2001), pp. 75 and European Central Bank (2000), pp. 181).

Since options far out-of-the-money or far in-the-money are not traded very actively und will therefore contain few reliable information on market participants' expectations we only use options with a time adjusted moneyness between +1.35 and -1.35 percent. Time-adjusted moneyness is here defined as $(\frac{K}{F_t} - 1)\frac{1}{\sqrt{T-t}}$, where F_t is the current futures price (compare Dumas, Fleming, and Whaley (2000)). This leads to a decline in the number of eligible options as their maturity shortens and reflects the fact that with a shorter maturity large changes in the underlying's price become less and less likely and therefore trading concentrates fewer exercise prices around the current level of the futures price.

Implied volatilities are then computed from the options selected by using Black's (1976) formula for pricing futures options.¹⁹ Since European Put-Call-Parity holds for the options in our sample there is no difference in using put or call options. Therefore, we decided to construct the volatility smile from in-the-money call and in-the money put options as in Aït-Sahalia, Wang und Yared (2001).²⁰

¹⁸Options of very short maturity often are not traded very actively and thus might not provide much information on market participants' expectations.

¹⁹Calculating implied volatilities requires apart from the exercise price and the current value of the underlying asset a risk-free interest rate with the same maturity as the option as an input factor. We proxy for this interest rate by using linear interpolations from Euribor rates with maturity of one to five months.

²⁰This avoids problems with using out-of-the money options for which the ratio of minimum tick size to price is relatively large. Since the maximum error introduced by discrete price quotation is equal to half the minimum tick size these options are suspect to carry large measurement errors (e.g. Bliss and Panigirtzoglou (1999)).

In some cases inspection of the resulting delta/volatility pairs showed delta to be nonmonotonous in strike price. This problem was most severe for option prices from 1999 with only short maturity. In these cases we eliminated all delta/volatility combinations on the outer edge of our considered strike price spectrum beginning with the nonmonotonous observation. Finally, we dropped all observations with delta below 0.025 or above 0.975 because when approaching zero or one, delta becomes quite insensitive to changes in strike price and the volatility smile with respect to delta becomes so steep that it is no longer possible to fit the spline polynomials to the volatility smile.

5 Comparison Methodology and Results

In this section we will illustrate our suggested approach to investigate changes in riskneutral densities by applying it to risk-neutral PDFs computed with the estimation procedure explained before.

Our estimation yields 184 different risk-neutral PDFs extracted from option prices. Since it is very difficult to compare complete density functions with each other we condense the information contained therein into a small set of statistics. To capture asymmetries in the implied PDF we calculate a skewness parameter as,²¹

$$sp2 = \frac{\left(\mathbf{E}^Q[(\log(S_T) - \mu)^3]\right)}{stddev^{\frac{3}{2}}},\tag{7}$$

where,

$$\mu = \mathbf{E}^{Q}[\log(S_T)] \tag{8}$$

$$stddev = \mathbf{E}^{Q}[(\log(S_T) - \mu)^2].$$
(9)

An alternative skewness measure suggested by Campa, Chang and Reider (1998) is relative intensity which is defined as the difference between the expectations of the underlying's price conditional on being outside symmetric thresholds $\underline{S}, \overline{S}$ around the futures price.

$$rel.int = int_{\bar{S},+} - int_{\underline{S},-},\tag{10}$$

²¹Theoretically, the mean of each of the implied distributions should equal the current futures price. All estimated mean values turned out to be very close to their theoretical values.

where

$$int_{\bar{S},+} = \int_{\bar{S}}^{+\infty} (S_T - \bar{S})q(S_T)dS_T,$$
 (11)

$$int_{\underline{S},+} = \int_0^{\underline{S}} (\underline{S} - S_T) q(S_T) dS_T.$$
(12)

(13)

We also compute a kurtosis parameter as,

$$wp3 = \frac{\left(\mathbf{E}^{Q}[(\log(S_{T}) - \mu)^{4}]\right)}{stddev^{2}} - 3.$$
 (14)

sp2 and wp3 are computed with respect to the logarithm of the underlying's price, as suggested by Levin, McManus and Watt (1998) and McManus (1999), to facilitate comparison with a lognormal benchmark, i.e. Black's (1976) model for pricing futures options.²²

In search for statistics that represent uncertainty in market participants' expectations, i.e. the dispersion of expectations, we decide to look at volatility, i.e. standard deviation scaled by time to maturity $(vol = stddev/\sqrt{T-t})$, in addition to standard deviation. Furthermore, we consider scaled interpercentile ranges as suggested in Melick and Thomas (1998).²³ We construct scaled interpercentile ranges (SIPR) by dividing the difference of the 90 and 10 percent percentiles of the risk-neutral PDF, and the difference between the 75 and 25 percent percentiles, respectively, by the futures price on that day.²⁴ Typically, the percentiles converge towards the mean (i.e. the futures price) of the implied PDF as the option's maturity shortens so that the scaled interpercentile ranges decline with time until we switch to a contract with longer maturity. The intuition behind this observation is that, all other things equal, as the time to maturity decreases the market becomes more certain about the future price of the underlying while extreme outcomes become less likely (Bahra (1997)). Therefore, as suggested by Melick and Thomas (1998), a constant maturity series of scaled interpercentile ranges is computed by regressing the scaled interpercentile series on a constant and the time to expiration of the option considered (measured in trading days). Then, the estimated

 $^{^{22}}$ For Black's (1976) model, i.e. a single lognormal PDF, sp2=0, and wp3=0.

²³For the use of percentiles computed from the risk-neutral PDF, see also Bahra (1997).

 $^{^{24}}$ On all days in our sample these percentiles fall well within the range of prices covered by our option data.

time trend is subtracted from the SIPR resulting in a series with constant maturity. Since the calculated implied moments might depend on maturity, too, we apply to them the same correction procedure. In table 1 the column marked "trend" shows whether the outlined procedure for adjusting for maturity has been applied or not.²⁵

To investigate characteristic features of these constructed statistics we split our sample into two sets. The first one contains the days of interest, in our case Wednesdays and Fridays of weeks with a meeting of the Governing Council of the ECB. The second subsample is our benchmark and comprises Wednesdays and Fridays of weeks without a council meeting. We then compare the distributional characteristics of the computed statistics for Wednesday or Friday with those of the respective day in our benchmark set by means of a Kolmogorov-Smirnov test (see Kendall and Stuart (1973) or Rinne (1997)).²⁶ By comparing only Wednesdays to Wednesdays and Fridays to Fridays we effectively avoid dealing with day-of-the-week effects. In addition to that we compare the changes in the statistics from Wednesday to Friday in order to investigate whether the decisions taken on Council meetings lead to significant revisions of expectations.

The Kolmogorov-Smirnov test is based on the difference between the empirical distribution functions of the statistic in question in both subsamples. The empirical distribution function of the relevant statistic (or of its change) is calculated for each of the two subsamples and is denoted as \hat{F}_{CM} for the weeks in which the Governing Council met and \hat{F}_{NCM} for the other weeks. The null hypothesis is that both samples of statistics are drawn from the same underlying distribution and that therefore, their distribution functions are equal $H_0: F_{CM} = F_{NCM}$. Two alternative hypotheses are in turn considered. In the first case $H_1: F_{CM} > F_{NCM}$, i.e. the statistic is stochastically smaller in CM than in NCM. In the second case $H_1: F_{CM} < F_{NCM}$, i.e. the statistic is stochastically larger in CM than in NCM.²⁷

 $^{^{25}}$ We only use the maturity adjusted results if the trend coefficient in the regression is significantly different from zero.

²⁶This test is suitable for applications of this type because it can cope with small sample sizes.

²⁷The 5 and 10 percent critical values for this sample size are 0.25283 and 0.22115, respectively. H_0 can be rejected in favour of H_1 in the center right column (D+) if D+ exceeds the critical value. H_0 can be rejected in favour of H_1 in the right column (D-) if D- is less than -1 times the critical value (e.g. Rinne (1997)).

statistic	trend	D+	<i>D</i> -	
		$H_1: F_{CM} > F_{NCM}$	$H_1: F_{CM} < F_{NCM}$	
sp2 (Wed)	n	0.15045	-0.09492	
sp2 (Fri)	n	0.04699	-0.21927	
$\Delta sp2$	n	0.00000	-0.26673^*	
rel.int (Wed)	У	0.17086	-0.10441	
rel.int (Fri)	У	0.04129	-0.23351**	
$\Delta rel.int$	n	0.00759	-0.19174	
$wp3 \pmod{1}$	n	0.18462	-0.05885	
wp3 (Fri)	n	0.07167	-0.07897	
$\Delta wp3$	n	0.06217	-0.13764	
stddev (Wed)	У	0.06882	-0.07119	
stddev (Fri)	у	0.16184	-0.06645	
Δ stdabw	n	0.11011	-0.12957	
vol (Wed)	У	0.06502	-0.08021	
vol (Fri)	У	0.16184	-0.08685	
$\Delta \mathrm{vol}$	n	0.10916	-0.13052	
SIPR9010 (Wed)	У	0.10679	-0.06028	
SIPR9010 (Fri)	У	0.12008	-0.05078	
$\Delta { m SIPR9010}$	n	0.21251	-0.17038	
SIPR7525 (Wed)	У	0.06217	-0.07784	
SIPR7525 (Fri)	У	0.09445	-0.03749	
Δ SIPR7525	n	0.28714^{*}	-0.04651	
* and $**$ denote significance at the 5% or 10%-level, respectively.				

Table 1: Kolmogorov-Smirnov tests for various statistics of implied PDF

	Δ stddev	$\Delta \mathrm{vol}$	Δ SIPR9010	Δ SIPR7525
Δ stddev	1.00000			
$\Delta \mathrm{vol}$	0.98319	1.00000		
Δ SIPR9010	0.80281	0.78846	1.00000	
Δ SIPR7525	0.18720	0.19632	0.59594	1.00000

Table 2: Correlation coefficients for various statistics of implied PDFs

The results in table 1 show evidence for significant differences in skewness between weeks with a ECB-council meeting and weeks without for the skewness measure sp2and for relative intensity. The change in sp2 is stochastically larger in weeks with a council meeting than in weeks without a council meeting. This means that skewness either increases more or decreases less from Wednesday to Friday in weeks in which the ECB-council met. The result is a relatively more positively skewed distribution and thus a relatively larger probability for strong increases in interest rates than for strong interest rate decreases. We also find that relative intensity is stochastically larger on Fridays in weeks with a council meeting than in the benchmark weeks indicating that particularly on Friday after a council meeting, market participants consider large increases in interest rates to be more probable than large declines.

Table 1 also shows some evidence suggesting that ECB-council meetings generally decrease the uncertainty in market expectations, because the change in the scaled interpercentile range (75% to 25%) is significantly stochastically smaller in weeks with a council meeting. That means, that this percentile range decreases more or increases less from Wednesday to Friday in weeks in which the ECB-council meet than in the benchmark weeks. We would expect this result if market participants were not completely certain in advance of the council meeting of what decisions to expect and if this uncertainty was resolved by the announcement of the council's decision. However neither the standard deviation, nor volatility or the wider scaled interpercentile range (90% to 10%) appear to exhibit significant differences between weeks with and without council meetings. To shed some light on these discrepancies we calculated correlation coefficients between the changes from Wednesday to Friday in standard deviation, volatility and scaled interpercentile differences.

Standard deviation, volatility and the wider scaled interpercentile range (90% to 10%) are highly correlated. Correlations between this group of statistics and the narrow scaled interpercentile range (75% to 25%) however, are much weaker. This shows that the first three statistics put more weight on changes in the tails of the risk-neutral PDF than SIPR7525, i.e. these statistics convey information more focused on expectations of extreme outcomes for the Euribor whereas SIPR7525 relates to uncertainty with respect to moderate interest rate changes.

In the time period covered by our sample there were seven council meetings on which official rates were increased while there was only one decrease in official rates. Many central banks follow interest rate smoothing policies that make it more likely that official interest rates will be increased further after an increase in rates than that they will be decreased. This type of policy in small steps should also show up in market expectations after a rate increase making higher interest rates in the future more likely than lower rates.²⁸

To investigate whether the result for skewness is predominantly caused by this effect we eliminate from the CM-subsample all observations for weeks in which the Governing Council decided to change official rates. The test statistics for the remaining sample are presented in table 3.²⁹

We no longer find significant test statistics for sp2 and relative intensity. This indicates that indeed most of the differences in the change in skewness from Wednesday to Friday and in the Friday level of relative intensity might be due to the seven observations when the council meeting resulted in an increase in interest rates.³⁰ Therefore the significant effects of central bank council meetings on the asymmetry in expectations seem to stem from market participant's anticipation of a policy of small steps by the ECB in which they expect more interest rate increases to follow the observed raises in interest rates.

Another interesting result is that while the change in SIPR7525 remains significantly

²⁸For interest rate smoothing and the behaviour of money market rates see Rudebusch (1995). Goodfriend (1991) discusses the implications of interest rate smoothing for the behaviour of the operating target. See also Goodhart (1999).

 $^{^{29}}$ For this reduced sample size the critical values are 0.26743 (5%) and 0.23156 (10%), respectively.

³⁰Because of the small sample size it is not possible to test the weeks in which official interest rates were changed against the weeks in which they were left unchanged effectively.

statistic	trend	D+	<i>D</i> -	
		$H_1: F_{CM} > F_{NCM}$	$H_1: F_{CM} < F_{NCM}$	
sp2 (Mi)	n	0.07347	-0.13878	
sp2 (Fr)	n	0.07755	-0.20000	
$\Delta sp2$	n	0.06531	-0.20408	
rel.int (Mi)	У	0.11837	-0.14694	
rel.int (Fr)	У	0.06122	-0.21244	
$\Delta rel.Int$	n	0.09796	-0.16327	
wp3 (Mi)	n	0.19592	-0.04490	
wp3~(Fr)	n	0.07347	-0.07347	
$\Delta wp3$	n	0.05306	-0.17551	
stddev (Mi)	у	0.12245	-0.09769	
stddev (Fr)	У	0.07347	-0.11429	
Δ stdabw	n	0.02041	-0.25714^{**}	
vol (Mi)	у	0.09388	-0.09796	
vol (Fr)	У	0.09388	-0.13469	
$\Delta \mathrm{vol}$	n	0.04490	-0.24082**	
SIPR9010 (Mi)	у	0.19184	-0.05714	
SIPR9010 (Fr)	У	0.08571	-0.08980	
$\Delta { m SIPR9010}$	n	0.09388	-0.24082**	
SIPR7525 (Mi)	у	0.07347	-0.08980	
SIPR7525 (Fr)	у	0.09388	-0.06939	
$\Delta SIPR7525$	n	0.23265**	-0.05714	
** denotes significance at the 10%-level.				

Table 3: Kolmogorov-Smirnov tests for various statistics of implied PDF (no interest rate changes

stochastically smaller in weeks with ECB-council meetings now SIPR9010, standard deviation and volatility are significantly stochastically larger in these weeks compared to the benchmark sample. In the light of the correlations presented in table 2 these results indicate that central bank council meetings appear to have increased uncertainty in market participants' expectations concerning to extreme interest rate movements in the future while causing decreasing uncertainty related to moderate interest rate movements. The result for SIPR7525 was robust with respect to the change in the sample, the other result was not. Since the standard deviation, volatility and SIPR9010 appear to be stochastically larger in CM after eliminating the central bank meetings where interest rates were changed this provides indirect evidence that over the council meetings that were eliminated uncertainty measured by these statistics declined strongly. Along broad lines the eliminated observations comprise unanticipated interest rate changes and anticipated interest rate changes. Most probably, the decline in uncertainty is due to the interest rate changes that were correctly anticipated by market participants.

The remaining sample consists of cases where the market correctly anticipated that interest rates would remain unchanged and of cases where market participants expected a change in interest rates which however, did not manifest after the ECB-council meeting. Thinking about these scenarios we would expect to observe increases in uncertainty about the future course of monetary policy if interest rate expectations were disappointed and market participants wonder about their ability to anticipate and understand central bank actions. On the other hand, for events where no change in interest rates were anticipated and the council's decision conformed to this expectation we would not expect to observe significant changes in uncertainty in market participants' expectations. The results for standard deviation, volatility and SIPR9010 in table 2 under these hypotheses would originate from a number of events where expectations of interest rate changes were disappointed.

Overall, these results suggest the presence of considerable heterogeneity with respect to changes in uncertainty about large interest rate changes between these different "types" of central bank council meetings covered by the sample.

6 Conclusion

In this paper we propose an approach to highlight significant changes in risk-neutral PDFs extracted from option prices. Since statistics calculated from implied risk-neutral PDFs exhibit large changes on a day-to-day basis it is difficult to use information provided by them to investigate how important news or events affect the expectations formed by participants in financial markets. Our approach uses a subsample as a benchmark in order to test for distributional differences in the statistics of a period of interest relative to it. In an application using LIFFE options on three month Euribor futures we use the weeks in which no meeting of the Governing Council of the ECB took place as the benchmark in order to examine the influence of ECB-Council meetings on market participants' expectations.

By focussing on distributional differences between the observations of interest and the benchmark observations the approach suggested deviates from the more common approach in interpreting risk-neutral densities which does not take the question into account how significant observed changes in densities are statistically. Provided a large enough sample size our approach might be able to highlight important differences in risk-neutral PDFs and their evolution in relation to important events. However, a limitation is that it cannot be applied to investigate single events but relies on some kind of recurring event or a set of events which are expected to affect risk-neutral densities in a similar way in order to construct empirical distribution functions of the statistics of interest.

References

- Abken, Peter A., Dilip B. Madan, and Sailesh Ramamurtie (1996), Estimation of Risk-Neutral and Statistical Densities by Hermite Polynomial Approximation: With an Application to Eurodollar Futures Options, Working Paper, Federal Reserve Bank of Atlanta, WP 96-5.
- Aït-Sahalia, Yacine, Yubo Wang and Francis Yared (2001), Do Option Markets Correctly Price the Probabilities of Movement of the Underlying Asset?, Journal of Econometrics 102, 67-110.
- Bahra, Bupinder (1997), Implied Risk-Neutral Probability Density Functions from Option Prices: Theory and Application, Working Paper, Bank of England.
- Bank for International Settlements (1999), Proceedings of a workshop on estimating and interpreting probability density functions, 14 June 1999.
- Bank of Japan (1995), Empirical Analyses of the Information Content of Implied Volatilities, Quarterly Bulletin, February, 63-88.
- Black, Fischer (1976), The Pricing of Commodity Contracts, Journal of Financial Economics 3, 167-79.
- Bliss, Robert B., and Nicolaos Panigirtzoglou (1999), Testing the Stability of Implied Probability Density Functions, Arbeitspapier, Federal Reserve Bank of Chicago, WP 99-21.
- Borio, Claudio E. V. (1997), The Implementation of Monetary Policy in Industrial Countries: A Survey, *BIS* Economic Papers 47, Bank for International Settlements, Basle.
- Breeden, Douglas, and Robert H. Litzenberger (1978), Prices of State-contingent Claims Implicit in Option Prices, Journal of Business 51, 621-51.
- Butler, Creon, and H. Davies (1998), Assessing Market Views on Monetary Policy: The Use of Implied Risk Neutral Probability Distributions, Working Paper, Bank of England.
- Campa, José M., and P.H. Kevin Chang (1995), Testing the Expectations Hypothesis on the Term Structure of Volatilities in Foreign Exchange Options, *Journal of Finance* 50, 529-47.
- Campa, José M., and P.H. Kevin Chang (1996), Arbitrage-based tests of target-zone credibility: Evidence from ERM cross-rate options, American Economic Review 86, 726-40.
- Campa, José M. und P.H. Kevin Chang (1998), ERM Realignment Risk and its Economic Determinants as Reflected in Cross-Rate Options, *Economic Journal* 108, 1046-66.
- Campa, José M., P.H. Kevin Chang and Robert L. Reider (1997), EMS Credibility: Exchange Rate Expectations Implied by Derivatives Prices, *Economic Policy* 5, 53-89.
- Campa, José M., P.H. Kevin Chang, and Robert L. Reider (1998), Implied Exchange Rate Distributions: Evidence from OTC Option Markets, *Journal of International Money and Finance* 17, 117-60.
- Canina, Linda, and Stephen Figlewski (1993), The Informational Content of Implied Volatility, Review of Financial Studies 6, 659-81.
- Chang, P.H. Kevin, and William R. Melick (1999), Background Note, in: Estimating and Interpreting Probability Density Functions – Proceedings of a Workshop held at the BIS on 14 June 1999, Bank for International Settlements.
- Chen, Ren-Raw, and Louis Scott (1993), Pricing interest rate futures options with futures-style margining, Journal of Futures Markets 13, 15-22.
- Cooper, Neil (1999), Testing Techniques for Estimating Implied RNDs from the prices of European-Style Options, in: Estimating and Interpreting Probability Density Functions – Proceedings of a Workshop held at the BIS on 14 June 1999, Bank for International Settlements.
- Cooper, Neil, and James Talbot (1999), The Yen/Dollar Exchange Rate in 1998: Views from Options Markets, Bank of England Quarterly Bulletin, February, 68-77.
- Coutant, Sophie, Eric Jondeau, and Michael Rockinger (1998), Reading Interest Rate and Bond Futures Options' Smiles: How PIBOR and Notional Operators Appreciated the 1997 French Snap Election, Working Paper HEC School of Management.

- Cox, John C., and Stephen A. Ross (1976), The Valuation of Options for Alternative Stochastic Processes, Journal of Financial Economics 3, 145-66.
- Cuthbertson, Keith (1996), Quantitative Financial Economics. Stocks, Bonds and Foreign Exchange, Wiley, Chichester etc.
- Day, Theodore E., and Craig M. Lewis (1992), Stock Market Volatility and the Information Content of Stock Index Options, *Journal of Econometrics* 52, 267-87.
- Dumas, Bernard, Jeff Fleming, and Robert E. Whaley (1998), Implied Volatility Functions: Empirical Tests, Journal of Finance 53, 2059-2106.
- European Central Bank (2000), Annual Report 1999.
- European Central Bank (2001), Monthly Bulletin, January.
- Fleming, Jeff (1998), The Quality of Market Volatility Forecasts Implied by S&P 100 Index Option Prices, Journal of Empirical Finance 5, 317-45.
- Fornari, Fabio, and Roberto Violi (1998), The Probability Density Function of Interest Rates Implied in the Price of Options, Temi di Discussione del Servizio Studi 339, Banca D'Italia.
- Galati, Gabriele, and William R. Melick (1999), Perceived Central Bank Intervention and Market Expectations: An Empirical Study of the Yen/Dollar Exchange Rate 1993-96, Working Paper No. 77, Bank for International Settlements.
- Goodfriend, Marvin (1991), Interest Rates and the Conduct of Monetary Policy, Carnegie-Rochester Conference Series on Public Policy 34, 7-30.
- Goodhart, Charles E. A. (1999), Central Bankers and Uncertainty, *Quarterly Bulletin*, February, 102-121, Bank of England.
- Harrison, J. Michael, and David M. Kreps (1979), Martingales and Arbitrage in Multiperiod Securities Markets, Journal of Economic Theory 20, 381-408.
- Jorion, Phillipe (1995), Predicting Volatility in the Foreign Exchange Market, Journal of Finance 50, 507-29.
- Kendall, Maurice, and Alan Stuart (1973), The Advanced Theory of Statistics, Vol. II, 3rd ed., Charles Griffin & Company, London.
- Lamoureux, Christopher G., and William D. Lastrapes (1993), Forecasting Stock-Return Variance: Toward an Understanding of Stochastic Implied Volatilities, *Review of Financial Studies* 6, 293-325.
- Levin, Alexander, Des McManus, and David Watt (1998), The Information Content of Canadian Dollar Futures Options, in: Information in Financial Asset Prices, Proceedings of a Conference held by the Bank of Canada, May 1998, 229-75.
- LIFFE (2000), Short Term Interest Rate Futures and Options, London.
- Malz, Allan M. (1996), Using Option Prices to Estimate Realignment Probabilities in the European Monetary System: The Case of Sterling-Mark, *Journal of International Money and Finance* 15, 717-48.
- Malz, Allan M. (1998), Option Prices and the Probability Distribution of Exchange Rates, in: Chen, Zhaohui (Hrsg.), Currency Options and Exchange Rate Economics, World Scientific, Singapur, 108-37.
- McManus, Des (1999), The Information Content of Interest Rate Futures Options, in: Estimating and Interpreting Probability Density Functions – Proceedings of a Workshop held at the BIS on 14 June 1999, Bank for International Settlements.
- Melick, William R. (1999), Results of the Estimation of Implied PDFs from a Common Dataset, in: Estimating and Interpreting Probability Density Functions – Proceedings of a Workshop held at the BIS on 14 June 1999, Bank for International Settlements.
- Melick, William R., and Charles P. Thomas (1997), Recovering an Asset's Implied PDF from Option Prices: An Application to Crude Oil during the Gulf Crisis, Journal of Financial and Quantitative Analysis 32, 91-115.
- Melick, William R., and Charles P. Thomas (1998), Confidence Intervals and Constant-Maturity Series for Probability Measures Extracted from Options Prices, in: Information in Financial Asset Prices, Proceedings of a Conference held by the Bank of Canada, May 1998, 293-320.

- Nakamura, Hisashi, and Shigenori Shiratsuka (1999), Extracting Market Expectations from Option Prices: Case Studies in Japanese Option Markets, Working Paper, Bank of Japan.
- Neuhaus, Holger (1995), Der Informationsgehalt von Derivaten für die Geldpolitik: Implizite Volatilitäten und Wahrscheinlichkeiten, Diskussionspapier der Volkswirtschaftlichen Arbeitsgruppe der Deutschen Bundesbank 3/95.
- Rinne, Horst (1997), Taschenbuch der Statistik, 2nd ed., Verlag Harri Deutsch, Thun, Frankfurt a.M.
- Rudebusch, Glenn D. (1995), Federal Reserve Interest Rate Targeting, Rational Expectations, and the Term Structure, *Journal of Monetary Economics* 35, 245-74.
- Scott, Louis O. (1992), The Information Content of Prices in Derivative Security Markets, *IMF Staff Papers* 39, 596-625.

Shimko, David (1993), Bounds of Probability, RISK Magazine 6, 33-37.

Söderlind, Paul (2000), Market Expectations in the UK before and After the ERM Crisis, *Economica* 67, 1-18.