Portfolio construction by volatility forecasts: Does the covariance structure matter?

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Abstract

This paper explores the performance of a global minimum variance (GMV) portfolio in dependence of the structure of the covariance matrix and the type of volatility model. Different information sets of time series are used to predict the future covariance matrix. We investigate diagonal portfolio strategies based on univariate and multivariate GARCH models for a portfolio consisting of the North America, Europe and the Pacific region. The evaluation is based on a daily out-of-sample comparison from 25th May 1998 until 3rd April 2000. We find that variance forecasts are more important than covariance forecasts and that multivariate volatility models yield better results than univariate volatility models.

Keywords: Asymmetric GARCH Model, BEKK Model, Volatility Forecasts, Mean-Variance Portfolio, Diagonal Portfolios.

1 Introduction

Markowitz [1] proposed the expected return (mean) and the variance of return of the portfolio as criteria for optimal portfolio selection. He showed that the expected return of the portfolio is a weighted average of the expected returns of individual securities and the variance of return of portfolio is a particular function of the variances of, and the covariances between, securities and their weights in the portfolio. The performance of optimal mean-variance (MV) portfolios depends on the quality of the forecasts of the first two moments, i.e. the future mean returns and their variance matrix. Chopra and Ziemba [2] have shown that estimation errors in the predicted returns are most influential for the portfolio performance. Errors in variances and covariances are less important. Pojarliev and Polasek [3] (PP henceforth) found in an empirical analysis that the weights of a global minimum variance (GMV) portfolio are very sensitive with respect to the inputs, i.e. the predicted variance matrix. Therefore the structure of the variance matrix and the selection of the appropriate volatility models will be important for the portfolio weights and determines the overall portfolio performance.

In practice, the portfolio construction is a process divided into two parts: the stock picking and the weights selection for the different stocks. Some portfolio managers are trying to track the benchmark by selecting a subset of stocks in order to reduce the transaction costs. Nevertheless, the number of assets in actively managed portfolios is usually high, which leads to dimensionality problems in the prediction of the variance matrix of the assets included in the portfolio.

The ordinary (classical) time series approach for portfolio construction will become numerically intractable for higher dimensional portfolios. Assuming a diagonal structure for the variance matrix simplifies the prediction process and would provide a solution for the high dimensional problem. What is the impact of such information losses (e.g. by setting covariances equal to zero) on the portfolio performance? We explore portfolios based on a diagonal structure for the variance matrix (of the assets which are included in the portfolio) and we evaluate this strategy by comparing the 'diagonal portfolio' with a portfolio based on the full variance matrix. Furthermore, we compare diagonal portfolio strategies based on univariate and multivariate GARCH models and we measure the "added value" of the multivariate modeling by the relative differences of the *Sharpe* ratios.

This paper is organized as follows: Section 2 describes the univariate and multivariate GARCH models used to predict the conditional volatility. Section 3 presents the methodology for constructing the different portfolios and compares the performance. Some concluding remarks are given in the final section.

2 Volatility forecasts

The volatility prediction of portfolio assets is one of the key factors for modern portfolio selection problems. Furthermore, it plays a significant role in derivative pricing. Many statistical models have been proposed to describe the behavior of stock markets volatility, including rolling variance estimates, ARCH models and non-parametric methods. Empirical research has provided a number of stylized facts about the volatility of the stock markets and a recent survey is given in Engle and Patton (2000) [4]. There exists a wide-spread consensus that volatility processes exhibits persistence (i.e. the conditional return variances have a lasting effect on the annualized variance over many periods ahead); these conditional variances are assumed to be 'mean reverting' such that there is a certain level of volatility to which the conditional variances will return. We investigate the volatility of the daily returns of the MSCI North America, MSCI Europe and MSCI Pacific indices from 1st May 1995 until 3rd April 2000. The first 800 observations (from 1st May 1995 until 22nd May 1998) are used as a "training" sample for the model selection and the rest for the out-of-sample evaluation.

The following Table 1 contains the annualized standard deviations (SD) of the daily returns of the three MSCI indices for the full sample and for the year

1998. Table 1 shows that the year 1998 was an exceptional volatile year: The

MSCI region	Pacific	Europe	North America
1998			
annualized SD	28.44%	20.65%	19.40%
Full sample			
annualized SD	20.73%	14.47%	16.97%

Table 1: Annualized standard deviations (SD) for the three MSCI indices for the full sample (from 1st May 1995 until 3rd April 2000) and for the year 1998. Annualized SD is computed by multiplying the standard deviation of the daily returns by $\sqrt{250}$.

annualized SD in 1998 is much higher for the three MSCI indices in comparison to the annualized SD over the last five years (1st May 1995 until 3rd April 2000). Presumably the Asia crisis in 1997/98 was responsible for the higher volatility in the stock markets in 1998. This leads us to consider an asymmetric GARCH model to forecast the volatilities of the three MSCI indices.

2.1 The asymmetric GARCH model

To predict volatilities we assume asymmetric GARCH models for the index returns and normally distributed errors

$$r_t | I_{t-1} \sim N[\mu, \sigma_t^2] \tag{1}$$

or

$$r_t = \mu + \epsilon_t, \quad t = 1, ..., T, \tag{2}$$

where I_{t-1} is the information set until time $t-1$. We assume a constant mean μ for the returns and for the errors ϵ_t a Gaussian distribution with mean zero and variance σ_t^2. We parameterize the conditional variances by an asymmetric GARCH model of orders p and q, which is denoted as AGARCH(p,q) model and has the form (see e.g. Glosten et al. [5])

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} (\alpha_i + \gamma_i S_{t-i}) \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 \tag{3}$$

where S_t is the dummy variable for the negative residuals and is defined as

$$S_t = \begin{cases} 1 & \text{if} \quad \epsilon_t < 0; \\ 0 & \text{if} \quad \epsilon_t \geq 0. \end{cases} \tag{4}$$

The idea behind the AGARCH model is that asymmetric behaviour of the negative shocks are sources for additional risk. We estimate an AGARCH(1,1) model for the daily returns of the three MSCI indices using the last 800 observations (approximately 3 years) of our time horizon. For the estimation of the (non-linear) model we use the BHHH algorithm of Berndt et al. [6].

The AGARCH(1,1) models for the daily returns of the MSCI regions (using the last 800 observations of our data set, from 1st May 1995 until 22 May 1998) are estimated as

1. MSCI Pacific index

$$\begin{array}{cccccccc} \hat{\sigma}_t^2 & = & 10^{-7}6.6 & + & (0.017 + 0.05 S_{t-1}) \epsilon_{t-1}^2 & + & 0.95 \sigma_{t-1}^2; \\ (t-val.) & & (1.87) & & (1.54) \quad (3.58) & & (78.62) \end{array}$$

2. MSCI Europe index

$$\begin{array}{cccccccc} \hat{\sigma}_t^2 & = & 10^{-7}4.1 & + & (0.043 + 0.02 S_{t-1}) \epsilon_{t-1}^2 & + & 0.94 \sigma_{t-1}^2; \\ (t-val.) & & (1.30) & & (1.96) \quad (0.78) & & (52.33) \end{array}$$

3. MSCI North America index

$$\begin{array}{cccccccc} \hat{\sigma}_t^2 & = & 10^{-6}3.3 & + & (0.003 + 0.18 S_{t-1}) \epsilon_{t-1}^2 & + & 0.86 \sigma_{t-1}^2. \\ (t-val.) & & (4.23) & & (0.19) \quad (7.27) & & (44.21) \end{array}$$

The comparison of the regional models exhibit several interesting results: First note that the asymmetry parameter can be estimated significantly for the Pacific and the North American region but not for Europe (for the period 1995-1998). The asymmetry coefficient γ_1 is the largest for the MSCI North America index but the α_1 and β_1 parameters are the smallest. This shows that the volatility of the daily returns follows different patterns in the 3 regions and Europe was exposed to a more balanced volatility (and risk) process than the rest of the world at the end of 1997 when there was the volatility shock induced by the Asian financial market crisis. This can be seen graphically from Figure 1 which plots the conditional SD of the returns of the three MSCI indices and we can see the higher volatility at the end of 1997 (between observations 650 and 750). How does this asymmetry affect the persistence behavior? The sum of the estimated ARCH coefficients $\alpha_1 + \beta_1$ is less than 1 for positive shocks but $\alpha_1 + \beta_1 + \gamma_1$ is larger than 1 for negative shocks. This means that negative residuals at time t tend to lead to a higher conditional volatility in the period $t + 1$.

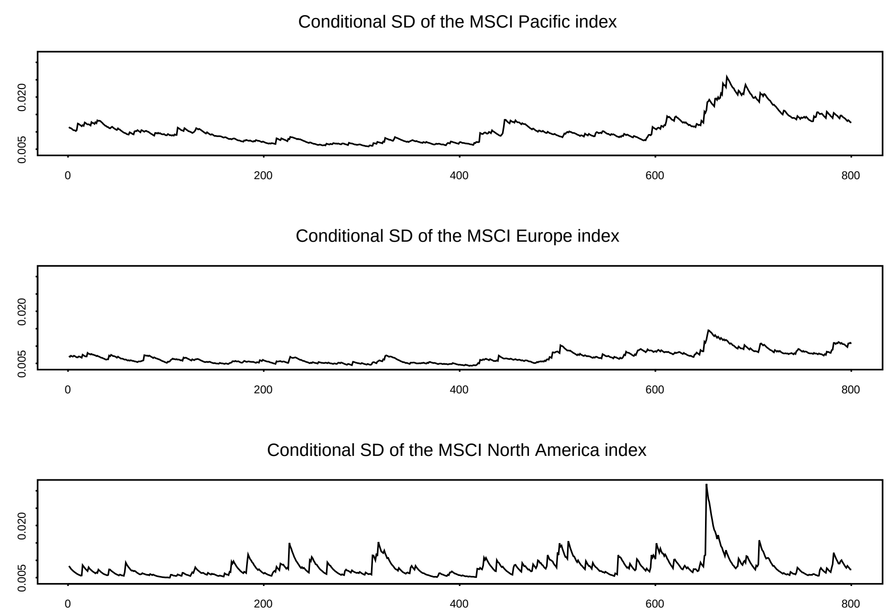

Figure 1: Conditional SD of the three MSCI indices from 1st May 1995 until 22nd May 1998 (see the estimation results above).

The goal of the volatility modelling is to use the GARCH approach for the portfolio construction. Using the AGARCH model we calculate the conditional volatility forecasts for the next trading day (25 May 1998) by:

$$\hat{\sigma}^2_{t+1} = \hat{\alpha}_0 + (\hat{\alpha_1} + \hat{\gamma_1} S_t)\epsilon^2_t + \hat{\beta}_1 \sigma^2_t. \tag{5}$$

This procedure is repeated 486 times using a rolling sample of 800 observations and re-estimating the AGARCH(1,1) models for each region. This was a very computationally intensive experience. We have used the S-GARCH module of SPlus [7]. The 486 re-estimations of each model have consumed about 2 hours. However it took more than 20 hours to re-estimate 486 times the multivariate GARCH model described in the next section. Thus, we forecast the conditional variances of the returns of the MSCI Pacific, MSCI Europe and MSCI North America indices from 25st May 1998 until 3rd April 2000. Figure 2 plots as an example the predicted conditional variance ($\hat{\sigma}^2_{t+1}$) of the daily returns of the MSCI North America index.

2.2 The Multivariate GARCH (or BEKK) Model

Let $r_t = (r^1_t, ..., r^N_t)'$ be a N dimensional vector of returns at time t and we specify the following multivariate GARCH model

$$r_t = \mu + \epsilon_t, \quad t = 1, ..., T, \tag{6}$$

with

$$\epsilon_t | I_{t-1} \sim N(0, H_t) \tag{7}$$

where μ is a constant mean vector of dimension N and the heteroskedastic errors ϵ_t are *conditionally* multivariate normally distributed. Each element of H_t depends on p lagged values of squares and cross-products of $\epsilon^l_t, l = 1, ..., N$ and on q lagged values of H_t.

Defining $h_t = vechH_t$ as the vectorisation of a symmetric matrix and $\eta_t = vec(\epsilon_t \epsilon'_t)$ then the multivariate $GARCH(p, q)$ parameterization of the variance matrix can be written as

$$h_t = a_0 + A_1 \eta_{t-1} + ... + A_p \eta_{t-p} + B_1 h_{t-1} + ... + B_q h_{t-q} \tag{8}$$

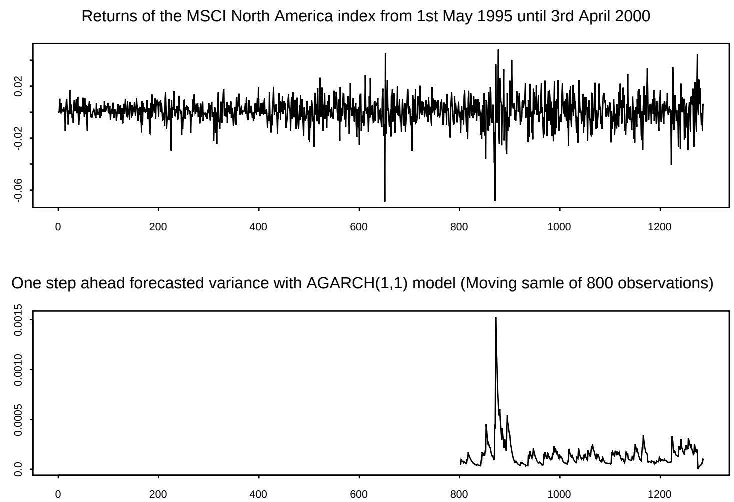

Figure 2: The out-of-sample analysis of volatility forecasts. The AGARCH (1,1) model is estimated using the past 800 observations of the returns of the MSCI North America index. After a volatility forecast is obtained for 25th May 1998, the parameters are re-estimated using another 800 past observations to obtain the forecast for the next day. This procedure is repeated 486 times.

where a_0 is a $n \times 1$ vector with $n = N(N+1)/2$ and the A_i's and B_i's are $n \times n$ parameter matrices. This parameterization is also called *vec* representation. Bollerslev et al. [8] have proposed a *diagonal* representation, in which each element of the variance matrix $h_{jk,t}$ depends only on past variances and the past values of $\epsilon_t^l \epsilon_t^k$. This means that the conditional variances depend on past own variances and past squared residuals; likewise the covariances depend on past own covariances and cross products of residuals. In the *vec* representation the *diagonal* model is obtained by assuming a diagonal structure of the matrices A_i and B_i.

In both representations it is difficult to impose the condition of a positive definite variance matrix for the estimation procedure. Engle and Kroner [9] propose the so-called BEKK representation which ensures the condition of a positive definite conditional variance matrix by a special matrix form. This BEKK representation parameterizes the variance matrix by the following way:

$$H_t = A_0 A_0' + \sum_{i=1}^{p} A_i (\epsilon_{t-i} \epsilon_{t-i}') A_i' + \sum_{i=1}^{q} B_i H_{t-i} B_i'. \tag{9}$$

For the order selection of the multivariate GARCH process we have calculated the AIC or BIC values in Table 2. Using the AIC criterion, we select a BEKK(2,1) model for the three MSCI indices. The BEKK(2,1) model was preferred to the BEKK(1,1) model because the parameter estimates in the lag 2 matrix in equation (14) contain significant coefficients on the main diagonal, i.e. will contribute to the forecasts of the conditional variances. Note that the return vector r_t in equation (7) is specified as $r_t = (r_t^P, r_t^E, r_t^A)'$ where the letters P, E and A stand for the MSCI Pacific, MSCI Europe and MSCI North America indices, respectively. We have used the geographical ordering (from east to west) and the closing values of the indices to compute the returns. Thus, the variance

	AIC	**BIC**
BEKK(1,1)	-16388	-16135^*
BEKK(2,1)	-16400^*	-16035
BEKK(2,2)	-16323	-15901
BEKK(3,2)	-16164	-15658

Table 2: AIC and BIC values for different lag orders of the BEKK model. The star (*) denotes the smallest values.

matrix forecast is obtained from

$$\hat{H}_{t+1} = \hat{A}_0 \hat{A}_0' + \sum_{i=1}^{p} \hat{A}_i E_t(\epsilon_{t+1-i} \epsilon_{t+1-i}') \hat{A}_i' + \sum_{i=1}^{q} \hat{B}_i H_{t+1-i} \hat{B}'{}_i, \tag{10}$$

where E_t is the conditional expectation operator. The estimated coefficients of the BEKK(2,1) model for the period of 800 observations are given by (t-values are in parenthesis):

$$\hat{\mu} = \begin{pmatrix} -0.00029(-0.90) \\ 0.00083(3.52) \\ 0.00073(2.49) \end{pmatrix} \tag{11}$$

$$\hat{A}_0 = \begin{pmatrix} 0.0004(0.30) & \% & \% \\ 0.0014(0.30) & -0.0016(-0.23) & \% \\ 0.0002(0.00) & 0.0012(0.00) & 0.0006(0.00) \end{pmatrix} \tag{12}$$

$$\hat{A}_1 = \begin{pmatrix} 0.22(5.95) & 0.08(2.19) & -0.01(-0.37) \\ 0.02(0.33) & 0.03(0.58) & -0.11(-1.77) \\ -0.18(-4.09) & -0.13(-4.48) & 0.12(4.05) \end{pmatrix} \tag{13}$$

$$\hat{A}_2 = \begin{pmatrix} -0.08(-1.35) & -0.10(-2.79) & -0.03(-0.90) \\ 0.11(1.49) & 0.22(4.93) & 0.09(1.57) \\ 0.04(0.67) & -0.01(-0.32) & 0.25(4.67) \end{pmatrix} \tag{14}$$

$$\hat{B} = \begin{pmatrix} 0.96(111.90) & -0.01(-1.22) & 0.00(0.13) \\ -0.06(-1.45) & 0.90(27.96) & 0.03(0.84) \\ 0.03(0.70) & 0.05(1.82) & 0.90(25.60) \end{pmatrix} \tag{15}$$

Note that all non-diagonal elements of the matrix $\hat{B}$ in (15) are not significant and that only a few non-diagonal elements of the matrices A_1 and A_2 are estimated significantly.

2.2.1 Diagnostics and Forecasting

As an overall diagnostics check we have calculated the residual autocorrelation function (ACF) of the sum of squared returns which corresponds to the squared norm of the return vector. The ACF of the squared norm of the returns of the three MSCI indices exhibits significant correlation and has motivated the use of a multivariate GARCH model. The second panel of Figure 3 shows the ACF of the squared norm of the standardized residuals for the fitted BEKK(2,1) model. Except for lag 1 all significant autocorrelations stay within the asymptotic (+/-2) standard error bounds (dotted lines).

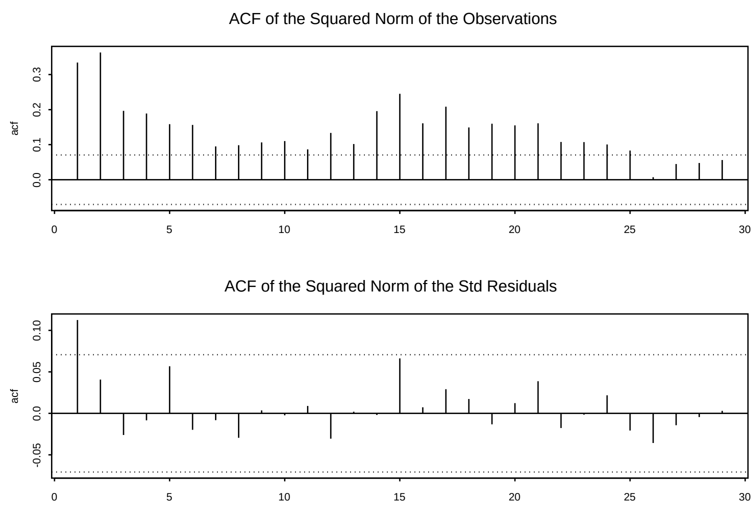

Figure 3: The ACF of the "squared norm returns" of the MSCI Pacific, MSCI Europe and MSCI North America indices and the squared norm of the standardized residuals. The norm is computed as the square root of sum of squares of the returns of the three MSCI indices.

With the estimated parameters of the BEKK model in (10) and (11) we calculate the one-step-ahead forecasts for the next trading day (25 May 1998). As for the AGARCH model, we repeat this procedure 486 times using a rolling sample of 800 observations where we re-estimate the BEKK(2,1) model each time. Figure 4 shows these forecasts for the variance of the MSCI Europe index in the lower panel (the second diagonal element of the forecasted variance matrix $\hat{H}_{t+1}$).

2.3 Volatility performance

In this section we address the problem of finding good volatility models for portfolios. The performance of a volatility model is measured by its ability to forecast future volatilities. To evaluate the actual predictive power of a model, we will use a back-testing procedure. Unfortunately, the actual volatility is unobservable. Pagan and Schwert [10] have proposed an auxiliary linear regression model to evaluate the forecasting performance of volatility models. In their model the actual volatility is approximated by the squared observed returns. Thus, we regress the squared returns r_t^2 on a constant and the forecasted volatility $\hat{\sigma}_t^2$:

$$r_t^2 = \alpha + \beta\hat{\sigma}_t^2 + \epsilon_t, \quad t = 1, ..., T. \tag{16}$$

The R^2 of this auxiliary regression is an overall measure of the forecasting performance. Table 3 summarizes the results of the auxiliary regressions for the volatilities forecast obtained from the univariate GARCH (1,1) model, the asymmetric AGARCH(1,1) and the multivariate BEKK model. While all the R^2 values are disappointingly low we can see small differences between the forecasting abilities for the MSCI indices. The daily volatility of the MSCI Europe shows the highest R^2 for the period 25th May 1998 - 3rd April 2000. The AGARCH model provides the best forecasting performance in this comparison, even for Europe where the asymmetry parameter was not estimated significantly.

	Pacific	Europe	North America
GARCH(1,1)	0.017	0.111	0.032
AGARCH(1,1)	0.028	0.126	0.058
BEKK(2,1)	0.026	0.111	0.034

Table 3: The R^2 from the auxiliary regression of the squared returns of the MSCI indices and the one step ahead variance forecast from 25th May 1998 until 3rd April 2000. The forecasts by the GARCH model are obtained in the same way as for the AGARCH model. In the multivariate case the forecasts of the volatilities are the diagonal elements of the forecasted conditional variance matrix.

Anderson and Bollerslev [11] are critical about the simple application of the auxiliary equation (16) to evaluate volatility forecasts. But our main point is that 'bad' volatility forecasts according to evaluation criteria don't necessarily imply that these volatility forecasts are not useful for improving portfolio performances (and vice versa). Ultimately the portfolio performance is important for the investors. This means that the relationship between volatility forecasts and portfolio performance needs to be more explored. Thus, in the next section we compare the performance of the portfolio based on the different volatility forecasts.

3 Portfolio construction and volatility forecasts

Assuming a portfolio which consists of the 3 MSCI indices, we have to compute the weights of the portfolio for each index. The weights of the *global minimum variance* (GMV) portfolio w_i depend only on the predicted variance matrix H_{t+1} (see [12]). Thus, by predicting the variance matrix for time $t+1$, we can compute the optimal (expected) weights of a GMV portfolio. We investigate the following questions: First, what is the 'portfolio gain' when we go from univariate forecasts to multivariate forecasts, i.e. can we quantify the value of the information gained by multivariate time series forecasts? Second, how do portfolio weights depend on a changing covariance structure?

1. **Univariate diagonal (UD) portfolio**

   We assume a diagonal variance matrix for the three MSCI indices to compute the optimal weights of portfolio one. The weights of the univariate diagonal portfolio are given by the share of an asset precision (inverse variance) over the sum of all precisions:

$$w_{t,i} = \frac{\hat{\sigma}^{-2}_{t+1,i}}{\sum_{j=1}^{3} \hat{\sigma}^{-2}_{t+1,j}}, \quad i = 1, 2, 3, \tag{17}$$

   where $\hat{\sigma}^2_{t+1,i}$ is the predicted conditional variance of the daily returns of the i^{th} MSCI index (for i = North America, Europe and the Pacific region) by the AGARCH(1,1) model as described in the previous section.

2. **Multivariate diagonal (MD) portfolio**

   We compute the weights for portfolio two using again equation (17), but the forecasted volatilities are the diagonal elements of the predicted variance matrix by the BEKK model. Figure 5 shows the weights of the MD portfolio.

3. **Global minimum variance (GMV) portfolio**

   The GMV portfolio is based on the full variance matrix forecasts from the BEKK model, hence it is the same as in PP [3] and the weights are computed as follow:

$$w_t = \frac{\hat{H}_{t+1}^{-1}\iota}{\iota'\hat{H}_{t+1}^{-1}\iota}. \tag{18}$$

   where ι is vector of ones.

We have also calculated the returns for the optimal *Sharpe* ratio portfolio, i.e. a portfolio with a riskless return (see Campbell et el. [12] p. 88). This portfolio produced in all our calculations the worst portfolio performance and therefore we have omitted it for the comparison. The bad performance of this portfolio has at least two reasons. Firstly, as mentioned before, the estimation errors in the predicted returns are very harmful for the portfolio performance and secondly, although daily asset returns are rather unpredictable, return volatilities are much more predictable. Ironically the *maximum Sharpe* ratio portfolio has a lower Sharpe ratio than the *global minimum variance* portfolio. Since the GMV portfolio depends only on the predicted variance matrix it does not depend on return forecasts. Figure 6 shows the dependence of the portfolio weights on the predicted variances. The first panel compares the variance forecasts of the AGARCH models and of the BEKK model for the returns of the MSCI Pacific index. The second panel plots the resulting weights of the univariate diagonal (UD) portfolio and multivariate diagonal (MD) portfolio for the Pacific region. We see that the weights fluctuate considerably since they are very sensitive to the input, i.e. the inverse of the predicted variance matrix $\hat{H}_{t+1}$. Therefore the selection of a reliable volatility model will mainly determine the portfolio performance.

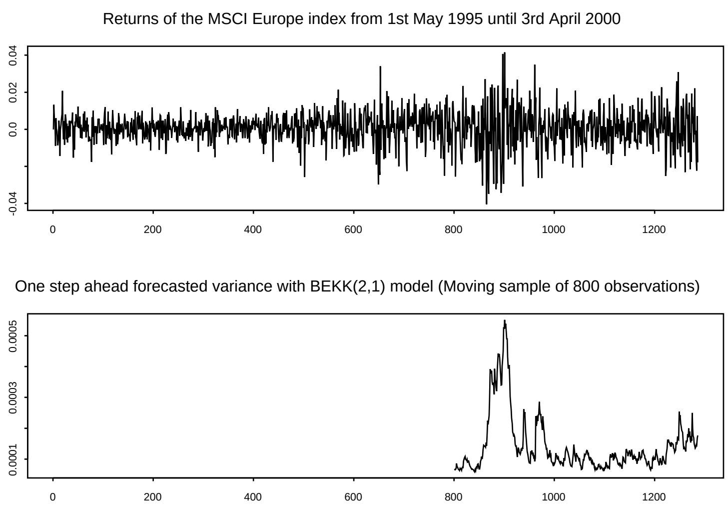

Figure 4: Out-of-sample volatility prediction for the 25th May 1998 and the following 485 days: The upper panel plots the daily returns for MSCI Europe; the lower panel plots the predicted conditional variance for MSCI Europe using the BEKK (2,1) model.

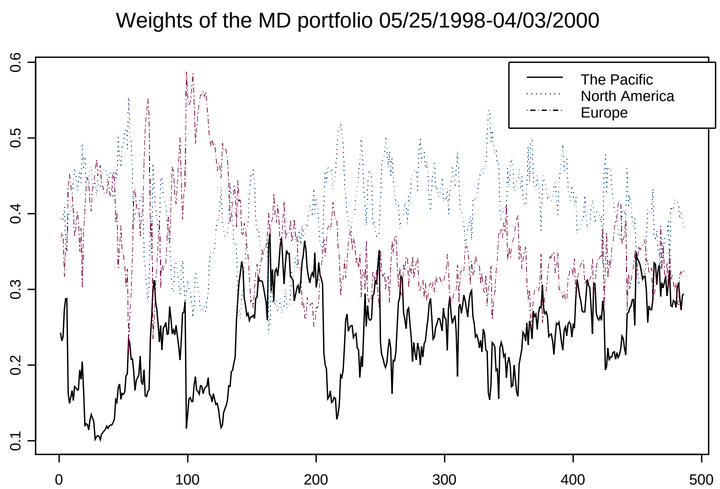

Figure 5: The figure plots the weights of the MD portfolio from 25th May 1998 until 3rd April 2000. The weights for 25th May 1998 are computed from equation (18) using the forecasted variance matrix by the BEKK model for 25th May 1998.

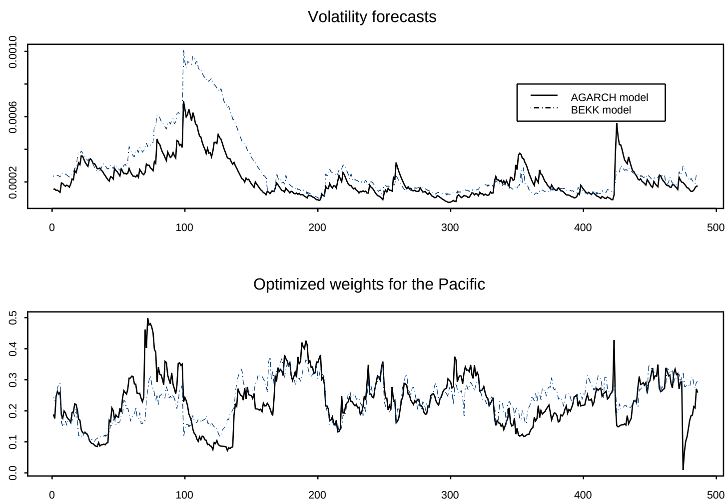

Figure 6: The first panel shows the one step ahead volatility forecast of the AGARCH and the BEKK model for the daily returns of the MSCI Pacific index from 25th May 1998 until 3rd April 2000. The second panel plots the weights of the UD and MD portfolios for the Pacific region, computed by equation (18).

3.1 Portfolio Evaluation

We compare the performance of the three portfolios with the MSCI World index from 25th May 1998 until 3rd April 2000. Table 4 summarizes the cumulative returns, the standard deviations of the returns, the resulting *Sharpe* ratios and the annualized returns and standard deviations. Although the Sharpe ratio is defined as the ratio of the *excess* portfolio return over volatility, many investment funds simply use the ratio of the cumulative portfolio return and the volatility for a given period. Therefore, we compute the *Sharpe* ratio as $SR_P = \mu_P/\sigma_P$ with μ_P the cumulative return and σ_P the standard deviation of the portfolio from 25th May 1998 until 3rd April 2000. Comparing the results for the portfolio returns and their SDs are quite interesting. We see that the MD portfolio and the GMV portfolio beat the benchmark, because they have larger annualized average returns and smaller annualized SDs. The univariate diagonal (UD)

	UD portfolio	**MD portfolio**	**GMV portfolio**	**bench-mark**
returns	20.02%	29.63%	31.85%	27.46%
SD	20.77%	20.38%	20.62%	21.11%
Sharpe ratio (RpV)	0.96	1.45	1.54	1.30
Annualized returns	10.30%	15.24%	16.39%	14.13%
Annualized SD	14.90%	14.61%	14.80%	15.14%

Table 4: Performance of the portfolios and the benchmark (MSCI World index) from 25th May 1998 until 3rd April 2000 (486 trading days). The returns are calculated as $\sum_{t=1}^{486} r_t$. The SD is computed by multiplying the standard deviation of the daily returns by $\sqrt{486}$. The annualized returns are computed as the mean portfolio returns multiplied by 250, the annualized SD as the standard deviation of the portfolio returns multiplied by $\sqrt{250}$.

portfolio has the lowest cumulative returns (20.02%) and the lowest *Sharpe* value (0.96). Thus, the R^2 obtained from the auxiliary regression model (see Table 3) does not provide a good measure for the performance of volatility prediction, because according to this criteria the univariate AGARCH model

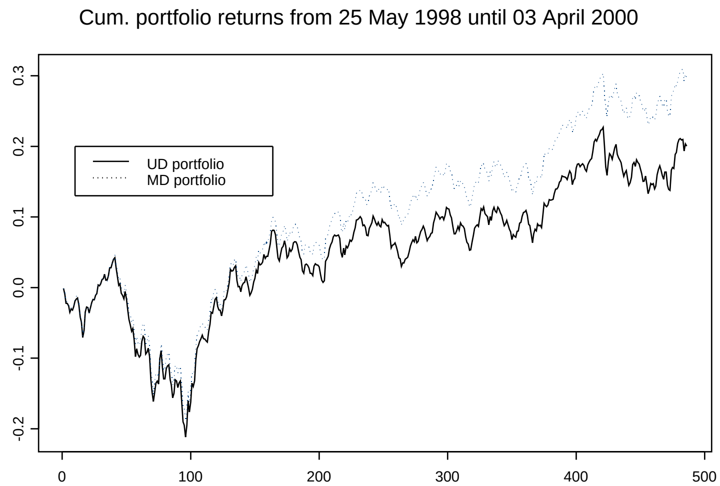

Figure 7: Cumulative returns for the univariate UD and multivariate MD portfolios from 25 May 1998 until 3 April 2000. The MD portfolio yields 9.6% more returns in this period. The MD portfolio uses multivariate BEKK volatility forecasts and the UD portfolio univariate AGARCH forecasts.

has outperformed the BEKK model. The multivariate MD portfolio yields 5% more returns p.a. and has a smaller standard deviation than the UD portfolio. The relative difference of the *Sharpe* ratios can be used as a measure for the improvement (value added) through the BEKK model. The MD portfolio has a *Sharpe* ratio which is about 50 percent larger than the *Sharpe* ratio of the UD portfolio. The best portfolio performance is obtained from the multivariate BEKK model forecasts with GMV weights where we see an annualized return of 16.39% (column GMV portfolio in Table 4) and the highest *Sharpe* ratio (1.54). This portfolio beats the benchmark by 4.5% for an evaluation period of two years. If we approximate the full GMV portfolio by the multivariate diagonal (MD) portfolio we see that the cumulative returns decrease only by 1.1% p.a. The *Sharpe* ratios also differ only slightly. This confirms that the covariances are much less important for the optimal weights than the variances. Thus, the information loss (covariances assumed to be zero) do not much harm the portfolio performance. The choice of the appropriate volatility model is much more important as it improves the *Sharpe* ratio by 50%.

4 Conclusion

The main motivation of this paper was to answer the question how portfolio managers should construct their portfolios. In previous research it was found that **1)** errors in the predicted returns are more harmful for the portfolio performance than errors in the predicted variances and covariances and **2)** volatilities are more reliable for prediction than returns, we have focused on the portfolio construction using only volatility models. This points to the question what kind of volatility models lead to better portfolio performance. The comparison between the different volatility models was also motivated by the *high dimensional* parameter problem (i.e. dimensionality restrictions in the prediction of the variance matrix) in the portfolio construction process.

Global minimum variance (GMV) portfolios based on volatility forecasts by the multivariate BEKK model dominate clearly the benchmark in the period from 25th May 1998 until 3rd April 2000. The MD portfolio based on the multivariate volatility model also dominates the benchmark and has a *Sharpe* ratio which is about 50% larger than the *Sharpe* ratio of a UD portfolio based on volatility prediction by the univariate AGARCH models. The performance of the volatility models can be measured by the *Sharpe* ratio of portfolios based on these forecasts. The relative differences of the *Sharpe* ratios can be used as an empirical measure for the performance of the volatility prediction model. The R^2 obtained from an auxiliary regression model should be interpreted with care in the sense that volatility models which lead to higher R^2 in the auxiliary regressions comparison will not necessary lead to better portfolios based on those forecasts.

There is also an important result for the construction of high dimensional portfolios based on time series forecasts. The performance evaluation of the MD portfolio and the 'full-information' GMV portfolio have shown that portfolios with an appropriate covariance structure can come close to the returns of the 'full-information' portfolio. We have found that volatility forecasts obtained by multivariate GARCH models are more important than those from univariate GARCH models and the optimum results of quantitative portfolio management can be approximated by appropriate partitioning of the information set.

References

[1] H. Markowitz. Portfolio Selection. *Journal of Finance*, 7:77–91, 1952.

[2] V. K. Chopra and W. T. Ziemba. The Effect of Errors in Means, Variances, and Covariances on Optimal Potfolio Choice. *Journal of Portfolio Management*, 19(2):6–11, 1993.

[3] M. Pojarliev and W. Polasek. Applying Multivariate Time Series Forecasts for Active Portfolio Management. *Financial Markets and Portfolio Management*, 15(2):201–211, 2001.

[4] R. Engle and A. Patton. What is a Good Volatility Model? University of California, San Diego, working paper [11 January 2001], September 2000.

[5] L. Glosten, R. Jagannathan, and D. Runkle. On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *Journal of Finance*, 48:1779–1801, 1993.

[6] E. Berndt, B. Hall, R. Hall, and J. Hausman. Estimation and Inference in Nonlinear Structural Models. *Annals of Economic and Social Measurement*, pages 653–665, 1974.

[7] Mathsoft. S+GARCH User's Manual, 1996. Data Analysis Products Division, MathSoft, Seattle.

[8] T. Bollerslev, R. Engle, and J. Wooldridge. A Capital Asset Pricing Model with Time-varying Covariances. *Journal of Political Economy*, (96):116–131, 1988.

[9] R. Engle and K. Kroner. Multivariate Simultaneous Generalized ARCH. *Econometric Theory*, 11:122–150, 1995.

[10] A. Pagan and W. Schwert. Alternative Models for Conditional Stock Volatility. *Journal of Econometrics*, (50):267–290, 1990.

[11] T. Andersen and T. Bollerslev. Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts. *International Economic Review*, (39):885–905, 1998.

[12] J. V. Campbell, A. W. Lo, and A. C. MacKinlay. *The Econometrics of Financial Markets*. Princeton University Press, 1997.