IMPLICATIONS OF A FIRM'S MARKET WEIGHT IN A CAPM FRAMEWORK

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Abstract

This paper derives the relationship between a stock's beta and its weighting in the portfolio against which its beta is calculated. Contrary to intuition the effect of this market weight is in general very substantial. We then suggest an alternative to the conventional measure of abnormal return, which requires an estimate of a firm's beta when its market weight is zero. We argue that the alternative measure is superior, and show that it can differ substantially from the conventional measure when a firm has non-trivial market weight. The difference in abnormal returns may be disaggregated into a "market return effect" and a "beta effect".

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1. Introduction

Since the development of the risk measure conventionally termed beta, by Sharpe (1963, 1964) and Lintner (1965), a number of papers have developed theoretical relationships between it and various underlying variables. These include Hamada (1972) with respect to financial leverage, Rhee (1986) concerning operating leverage, and Ehrhardt and Shrieves (1995) for the impact of warrants and convertible securities. Others, such as Rosenberg and Guy (1976) have empirically identified variables correlated with beta. Given that the portfolio against which an asset's beta is calculated includes that asset, then beta must also be sensitive to the weighting of that asset in the portfolio. This paper models the relation between an asset's beta and its market weight, and then discusses its implications for measures of abnormal returns.

The significance of this issue will depend upon the extent to which single assets achieve non-trivial market weights. Assuming that betas are defined against domestic "market" portfolios, a non-trivial market weight occurs for a number of assets in different financial markets[1]. Furthermore, the smaller the market, the more important is the phenomenon. An example of this is Hong Kong's Hang Seng Index, in which China Telecom represents 24%, and two others exceed 10%. Similar situations arise in New Zealand's NZSE40 Index, with Telecom NZ representing 32%, and the next two lying in the 5-10% range, and in The Netherlands where Royal Dutch Petroleum constitutes 20% of the CBS Index and the next four lie in the 8-10% range. Most remarkable of all is Finland, in which Nokia represents 69% of the HEX General Index and a second firm represents a further 10%. Even in the UK, which constitutes the world's second largest equity market, the two largest stocks each represent around 10% of the FTSE 100 (all data courtesy of Ord Minnett).

[1] It is standard practice to estimate betas against domestic market portfolios rather than the world market portfolio. A common motivation for doing so is estimation of a firm's cost of equity using some version of the Capital Asset Pricing Model that assumes that national capital markets are segregated. This assumption of segregation is broadly consistent with the observation that investor portfolios are strongly tilted towards domestic assets (see, for example, Cooper and Kaplanis (1994) and Tesar and Werner (1995)).

The paper begins by developing the relationship between a stock's beta and its weighting in the market proxy. Clearly, as a stock's weight in the market goes to 1, its beta against that market also approaches 1. Intuition might suggest that the progression is monotonic, and possibly even proportional to the weight. However subsequent examination shows that this intuition would be incorrect, and instead finds a non-linear relationship. Moreover, the analysis suggests that even if a firm's set of risk characteristics remains constant, changes in market weight can cause dramatic shifts in its beta. The paper then goes on to explore this phenomenon's implications for the process of measuring abnormal returns. The analysis offers an alternative method to using the standard market model, and shows how the difference in estimated abnormal returns can be disaggregated into a "market return effect" and a "beta effect."

2. Modeling the Relationship between an Asset's Beta and Market Weight

The following analysis derives how the beta of a stock j varies as its market weight changes. Let R denote an asset's return, m the market proxy, w the weight of j in m, n the market proxy exclusive of j, and β_{jn} the beta of j against n. Then, the beta of j against m (i.e., the standard definition of an asset's β) is:

$$\beta_{jm} = \frac{Cov(R_j, R_m)}{Var(R_m)}$$

$$= \frac{Cov\left[R_j, wR_j + (1-w)R_n\right]}{Var\left[wR_j + (1-w)R_n\right]}$$

$$= \frac{w\sigma_j^2 + (1-w)\sigma_{jn}}{w^2\sigma_j^2 + (1-w)^2\sigma_n^2 + 2w(1-w)\sigma_{jn}} \qquad (1)$$

Dividing through by σ_n^2 yields

$$\beta_{jm} = \frac{w\frac{\sigma_j^2}{\sigma_n^2} + (1-w)\beta_{jn}}{w^2\frac{\sigma_j^2}{\sigma_n^2} + (1-w)^2 + 2w(1-w)\beta_{jn}} \tag{2}$$

Assuming that σ_j^2, σ_n^2 and β_{jn} are independent of w, it follows that the relationship between β_{jm} and w is neither proportional nor monotonic. To prove this, we show that the slope of β_{jm} changes sign between the two market weighting extremes. Defining V as the denominator on the right hand side of (2), then differentiating β_{jm} with respect to w and using the quotient rule yields

$$\frac{d\beta_{jm}}{dw} = \frac{V\left[\frac{\sigma_j^2}{\sigma_n^2} - \beta_{jn}\right] - \left[w\frac{\sigma_j^2}{\sigma_n^2} + (1-w)\beta_{jn}\right]\left[2w\frac{\sigma_j^2}{\sigma_n^2} - 2 + 2w + 2\beta_{jn} - 4w\beta_{jn}\right]}{V^2} \tag{3}$$

If asset j constitutes the entire market ($w = 1$), then this reduces to

$$\frac{d\beta_{jm}}{dw} = \frac{\beta_{jn} - \frac{\sigma_j^2}{\sigma_n^2}}{\frac{\sigma_j^2}{\sigma_n^2}}$$

$$= \frac{\beta_{jn}\left(1 - \frac{\sigma_j^2}{\sigma_{jn}}\right)}{\frac{\sigma_j^2}{\sigma_n^2}} \tag{4}$$

$$< 0 \text{ for } \sigma_j^2 > \sigma_{jn} \text{ and } \beta_{jn} > 0$$

Also, if $w = 0$

$$\frac{d\beta_{jm}}{dw} = -2\beta_{jn}^2 + \beta_{jn} + \frac{\sigma_j^2}{\sigma_n^2} \tag{5}$$

> 0 for

$$.25 \; - \; \sqrt{.0625 + .5\frac{\sigma_j^2}{\sigma_n^2}} < \beta_{jn} \; < \; .25 \; + \; \sqrt{.0625 + .5\frac{\sigma_j^2}{\sigma_n^2}} \qquad (6)$$

If, as in (4), it is assumed that $\beta_{jn} > 0$, then (6) simplifies to

$$\beta_{jn} \; < \; .25 \; + \; \sqrt{.0625 + .5\frac{\sigma_j^2}{\sigma_n^2}} \qquad (7)$$

The above derivations make it clear that the exact curvature of the path for β_{jm} between $w = 0$ and $w = 1$ depends upon the boundary conditions involving β_{jn}, σ_{jn}, σ^2_j and σ^2_n. A typical stock has variance four times that of the market (Fama, 1976, pp. 252-254). So, if $\sigma_j^2 = 4\sigma_n^2$ then (7) becomes $\beta_{jn} < 1.69$. Even if $\sigma_j^2 = 2\sigma_n^2$ then (7) reduces to $\beta_{jn} < 1.28$, a requirement likely met by many stocks. The other requirements noted in (4) are implied by $\sigma_j > \sigma_n$ and $\beta_{jn} > 0$, and these should be satisfied by most stocks. Thus, for the typical stock j,

$$\left.\frac{d\beta_{jm}}{dw}\right|_{w=0} \; > \; 0 \qquad \text{and}$$

$$\left.\frac{d\beta_{jm}}{dw}\right|_{w=1} \; < \; 0$$

Since the slope changes from positive to negative as the market weight of asset j increases, it follows that β_{jm} reaches its maximum at $0 < w < 1$. This implies non-monotonicity, as claimed.

An illustration of the non-monotonic relationship between β_{jm} and w appears in Figure 1. Consider a stock where $\beta_{jn} = .5$ and $\sigma_j^2 = 4\sigma_n^2$. From equation (2)

$$\beta_{jm} \; = \; \frac{4w + .5(1-w)}{4w^2 + (1-w)^2 + 2w(1-w)(.5)}$$

As the firm's market weight increases from zero to .125, β_{jm} doubles from .50 to 1. As the market weight increases further, β_{jm} reaches its maximum value of 1.53 (at a market weight of .41) and then falls to 1 as w approaches 1.

The non-monotonic pattern illustrated in Figure 1 has at least two important implications. First, although β_{jm} goes to 1 as w goes to 1, it will diverge from 1 over some range of values for w. Second, the non-monotonic pattern implies that a stock with a low value for β_{jm} (i.e., less than 1) when its market weight is low will have a high value for β_{jm} (i.e., greater than 1) at certain higher market weights. All of this implies that modest changes in w can produce quite dramatic shifts in β_{jm}. This is most pronounced when β_{jn} is less than 1.

The full thrust of the second implication is that, depending on a firm's market weight, it may be a high or low beta stock. Put another way, when market weight is non-trivial, a given set of firm risk characteristics (i.e., a given σ_j^2 and σ_{jn}) does not imply a unique value for systematic risk. Thus, it is possible for a firm to go from a low beta stock to a high beta stock with little change in it's volatility (σ_j^2), the volatility of the rest of the market (σ_n^2) or its correlation with other stocks (σ_{jn}).

At the same time portfolio n (i.e., the rest of the market portfolio) also has a beta against the market, which complements that of j, i.e.

$$w\beta_{jm} + (1-w)\beta_{nm} = 1 \tag{8}$$

which implies

$$\beta_{nm} = \frac{1 - w\beta_{jm}}{1-w} \tag{9}$$

Thus, β_{nm} also varies with w, i.e., the average stock within n experiences a market weight effect governed by this complement law. For the values from the example, Figure 1 plots the path of β_{nm}. At w = .25, asset j's beta has increased from .5 to 1.375 while the beta of the remaining assets, in aggregate, has fallen from 1 to .875.

3. Implications for Abnormal Returns

The effect of an asset's market weight on its beta has important empirical implications for measuring abnormal returns. The conventional measure of the abnormal return on asset j in period t is the excess of the asset return in that period over its market model counterpart,

$$AR_{jm,t} = R_{jt} - \left[\hat{\alpha}_{jm} + \hat{\beta}_{jm} R_{mt}\right] \tag{10}$$

where $\hat{\alpha}_{jm}$ and $\hat{\beta}_{jm}$ are estimated from a time series regression of R_j on market return R_m (see, for example, Brown and Warner, 1985). The "normal" return [·] is typically interpreted as an estimate of R_{jt} in the absence of events specific to firm j, so that the abnormal return is a measure of firm specific events. However, if asset j is included in the market index m, then R_{mt} and hence the normal return includes R_{jt}. Consequently the normal return includes firm j specific events, and so the abnormal return fails to represent firm specific events. If asset j's weight in m is non-trivial, this error may be substantial.

A possible solution to this problem is to replace R_{mt} by R_{nt}, which does not include R_{jt} and hence does not include the event specific to firm j. As a further consequence, $\hat{\alpha}_j$ and $\hat{\beta}_j$ must be estimated by a time-series regression of R_{jt} on R_{nt}. This leads to a measure of abnormal return involving an alternative version of the market model,

$$AR_{jn,t} = R_{jt} - \left[\hat{\alpha}_{jn} + \hat{\beta}_{jn} R_{nt}\right] \tag{11}$$

where the time-series regression of R_{jt} on R_{nt} yields the estimated coefficients $\hat{\alpha}_{jn}$ and $\hat{\beta}_{jn}$. The standard market model is inferior in two respects. First, it uses a market index, and hence a measure of "normal" return, that partly includes the very shock one is trying to measure. Thus the measure of abnormal return must be biased. Second, in so far as a firm experiences a non-trivial change in its market weight over the beta estimation period, the estimate of beta in the conventional market model will be biased, and this flows through to the estimate of the abnormal return. By contrast, the proposed abnormal return measure is free of both concerns. The firm's shock is not included in the measure of "normal" return. Furthermore, if the firm experiences a non-trivial shift in market weight over the beta estimation period, this has no effect upon the estimate of its beta β_{jn}, because the latter is invariant to the firm's market weight.

The difference in these two abnormal return measures is then

$$AR_{jm,t} - AR_{jn,t} = \hat{\alpha}_{jn} + \hat{\beta}_{jn} R_{nt} - \hat{\alpha}_{jm} - \hat{\beta}_{jm} R_{mt}$$

$$= \left(\hat{\alpha}_{jn} - \hat{\alpha}_{jm}\right) - \hat{\beta}_{jm}\left(R_{mt} - R_{nt}\right) + R_{nt}\left(\hat{\beta}_{jn} - \hat{\beta}_{jm}\right) \qquad (12)$$

The first of the three terms on the right hand side of (12) is generally trivial because estimated alphas are typically close to zero for the short periods used in abnormal return analysis[2]. The second term (the "market return effect") arises from the difference in market return measures and the third term (the "beta effect") arises from the difference in betas. Since the first of these two betas in the last term (β_{jn}) can be viewed as the conventional beta when the asset's market weight is zero, and the previous section has shown that the conventional beta can vary significantly with market weight shifts, then the "beta effect" can be substantial if the asset's market weight is non-trivial.

To illustrate this issue we examine the trading history of Telecom Corporation of New Zealand, the largest firm on the New Zealand Stock Exchange. From its initial public offering on July 18, 1991 to June 30, 1997, Telecom's market weight in the New Zealand index (NZSE40) fluctuated between 16 and 29 percent. To further appreciate Telecom's significance in the New Zealand stock market, its correlation coefficient with the NZSE40 over the six-year period is .74. If Telecom is excluded from the index, which is then denoted the NZSE39, the correlation coefficient drops to .49.

Across this period of time we select the two largest absolute daily returns for Telecom, and these are shown in Table 1. The first of these is February 16, 1993, when Telecom announced strong earnings, well above its profits from the previous year, and its stock price increased 11.4% (compared with the return for the rest of the market of .9%). The standard measure of abnormal return in equation (10) yields an abnormal return equal to 8.5%, whereas estimating AR_{jn} yields

[2] If expectations are applied to the market models in equations (10) and (11), then the alphas are the intercepts in expected return models. Over short periods of time (such as days, as examined below), expected returns are very small; accordingly, so too are the alphas. The estimated alphas shown in Table 1 are consistent with this.

the more appropriate result of 10.9%. The difference of 2.4% in these abnormal return measures is due primarily to the “market return effect” (2%). Here the “beta effect” is modest (.3%), in spite of the substantial difference in the two beta estimates (.513 versus .891), simply because R_{nt} is close to zero.

The second of these days is Monday, November 8, 1993, and this provides an even more striking example of the difference in the two abnormal return methodologies. Over the preceding weekend, a national election took place in New Zealand and a party which was widely perceived as anti-business, and which had threatened to buy back recently spun-off public assets, performed well enough to potentially hold the balance of power. Investors interpreted the election result as bad news for the market, and particularly bad news for Telecom, the largest of the recently sold public assets. Table 1 shows that Telecom's return for the day was –9.3% compared to the rest of the market's return of –5.0%. Using the traditional measure of abnormal return in equation (10), Telecom's abnormal return was –1.5%. However, AR_{jm} uses a market return that significantly depends upon Telecom's return, and therefore does not yield an abnormal return measure that disentangles the election's separate effect on Telecom. AR_{jn} provides a better measure of the election's specific effect on Telecom and equals –5.7%, nearly four times the size of AR_{jm}. The difference of 4.2% in these abnormal return measures is now largely due to the “beta effect” (2.7%) with most of the remaining difference attributable to the “market return effect” (1.3%). This beta effect arises because the two beta estimates are again substantially different (.769 versus 1.31). All of these calculations presume that the betas are accurately estimated. However, as we have noted earlier, the estimate of β_{jm} will be biased if the firm experiences a non-trivial shift in this over the estimation period. Estimation of β_{jn} is free of this problem.

The two examples provide evidence that the two abnormal return models can generate substantially different results, that the difference in betas may be substantial, and that the latter can contribute significantly to the difference in abnormal returns. In addition to all of this, the difference in betas conforms to the predictions of the theoretical analysis, i.e., if beta is significantly less than one when market weight is zero (the beta estimates of this kind are .513 for the first event in Table 1 and .769 for the second) then the effect of significantly increasing market weight is to significantly increase the beta (to .891 for the first event in Table 1 and 1.31 for the second). Finally, with respect to the second event in Table 1, the shift in beta from .769

to 1.31 illustrates the point made earlier that a substantial change in market weight can alter a stock from a low beta one to a high beta one.

4. Conclusion

This paper derives the relationship between a stock's beta and its weighting in the market index. Contrary to intuition, the relationship is in general non-monotonic, with the result that market weight variations within the observed range can have dramatic effects on a firm's beta. We then suggest an alternative to the conventional measure of abnormal return, which requires an estimate of a firm’s beta when its market weight is zero. We argue that the alternative measure is superior, and show that it can differ substantially from the conventional measure when a firm has a non-trivial market weight. The difference in abnormal returns may be due in large part to the difference in betas, which will arise if the firm's market weight is non-trivial.

The fact that market weight changes can substantially affect a stock’s beta also has implications for the cost of capital when the Capital Asset Pricing Model is invoked. In particular, the analysis suggests that the traditional measure of systematic risk, and thus the opportunity cost of capital, may change even when a firm's line of business remains the same. We leave the implications of market weight on a firm's cost of capital for future research.

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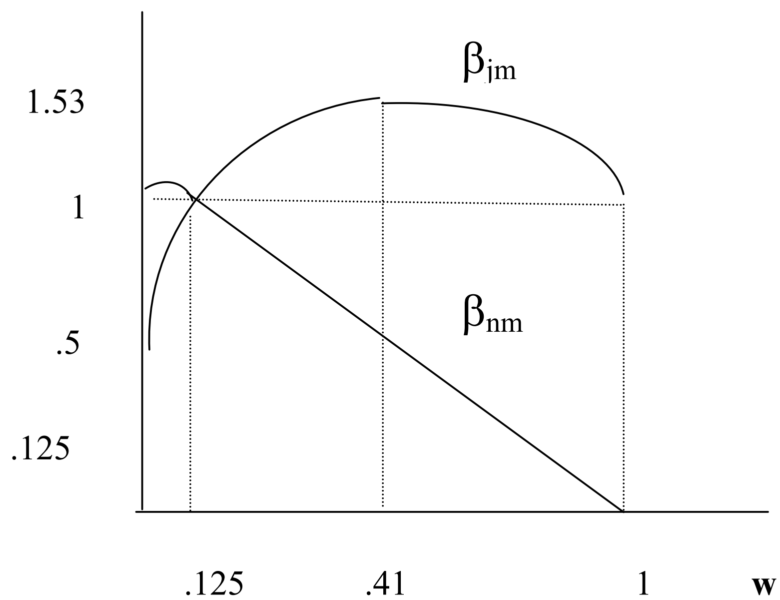
Figure 1

The Relationship Between Beta and Market Weight



This figure shows the relationship between asset *j*'s market weight *w* and its beta against the market portfolio, β_{jm}, and also the relationship between *w* and the beta of portfolio *n* (the market exclusive of *j*) against the market portfolio, denoted β_{nm}. The figure assumes that asset *j*'s beta against *n*, β_{jn}, is 0.5 and that *j*'s variance is four times that of *n*.

Table 1

Differences in Abnormal Return Measures: AR_{jm} versus AR_{jn}

Examples from Telecom NZ Stock Returns

Trading Date	Telecom Return	NZSE40 Return	$\hat{\alpha}_{jm}$	$\hat{\beta}_{jm}$	$AR_{jm,t}$	NZSE39 Return	$\hat{\alpha}_{jn}$	$\hat{\beta}_{jn}$	$AR_{jn,t}$
Feb 16 1993	.114	.031	.001	.891	.085	.009	.002	.513	.108
Nov 8 1993	–.093	–.060	.000	1.31	–.015	–.050	.002	.769	–.057

This table computes the abnormal returns for Telecom on two dates, and applies two measures of the market index - the NZSE40 and the NZSE39. The resulting abnormal returns are designated $AR_{jm,t}$ and $AR_{jn,t}$ respectively. Designating the trading date as $t = 0$, the α and β coefficients are estimated from OLS regressions based on daily returns from $t = –160$ to $t = –11$.