

The Decision of Venture Capitalists on Timing and Extent of IPOs

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Abstract

We analyze the venture capitalist's decision on the timing of the IPO, the offer price and the fraction of shares he sells in the course of the IPO. A venture capitalist may decide to take a company public or to liquidate it after one or two financing periods. A longer venture capitalist's participation in a firm (later IPO) may increase its value while also increasing costs for the venture capitalist. Due to his active involvement, the venture capitalist knows the type of firm and the kind of project he finances before potential new investors do. This information asymmetry is resolved at the end of the second period. Under certain assumptions about the parameters and the structure of the model, we obtain a single equilibrium in which high-quality firms separate from low-quality firms. The latter are liquidated after the first period, while the former go public either after having been financed by the venture capitalist for two periods or after one financing period using a lock-up. Whether a strategy of one or two financing periods is chosen depends on the kind of project (simple vs. complex). In the separating equilibrium, the offer price corresponds to the true value of the firm.

JEL Codes: D80, G24, G32

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1 Introduction

The commercial exploitation of innovative ideas is the driving force of the market economy. New firms with pioneering ideas and a flexible structure can often react to customers' needs more appropriately than old, established enterprises. Many start-ups in innovative branches, however, require a substantial amount of capital. Their founders may not have sufficient funds to finance the projects alone and, therefore, look for external financing. Furthermore, new firms typically need not only capital but also managerial advice. Because the banking sector generally does not want to take the extreme risks of young, innovative firms or carry out advisory functions and because, due to high costs and information opacity, very young firms do not tend to engage in public equity and debt underwriting, private equity is left as the most appropriate finance source. A special subgroup of private equity designed for young, innovative, high-risk, potentially high-reward firms is venture capital. Numerous hi-tech companies (including Apple Computer, Cisco Systems, Microsoft and Intel) and many successful service firms (e.g. Federal Express, Starbucks or Staples) in the US have received venture financing. Venture capitalists serve as specialized financial intermediaries who use various mechanisms (e.g. staged financing, convertible securities, delegation of control and decision rights, etc.) that mitigate agency conflicts.

The life of a venture capital fund is limited: in the US it is usually set at a maximum of ten years (see Sahlman [1990]). After this period, the capital providers (typically large institutional investors) want to harvest the revenues from their investments in venture capital funds and evaluate the venture capitalists. Therefore, the investment period of venture capitalists in young firms is short, usually 3 to 7 years (see Barry [1994]). The returns from their investment are the capital gains raised after a successful build-up of the business rather than regular dividend returns (see OECD [2002]). Hence, the development of a viable venture capital market essentially depends on the existence of suitable exit routes, which provide opportunities for high returns. Initial public offerings (IPOs) are generally considered to be the most profitable exit route (see Bygrave and Timmons [1992]). Second-tier or parallel markets are the place where most of the venture backed firms that go public are initially listed. The existence of such markets for young dynamic firms is considered to be essential for the venture capital industry (see e.g. Black and Gilson [1998]). Several empirical papers confirm the positive role of a viable IPO market on venture capital activity: Jeng and Wells [2000] found out that IPOs were the most powerful driver of venture capital investing. Gompers [1998] saw a surging market for venture-backed IPOs as one of the main reasons for the dramatic increase in venture capital commitments in the US. In our paper, we concentrate on this exit channel. In practice, we observe that (1) venture-backed firms go public at different times in the firms life and after being financed by venture capital for different time horizons; (2) the venture capitalists usually exit only partially at the IPO and commit themselves to hold part of their shares for several months beyond the IPO (lock-up) and (3) there are large differences in the level of these post-IPO shareholdings by venture capitalists. Our aim is to shed light

on these issues. Within a single model, we analyze the venture capitalist's decision on the timing, the offer price and the fraction of shares he sells in the course of the IPO.

Venture capitalists offer a combined provision of capital and managerial experience (see e.g. Casamatta [1999] for a theoretical analysis or Kaplan and Strömberg [2001] for an empirical one). They monitor strategic and managerial decisions and tend to take an active role in advising the firm and providing it with valuable business contacts (e.g. with consultants, lawyers, investment bankers) or qualified staff. Based on US data, Hellmann and Puri [2000] found out that companies that obtain venture capital are more likely and/or quicker to become professional than their non venture-backed counterparts. In our model, we consider two periods and two types of firms (high- and low-quality). Each firm carries out a single project. If the firm has high quality, two kinds of projects (complex and simple) exist. For complex projects, the venture capitalist's managerial activities enhance the firm's value in both periods. For simple projects the venture capitalist's managerial contribution adds value only in the first period.

As active investors, venture capitalists know their young portfolio firms better than the potential new investors. The timing and the extent of the IPO influences the new investors' assessment of the quality of the firm. The younger the firms are, the higher the information asymmetry is between the venture capitalist and the new investors. Here we assume that the VC knows the firm and project characteristics in $t=1$, with the NIs receiving the information in $t=2$. In $t=1$, the NIs only know the probabilities that a firm has high quality and that a project is complex. Hence, if the VC takes a high-quality firm public in $t=1$, he may want to signal that the IPO is not a means of getting rid of low-quality stocks. Rather, he would like to demonstrate that his reason for selling the company is a low benefit of his managerial support because the project is not complex. A costly obligation of not selling a part of his shares for a certain period of time (lock-up) can serve as such a signaling device.

Our paper is different from the existing theoretical research on venture capital and venture capitalist's exit, which typically deals with the choice of the optimal exit channel assuming (and analyzing) a conflict between the venture capitalist and the entrepreneur (see Berglöf [1994] or Bascha and Walz [2001]). We do not consider this kind of conflict. Instead, our paper analyzes the information asymmetry between the old investor (the venture capitalist) and the potential new investors. We introduce a new aspect: the timing of the exit.

Our model points out the role of the value added by the venture capitalist. One of the differences between the venture capitalist and the new investors who buy the firm at the IPO is that the former offer not only money but also managerial support that enhances the value of the high-quality firm. Since the IPO reduces the venture capitalist's stake in the firm, incentives to be active in the management of the company are diminished after the IPO. The potential value added from the non-monetary contribution by the venture capitalist depends on the firm type and the kind of project. For low-quality firms in both periods and for simple projects in the second period, the potential value added from the non-monetary contribution is zero. Thus, the venture capitalist wants to exit his firm as

soon as possible, since binding his resources there is costly. However, the new investors who purchase shares from venture capitalists do not observe the quality of young firms and the kind of projects. In order to receive a higher price for his shares, the venture capitalist who owns a high-quality firm has to prove its quality. He must either finance his firm longer (until the firm gets older and the information asymmetries are mitigated), which might be too costly for simple projects, or signal the quality of the young firm going public. We will show under which conditions a separating equilibrium holds in which low-quality firms are liquidated and high-quality firms go public either after one period with a lock-up (simple projects) or after two periods without a lock-up (complex projects).

Considering the possibility of signaling through a lock-up, our paper contributes to the existing literature on the IPO and signaling (see Allen and Faulhaber [1989], Grinblatt and Hwang [1989] or Welch [1989]). A seminal paper on signaling as a means of information transfer between the informed entrepreneur and the uninformed new investors is Leland and Pyle [1977]. They show that the entrepreneur's willingness to invest in his own project can serve as a signal of the project quality. In contrast to our model, the above-mentioned models deal with investors in general, instead of venture capitalists in particular. In our model, we try to capture the following three specific features of venture capitalists and venture-backed IPOs: (1) the combined provision of capital and management experience, (2) the superior information of venture capitalists on their portfolio companies and (3) the limited investment horizon of venture capitalists. A paper related to ours that considers lock-up and venture capital is Neus and Walz [2002]. They consider the timing of IPOs as given and ask whether or not the venture capitalists use a lock-up. In contrast, the central issue of the model we develop in the subsequent sections concerns the timing of the IPO.

The structure of the rest of the paper is as follows: section 2 will present the structure of the model, the analysis of the exit decision will be carried out in section 3, and, finally, section 4 will conclude and offer some empirically testable implications derived from the theoretical analysis.

2 Setup of the Model

Our model consists of two periods, two types of firms and two kinds of projects. Each venture capitalist (VC) finances a single firm from the start of the first period ($t=0$). Each firm carries out a single project. The value of the firm depends on its type, kind of project and VC's monetary investment and non-monetary contribution. At the end of each period there are numerous new investors (NIs) who are interested in buying firms in public markets from the VCs. The issues we want to analyze are in which period a VC exits his investment and what divestment strategy he chooses. All VCs and NIs are risk neutral and rational investors.

Further, we assume that:

1. the parameters, functional forms, structure of the game and players' rationality are common knowledge among the players;

and the VC may sell some or all of his old shares. The number (and the fraction) of the original shares sold by the venture capitalist at the IPO is denoted by a (no lock-up: $a=1$, lock-up: $0 \leq a < 1$). The number of new shares issued (and bought by the NIs) is d . Hence, the total number of shares after the IPO is $1 + d$.

Venture capitalists are actively involved in their portfolio companies. The higher the shareholdings of the venture capitalist in a period, the higher his effort costs in this period. In the first period, when he holds the complete firm, the effort cost equals B . In the second period, when he holds $(1 - a)$ shares, the cost amounts to

$$B \cdot (1 - a) . \quad (1)$$

B is the parameter of the non-monetary contribution. When he does not sell any shares in $t=1$, the effort cost of the VC in the second period equals B . The more shares sold in $t=1$ (higher a), the lower the shareholdings and, obviously, the effort cost of the VC in the second period.

Two types σ of firms exist. The firms have either high or low quality ($\sigma \in \{H, L\}$) with equal probability in $t=0$ ($Prob(\sigma = H) = Prob(\sigma = L) = 0.5$). In $t=0$ neither the VC nor the NIs know the true type of firm; they only know the percentage of high-quality firms. Since VCs are deeply involved in firm activities they recognize the type earlier than the potential NIs do. We assume that the VC knows the type of firm in $t=1$ and the NIs in $t=2$ when the firm grows older and more track record is available.

Before a new period starts (in $t=0$, $t=1$ and $t=2$), this dynamically growing firm needs a monetary investment I (exogenously given) to finance its activities in the coming period. Without this investment the firm cannot survive. Prior to the IPO, these investments are financed by the VC as the sole investor. In the period that follows the IPO, the revenues raised for the new issue cover the investment demands of the expanding firm. After being listed, the firm gains access to the credit markets, which finance any further investments in later periods.

At the IPO new and old shares are offered. The VC gets the revenues from selling the original shares since he has been their sole owner. The firm obtains the revenues from the new issue. We assume that the new issue exactly covers the investment demands of the firm in the period following the IPO. The number of new shares issued d is then determined by the exogenously given investment demand I and the share price $p(\cdot)$ that the VC sets

$$d = \frac{I}{p(\cdot)} . \quad (2)$$

In front of the IPO process, the VC chooses the price he wants to receive per share and the fraction of his shares he wants to sell and announces this information to the new investors. The number of new shares is determined through (2). The issue of how prices are set by venture capitalists will be explained later in this section. In the course of the IPO, any new investor may buy shares at this price. If the new shares were not sold, the

investment demands of the firm could not be met and the firm would be liquidated.¹ As the VC is rational and he maximizes his utility, this would never happen. The VC knows the expectation building of the NIs and, hence, the combination of price and fraction of shares they will accept. The VC would never spend the IPO costs and then let the IPO fail by choosing a non-acceptable combination of price and number of shares he offers. Liquidation would be the more profitable strategy for him.

Each firm carries out a single project. The projects of high-quality firms differ in their sophistication. For simplicity, we assume that only two kinds r of projects exist. For more complicated projects, the non-monetary contribution of the VC is valuable in both periods (“complex” project: $r = \lambda$). For less complicated projects, the business can be built up rather quickly, and the managerial contribution of the VC in the second period is no longer value enhancing (“simple” project: $r = \theta$). The probability of each kind of project in $t=0$ is 0.5. The information structure here is the same as for the type of firm. In $t=1$ the VC can observe the kind of project that is being financed and, therefore, whether or not his further non-monetary contribution adds any value to the firm. The NIs cannot see this feature in $t=1$. They know only the probability distribution.

The value of a firm depends on the firm’s type. The value of an L-firm is always zero. Neither the monetary investment nor the non-monetary contribution of the VC can increase the value of the type L.

$$W^L = 0 \quad t \in \{0, 1, 2\}. \quad (3)$$

The VC invests exogenously given amounts I (as a monetary investment) and B (as a non-monetary contribution) in each period in which he completely finances the firm. The value of an H-firm depends on the sum of the monetary investments and the non-monetary contributions as well as the kind of project. The parameter k measures the rate of return (which equals $k-1$; $k > 1$) on the investment. If a venture capitalist is present in an H-firm, his non-monetary contribution adds value kB in the first period. For complex projects, the VC’s full engagement in the second period raises the value of the type H further by kB . If a simple project is completely financed by the VC for both periods (IPO or liquidation in $t=2$), the VC’s managerial involvement in the second period does not generate any value added. If a VC takes his portfolio firm public in $t=1$ and retains some or all of the original shares (lock-up), his fraction of the firm shrinks after the IPO. Therefore, his claim to actively manage the company is diluted and his incentives are diminished. For convenience, we assume that if the firm goes public in $t=1$ and the VC retains some shares until $t=2$, he no longer adds value to the company. He still has some effort costs (see (1)) related to his presence in the firm, however. The costs and benefits from the VC’s non-monetary contribution to an H-firm are summarized in Table 1.

¹In our model, either all (announced fraction of old shares plus the new issue) or no offered shares are sold because infinitely many identical new investors exist. If the price is “reasonable”, all offered shares are sold because the demand is infinite. If the price was “too high”, nobody would buy any shares.

Table 1: The managerial contribution by the VC to an H-type firm

(net benefits = benefits - costs)

Exit channel	Complete IPO in $t=1$		Complete IPO in $t=2$		Lock-up	
Kind of project	simple	complex	simple	complex	simple	complex
First Period: NET BENEFITS	$kB-B$	$kB-B$	$kB-B$	$kB-B$	$kB-B$	$kB-B$
Second Period: NET BENEFITS	0-0	0-0	0- B	$kB-B$	0- $B \cdot (1-a)$	0- $B \cdot (1-a)$

Every investment I that is carried out either by the VC or by the NIs (through buying the new stock) increases the value of the H-firm by kI . The benefits from the non-monetary contribution of the venture capitalist raise the value of the firm additively. The value of an H-type firm at the IPO is thus the sum of the benefits from the monetary investments by the venture capitalist (their number equals t), the benefits from the investment carried out by the NIs at the IPO and the benefits from the non-monetary contribution of the VC before the IPO. If the IPO is conducted in $t=1$, the value of an H-firm is

$$W^H(1) = k \cdot (2I + B) . \quad (4)$$

If an H-firm is completely financed by the VC for two periods, its value at the end of the second period depends on the kind of project. The value of a simple project after two periods of venture capital financing is

$$W^{H\theta}(2) = k \cdot (3I + B) \quad (5)$$

and the value of a complex project is

$$W^{H\lambda}(2) = k \cdot (3I + 2B) . \quad (6)$$

We assume that there are many new investors who want to buy the new stock. The VC has all the bargaining power, which enables him to extract the entire surplus. The NIs do not know the value of the firm in $t=1$. The verification of the firm type and the project kind is impossible (i.e. very costly). We denote the value of the firm that the NIs expect after the IPO in t by $E[W^{\sigma r}(t)]$. In $t=2$, the NIs have full information so that $E[W^{\sigma r}(2)]$ equals the true value, which is 0 for an L-firm (see (3)) and $k \cdot (3I + B)$ or $k \cdot (3I + 2B)$ for an H-firm depending on the kind of project (see (5) or (6)).

After the IPO, the total number of shares consists of 1 old share plus d new shares. The value of a share that the NIs expect during and after the IPO equals the expected value of the firm divided by the (known) number of shares

$$\frac{E[W^{\sigma r}(t)]}{1 + d} . \quad (7)$$

If $E[W^{\sigma r}(t)] < I$, the NIs do not buy any shares (see Appendix A for the proof.) The intuition is the following: if the NIs bought new shares and hence invested the amount of

I , their investment would generate a negative net expected value due to the fact that after this investment, the expected value of the firm would be lower than the investment itself. Therefore, the NIs prefer to retain their funds and do not buy any shares. If $E[W^{\sigma r}(t)] \geq I$, the NIs are willing to pay the price of (7) per share. Combining (7) and (2) we get the maximum price per share $p(E[W^{\sigma r}(t)], I)$ that the NIs are willing to pay at the IPO

$$p(E[W^{\sigma r}(t)], I) = \max\{E[W^{\sigma r}(t)] - I, 0\} . \quad (8)$$

For this price, the VC may sell as many shares as he wants since there are infinitely many investors with identical beliefs. (However, the number of shares sold in $t=1$ influences the expectation building by the NIs.) If the venture capitalist were to set a higher price, the demand for shares would be 0. If he were to set a lower price than the maximum price per share $p(E[W^{\sigma r}(t)], I)$ that the NIs were willing to pay, he would only diminish his own profit, which equals the revenue from selling the shares minus the cost.² The issue of how NIs form their expectations in $t=1$ will be analyzed in the next section. We will show that the choice of the price alone by the VC has no impact on expectation building by the NIs. The venture capitalists, as profit maximizers and sophisticated investors who understand the expectations' building process of the NIs, set the maximum possible price they can receive when they exit via IPO (complete IPO in $t=1$, complete IPO in $t=2$, lock-up). If the IPO takes place in $t=2$, the NIs are fully informed. Then, the share price of an L-firm in $t=2$ would be 0, while the share price of an H-type firm at the IPO would reach $k \cdot (3I + B) - I$ or $k \cdot (3I + 2B) - I$, depending on the kind of project.

In the case of a lock-up, in $t=1$ a shares are sold by the VC for the price $p(E[W^{\sigma}(1)], I) = \max\{E[W^{\sigma}(1)] - I, 0\}$.³ The rest of the shares $(1 - a)$ is sold in $t=2$ for the price $p(E[W^{\sigma}(lock)], I)$. Since there is full information in $t=2$, the maximum possible price that the NIs are willing to pay corresponds to the true value of the firm, which for type H equals $E[W^{\sigma}(1)] = W^H(1) = k \cdot (2I + B)$. Hence, the maximum price per share which the NIs are willing to pay in $t=2$ for the rest $(1 - a)$ of the shares depends on the value of the firm and the number of shares issued in the first period (see (2) and (7)).

$$p(E[W^{\sigma}(lock)], I) = \begin{cases} \frac{W^H(1)}{1+d} = \frac{p \cdot W^H(1)}{p+I} = \frac{p \cdot k \cdot (2I+B)}{p+I} & \text{for an H-type,} \\ \frac{W^L(1)}{1+d} = 0 & \text{for an L-type.} \end{cases} \quad (9)$$

where $p > 0$ is the price per share in $t=1$.

If the VC completely finances the firm until $t \in \{1, 2\}$, he finances t -times the investment I and spends effort costs of t -times B . The total financing cost then amounts to $t \cdot (I + B)$. In the case of a lock-up, he spends $I + B(2 - a)$ because he completely finances the first period causing costs of $I + B$, and he retains $(1 - a)$ shares between $t=1$ and $t=2$ causing costs of

²In the lock-up case, a lower price at the IPO in $t=1$ would additionally lead to a dilution of the remaining VC's holdings and hence to a lower price per share and lower revenues in $t=2$.

³The type of project does not play any role in the value of the firm in the case of a lock-up. In $t=1$ the NIs enter, and the VC's presence in the firm no longer increases the firm value in the second period.

$B(1 - a)$. Further, there are some negligible IPO costs of ϵ that make an IPO slightly more expensive than a liquidation (for every $\gamma \in R^+ : 0 < \epsilon < \gamma$). The consequence is that if the maximum possible price the VC can get at the IPO in t is zero, he prefers liquidation in t .

The VC chooses that exit strategy which maximizes his profit. We have already argued why he chooses the maximum possible price he can receive. Under different exit channels, the VC then reaches the following levels of profit:

1. Since there are no revenues in the case of liquidation, the profit $\pi_{liq}^\sigma(t)$ of the VC who liquidates his portfolio firm in $t \in \{1, 2\}$ will be the negative sum of the monetary and non-monetary expenditures

$$\pi_{liq}^\sigma(t) = \pi_{liq}^H(t) = \pi_{liq}^L(t) = -t \cdot (I + B) . \quad (10)$$

2. If an IPO takes place in $t \in \{1, 2\}$, the profit of the VC amounts to the difference between the revenues from the IPO and the costs.

- Without a lock-up (complete IPO) the profit is

$$\pi_{ipo}^{\sigma r}(t) = E[W^{\sigma r}(t), I] - (t + 1) \cdot I - t \cdot B - \epsilon , \quad (11)$$

- with a lock-up (partial IPO) the profit reaches

$$\pi_{lock}^\sigma = a \cdot p(E[W^\sigma(1)], I) + (1 - a) \cdot p(E[W^\sigma(lock)], I) - I - (2 - a)B - \epsilon . \quad (12)$$

Under certain assumptions about the parameters of the model, we obtain a single equilibrium in which high-quality firms separate from low-quality firms. The latter are liquidated after the first period. The former go public either after having been financed by venture capital for two periods or after one financing period using a lock-up. The VC's decision between one or two periods depends on the kind of project. For firms with simple projects, for which the managerial contribution of the VC in the second period is no longer value enhancing, the lock-up serves as a signaling device to indicate high quality.

3 Exit Decision

3.1 Information Symmetry

In order to demonstrate how the model is designed and to get a benchmark for the more realistic case of information asymmetry, we first consider the case of symmetric information. Under symmetric information everybody already knows the type of firm in $t=1$. Hence, the new investors' expectations in $t=1$ are the following:

$$E[W^\sigma(1)] = \begin{cases} W^H(1) = k \cdot (2I + B) & \text{for an } H\text{-type,} \\ W^L(1) = 0 & \text{for an } L\text{-type.} \end{cases}$$

Proposition 1 (SYMMETRIC INFORMATION):

Let us denote $\frac{B+I}{I}$ with \bar{k} . We assume that $\bar{k} > k$.⁴

If everybody knows the firm type in $t=1$, then a type L will be liquidated in $t=1$ and a type H will go public in either $t=1$ (simple projects) or in $t=2$ (complex projects). The VC sells all his shares at the IPO (no lock-up). The price per share for a simple project equals $k \cdot (2I + B) - I$, the price per share for a complex project is $k \cdot (3I + 2B) - I$.

Proof**L-type**

The liquidation of an L-firm in $t=1$ is the profit maximizing strategy for the VC (with the profit $\pi_{liq}^L(1) = -I - B$). The reason is that since the NIs know that the type of the firm is L (and its value equals 0), they never pay a positive price for its shares. Hence, no other strategy would generate higher profits (in this case a lower loss) for the VC than liquidation in $t=1$ (see Appendix B): *Further financing* in the second period generates additional costs of $I + B$ and no revenues and, therefore, will not be chosen by the VC. The *lock-up* is not possible since the NIs would not finance the necessary investment I in $t=1$ (because $E[W^\sigma(1)] = W^L(1) = 0 < I$). The *complete IPO* in $t=1$ generates no revenues compared to the liquidation but causes additionally IPO costs of ϵ . Hence, liquidation in $t=1$ is the profit maximizing strategy.

H-type

The VC who finances an H-firm can earn profits by selling his shares. If the IPO takes place in $t=1$ (when $E[W^\sigma(1)] = W^H(1) = k \cdot (2I + B)$), the profit of the VC is equal to (see (8) and (11))

$$\pi_{ipo}^H(1) = (k - 1) \cdot (2I + B) - \epsilon. \quad (13)$$

If the IPO takes place in $t=2$ (when $E[W^{\sigma r}(2)] = W^{Hr}(2)$), the profit of the VC amounts to (see (5), (6), (8), and (11))

$$\pi_{ipo}^{Hr}(2) = \begin{cases} \pi_{ipo}^{H\theta}(2) = k \cdot (3I + B) - 3I - 2B - \epsilon & \text{for the simple project,} \\ \pi_{ipo}^{H\lambda}(2) = (k - 1) \cdot (3I + 2B) - \epsilon & \text{for the complex project.} \end{cases} \quad (14)$$

The VC prefers to sell all his shares at the IPO since his profit from the lock-up would only be $(k - 1) \cdot (2I + B) - B \cdot (1 - a) - \epsilon$ (see Appendix C), which is less than the profit from the complete IPO in $t=1$ (see (13)). Lock-up generates no additional revenues but raises additional costs of $B \cdot (1 - a)$ for the venture capitalist's involvement in the second period. The VC takes firms with simple projects in $t=1$ public ($\pi_{ipo}^{H\theta}(2) < \pi_{ipo}^H(1)$) and firms with

⁴The other case $\frac{B+I}{I} \leq k$ is not interesting because in this case even a simple project would be held by the VC until $t = 2$ due to the high rate of return.

complex projects in $t=2$ ($\pi_{ipo}^{H\lambda}(2) > \pi_{ipo}^H(1)$). The profit levels are $\pi_{ipo}^H(1)$ (see(13)) for the simple and $\pi_{ipo}^{H\lambda}(2)$ (see(14)) for the complex project. Whether or not the NIs already know the kind of project in $t=1$ does not affect the result. ■

3.2 Information Asymmetry

Let us consider a more relevant distribution of information where the VC, as an active investor, has an information advantage over the NIs. He knows the type of firm and the kind of project he finances earlier than the NIs. In $t=0$ nobody knows the value of either of the two parameters. We assume that if the firm has been financed by the VC only for one period ($t=1$), the NIs recognize neither its type nor the kind of project at the time of the IPO. At this time they know only the probability distribution over the two firm types and the two kinds of projects. The VC knows the value of both of these parameters in $t=1$. The information advantage of the VC disappears in $t=2$ when the NIs can observe the true value of the firm.

In what follows, we investigate the decision of the profit maximizing VC in $t=1$ for an L-firm, an H-firm with a simple project and an H-firm with a complex project. Under certain assumptions, which will be analyzed here, a unique (separating) equilibrium exists.

Proposition 2 (ASYMMETRIC INFORMATION - H with a complex project):
An H-firm with a complex project goes public in $t=2$, and the VC sells the whole firm at the IPO. The price per share is $k \cdot (3I + 2B) - I$.

Proof

If the complex project is sold in $t=2$ for the maximum possible price, the profit of the VC is $(k - 1) \cdot (3I + 2B) - \epsilon$. This is exactly the same profit as in the information symmetry case (see section 3.1), since in $t=2$ the information asymmetries between the VC and the NIs are dissolved. Under information asymmetry, none of the other exit channels (an IPO in $t=1$ with or without a lock-up or liquidation) can generate higher profits than in the case of the symmetric information. The reason is that in all these cases the costs remain unchanged while in neither case the revenues can be increased. If the VC were to choose a higher price per share, no shares would be sold. If the price were lower, the profit would shrink. Therefore, under asymmetric information in $t=1$, the VC takes a type H with a complex project in $t=2$ public for the price per share of $k \cdot (3I + 2B) - I$. He chooses the same strategy as in the reference case of information symmetry. ■

Let us now analyze which strategy is chosen by the VC for an L-firm and for an H-firm that has a simple project. Financing a simple project, the VC has no comparative advantage against the NIs in the second period. Under information symmetry, a type H with a simple project, therefore, goes public in $t=1$. Type L is liquidated in $t=1$. If there is asymmetric information, observing a complete IPO in $t=1$, the NIs cannot distinguish between an

H-firm and an L-firm in $t=1$. The VC who finances a type L prefers a complete IPO in $t=1$ to a liquidation whenever he can get a share price which is higher than the IPO costs ϵ . The maximum price that the NIs are willing to pay when a complete IPO takes place in $t=1$ may be "too low" for an H-firm. Waiting until the information asymmetries are resolved or selling only a part of his shares in $t=1$ may be more profitable for the VC. In order to determine the price that the NIs will be willing to pay in $t=1$, we have to analyze the NIs' expectation building. To do this, we have to look at the VC's payoffs from different strategies under asymmetric information and find out the profit maximizing strategy.

Each firm has five possible exit channels: IPO in $t=1$ with a lock-up, IPO in $t=1$ without a lock-up, IPO in $t=2$, liquidation in $t=1$ and liquidation in $t=2$. Obviously, liquidation in any period is strictly dominated by the IPO in $t=2$ for an H-firm. Hence, an H-firm goes public. For an L-firm, further venture capital financing in the second period (an IPO or a liquidation in $t=2$) is strictly dominated by the liquidation in $t=1$. After the elimination of these strictly dominated strategies of both types, we have three exit channels for each type to be considered.

The following three aspects of the IPO process can be distinguished: (1) the timing ($t=1$ or $t=2$), (2) the quantity of shares sold at the time of IPO ($a=1$ or $a < 1$) and (3) the price per share $p(\cdot)$. If we assume that a firm goes public in $t=1$ and that the VC offers all of his shares at the IPO ($a=1$), then we have already determined the first two of the three characteristics. In this case, there is no possibility for H to separate from L. The reason is that any price higher than ϵ that the VC demands for a type H in $t=1$ would result in an IPO of an L-firm in $t=1$ as well.⁵ Assuming that both types may participate in the IPO in $t=1$, the NIs would expect the following firm value:

$$\begin{aligned} E[W^\sigma(1)]_{pooling} &= \frac{Prob(r = \theta) \cdot Prob(\sigma = H) \cdot W^H(1) + Prob(\sigma = L) \cdot W^L}{Prob(r = \theta) \cdot Prob(\sigma = H) + Prob(\sigma = L)} = \\ &= \frac{k \cdot (2I + B)}{3}. \end{aligned} \quad (15)$$

The pooling price per share would then be $\max\{\frac{k \cdot (2I + B)}{3} - I, 0\}$. If the VC demanded a higher price for a complete IPO, NIs would not buy any shares. Hence, the maximum profit of the VC who would sell a type H with a simple project in $t=1$ via IPO without a lock-up would be $\max\{\frac{k \cdot (2I + B)}{3} - 2I - B - \epsilon, -I - B - \epsilon\}$.

Proposition 3 (ASYMMETRIC INFORMATION - optimal lock-up strategy):

Let us assume that a profit maximizing VC finances an H-firm with a simple project. If he prefers the lock-up to the complete IPO, then he sets the share price at $p^ = k \cdot (2I + B) - I$, and he sells a fraction $a^* = \frac{B}{k(2I + B) - I + B}$ of old shares in $t=1$.*

⁵ A price per share lower than ϵ that would prevent an L-firm from the IPO will not be chosen by an H-firm since this strategy for him is strictly dominated by the IPO in $t=2$.

Proof

Under p^* and a^* , a VC who finances an L-firm prefers liquidation in $t=1$ to the lock-up in $t=1$ (see Appendix D). Therefore, if $a = a^*$, the new investors are willing to pay the price p^* per share that corresponds to the value of an H-type. Now we will show that a profit maximizing VC who finances a simple project will not choose another lock-up strategy (by setting other levels of p and/or a).

If the VC with an H-firm and a simple project chose in $t=1$ a price \bar{p} that was higher than p^* , no NI would buy any shares in $t=1$ because the price would be higher than the firm value. The investment demand of the growing firm would not be met, and the firm would not survive. The VC would earn no revenues and, hence, his profit would be lower than if he chose a^* and p^* . If the VC chose the same price per share p^* and would offer a lower fraction of shares a ($a < a^*$) in $t=1$, his profit would be lower due to higher costs of the non-monetary contribution, which are decreasing in a . If he chose a higher $a \in (a^*, 1)$, the VC who finances a type L would be interested in the lock-up as well. Hence, the NIs would not be willing to pay p^* . If the VC opted in $t=1$ for a price per share \tilde{p} that was lower than p^* , he would have to issue more new shares in $t=1$ than under p^* in order to finance the investment demands I of the young firm in the second period. Since the value of the firm would remain the same but more new shares would be issued in $t=1$, the maximum possible share price the VC could get in $t=2$ for the rest of his shares would be lower than under p^* and a^* . If the VC chose \tilde{p} and $a \in [0, \frac{B}{\tilde{p}+B}]$, an H-firm would separate from an L-firm. The VC's profit would be lower than the profit under p^* and a^* (see Appendix E). If $a \in (\frac{B}{\tilde{p}+B}, 1)$, an L-firm would be interested in the lock-up as well, and we would have a situation with pooling that, for the H-firm, is strictly dominated either by lock-up with p^* and a^* or by the complete IPO in $t=1$ (see Appendix E). Therefore, if the profit maximizing VC with a type H and a simple project prefers a lock-up to a complete IPO, his strategy will be to sell a fraction $a^* = \frac{B}{k(2I+B)-I+B}$ of old shares in $t=1$ for the price per share of $p^* = k \cdot (2I + B) - I$. ■

Now we analyze if the lock-up with a^* and p^* may be an equilibrium strategy for the type H with a simple project (and if liquidation in $t=1$ may be an equilibrium strategy for the type L) or if other exit channels may generate higher profits.

Proposition 4 (ASYMMETRIC INFORMATION - separating equilibrium):

Let us assume that the following condition holds: $\frac{I+B \cdot \frac{B}{k(2I+B)-I+B}}{I} > k$.

Then, a separating equilibrium will emerge in which an L-firm is liquidated in $t=1$, an H-firm with a complex project goes public in $t=2$ for a price per share of $k \cdot (3I + 2B) - I$ and, finally, an H-firm with a simple project goes public in $t=1$ for a share price of $p^* = k \cdot (2I + B) - I$. The fraction of old shares the VC sells at the IPO in $t=1$ is $a^* = \frac{B}{k(2I+B)-I+B}$ (lock-up).

Proof

If the VC finances an H-firm with a complex project, he takes it public in $t=2$ (see Proposition 2). When the condition from the Proposition 4 holds, the VC with a type H and a simple project prefers a lock-up with a^* and p^* to an IPO in $t=2$ (see Appendix F). Hence, he never waits until $t=2$ but takes the firm public in $t=1$. The lock-up with a^* and p^* is preferred by the VC who finances an H-firm to the complete IPO in $t=1$ (see Appendix F). Thus, for the VC with a type H and a simple project, lock-up with a^* and p^* strictly dominates other strategies (see Proposition 3). The NIs know this and would indicate any other pair ($p \neq p^*$ and $a \neq a^*$) as an action by an L-firm. The consequence is that the price per share they would be willing to pay in $t=1$ if $a \neq a^*$ would equal 0. Hence, due to the IPO costs, the IPO in $t=1$ is strictly dominated by the liquidation in $t=1$ for an L-firm. Our candidate for an equilibrium with p^* and a^* for the type H with a simple project and the liquidation in $t=1$ for an L-firm is an equilibrium. ■

3.3 Comparative Statics

Let us look at some comparative statics results concerning the existence of the equilibrium and its features. If the condition $\frac{I+B \cdot \frac{B}{k(2I+B)-I+B}}{I} > k$ (see Proposition 4) holds, we have a separating equilibrium described in section 3.2 where high-quality firms go public in $t=1$ or $t=2$ (depending on the kind of project) and low-quality firms are liquidated in $t=1$. If this condition does not hold, waiting until $t=2$ is more profitable for a type H with a simple project than any lock-up strategy. Hence, if $\frac{I+B \cdot \frac{B}{k(2I+B)-I+B}}{I}$ were lower than k , another separating equilibrium would emerge in which all high-quality firms would go public in $t=2$ and low-quality firms would be liquidated in $t=1$.

Now we will concentrate on the features of the separating equilibrium with lock-up discussed in section 3.2, particularly on the fraction sold at the IPO (extent of a lock-up) and the share price.

Proposition 5 (ASYMMETRIC INFORMATION - comparative statics)

- **Consulting intensity:** *The higher the costs and benefits from the non-monetary contribution B by the venture capitalist, the higher the fraction of old shares sold by the VC in $t=1$ (simple projects) and the higher the share prices (both kinds of projects).*
- **Investment demands (size):** *The higher the investment demands I of the firm, the lower the fraction sold by the VC in $t=1$ and the higher the share prices.*
- **Rate of return:** *The higher the rate of return to the investment ($k - 1$), the lower the fraction sold by the VC in $t=1$ and the higher the share prices.*

Proof

See Appendix G. ■

4 Conclusion

After a certain period of time, venture capitalists have to exit their investments. The purpose of this paper has been to examine some important aspects of the divestment process of venture capitalists. Hereby, we have concentrated on the IPO as the most important exit channel and have analyzed how the timing of the IPO is chosen by the venture capitalists. Particularly we have offered answers on the following questions: (1) when do venture capitalists publicly offer their portfolio firms, (2) under which circumstances do they retain some shares beyond the IPO, (3) how large is the fraction of the retained shares and (4) how the share price is determined.

The difference between the venture capitalists and the new investors who buy shares at the IPO is that the former offer not only money but also managerial support that may enhance the value of the firm. This non-monetary contribution is lost after the IPO. For high-quality firms with complex projects, the venture capitalist's support raises the value of the firm in every period and, therefore, increases the venture capitalist's revenues when selling the firm. When deciding about the timing of the IPO, the venture capitalist compares his future benefits from his managerial and financial support with the future costs of spending his money and effort on his involvement in the firm. Another difference between the two types of investors is that the venture capitalist, being an active investor, is informed earlier about the value of the firm he has been financing than the potential new investors. Hence, the VC who finances a high-quality firm profits from the mitigation of the information asymmetry when the IPO takes place later.

We have analyzed the VC's decision within a single model. It has consisted of two periods, two types of firms (high and low quality) and two kinds of projects (simple and complex). At the end of the second period the VC has not been allowed to retain any shares. We have considered the following five possible exit channels: (1) a complete IPO after the first period (= the VC sells all his shares), (2) an IPO after the first period with a lock-up (= the VC commits himself to retain some of his shares until the end of the second period), (3) a complete IPO after the second period, (4) a liquidation after the first period and (5) a liquidation after the second period. We have questioned the sort and features of exit emerging under the different firm and project characteristics.

An important issue of our model has been the venture capitalist's option to retain some shares beyond the IPO (lock-up). We have analyzed under which circumstances the venture capitalist uses this option and how large the fraction of the retained shares is. Our main finding has been that the venture capitalist holds some shares in order to signal to the new investors that the firm is of high quality. The signaling mechanism is the following: At the end of the second period when the remaining shares are to be sold, the new investors can observe the quality of the firm. If the quality is low, they do not buy any

shares in $t=2$. In this case, the venture capitalist has raised revenues only at the IPO in $t=1$. Beyond a certain level of retained shares ($1 - a^*$), low-quality firms are not interested in the lock-up since the revenues from the partial IPO in $t=1$ are not higher than the IPO expenses plus the costs spent in the second period. Thus, a liquidation in $t=1$ is more profitable than a lock-up for low-quality firms.

Given rational beliefs of the new investors, the venture capitalists maximize their profits which equal the revenues from selling the shares minus the cost. After the elimination of the strictly dominated strategies, the following (separating) equilibrium emerges: High-quality firms with simple projects use a lock-up and the VC sells a fraction a^* in $t=1$ for the price per share that equals the share value in $t=1$. High-quality firms with complex projects go public in $t=2$ at the share price that equals the share value in $t=2$. Low-quality firms are liquidated in $t=1$.

Our model provides a number of empirically testable implications:

- Firms with complex projects will be financed longer by venture capital than firms with simple projects. This finding could, for example, help explain different lengths of financing periods in different industries.
- The high quality of young firms with a short venture capital financing period will be signaled through a lock-up. Since lock-up is a typical feature of almost every IPO in the US and most European markets, the fact that it is used will in practice have no signaling function. Further more, the length and the extent of the lock-up can be considered the signaling device.
- After the expiration of the lock-up period, the information asymmetry between the venture capitalist and the new investors is resolved. Hence, we can conclude a prediction concerning the length of the lock-up period: the higher the opacity of the firm and the greater the uncertainty, the longer the contracted lock-up period.

The extent of the lock-up (fraction of shares retained by the venture capitalist) and the share price depend on the characteristics of the firm.

- The higher the costs and benefits from the non-monetary contribution by the venture capitalists (consulting-intensive projects), the lower the extent of the lock-up and the higher the price per share.
- The higher the investment demands (size) of the firm and/or the greater the profitability, the lower the fraction sold in $t=1$ and the higher the price per share.

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Appendix A

Expected Value the NIs' Participation in an IPO

Using a proof by contradiction, we will show that if $E[W^{\sigma r}(t)] < I$, it is not profitable for the NIs to invest in the firm and finance the necessary investment I .

The value of a share that the NIs expect during and after the IPO (and, hence, the maximum price they are willing to pay per share) equals the expected value of the firm divided by the known number of shares: $\frac{E[W^{\sigma r}(t)]}{1+d}$, where $d = I/p(\cdot)$. Let us assume that $E[W^{\sigma r}(t)] < I$, and, further, that the NIs buy shares and finance $I > 0$ (which implies $p(\cdot) > 0$). Then the maximum price $p(\cdot)$ that the NIs are willing to pay is:

$$p(\cdot) = \frac{E[W^{\sigma r}(t)]}{1+d} < \frac{I}{1+d} = \frac{p(\cdot)I}{p(\cdot) + I}.$$

This implies: $p(\cdot) + I < I$, hence $p(\cdot) < 0$. This, however, contradicts the assumption that NIs buy shares and finance $I > 0$ ($p(\cdot) > 0$).

Appendix B

Type L: Decision Under Information Symmetry

The VC's profit from different exit strategies when financing an L-firm:

$$\text{Liquidation in } t=1: \pi_{liq}^L(1) = -I - B$$

$$\text{Liquidation in } t=2: \pi_{liq}^L(2) = -2I - 2B$$

The price per share at the IPO is determined by (8).

$$p(\cdot) = \max\{E[W^{\sigma r}(t)] - I, 0\} = \max\{E[W^L(t)] - I, 0\} = \max\{-I, 0\} = 0 \quad \text{for } t \in \{1, 2\}$$

$$\text{Complete IPO in } t=1: \pi_{ipo}^L(1) = p(\cdot) - I - B - \epsilon = -I - B - \epsilon$$

$$\text{Complete IPO in } t=2: \pi_{ipo}^L(2) = p(\cdot) - 2I - 2B - \epsilon = -2I - 2B - \epsilon$$

A liquidation in $t=1$ is the most favorable decision because it causes the least possible amount of loss.

Appendix C

Type H: Decision Under Information Symmetry

The VC's profit when financing an H-firm:

Liquidation in $t=1$: $\pi_{liq}^H(1) = -I - B$
 Liquidation in $t=2$: $\pi_{liq}^H(2) = -2I - 2B$

Let us consider the IPO. The VC maximizes his profit and therefore chooses the highest possible price per share that the NIs are willing to pay. This equals the true value of the share (see (8)).

$$p(E[W^\sigma(1)], I) = \max\{W^H(1) - I, 0\} = k \cdot (2I + B) - I$$

$$p(E[W^{\sigma r}(2)], I) = \max\{W^{Hr}(2) - I, 0\} = W^{Hr}(2) - I = \begin{cases} k \cdot (3I + B) - I & \text{for the simple p.} \\ k \cdot (3I + 2B) - I & \text{for the complex p.} \end{cases}$$

The profits are:

$$\text{IPO in } t=1: \pi_{ipo}^H(1) = p(E[W^\sigma(1)], I) - I - B - \epsilon = (k - 1) \cdot (2I + B) - \epsilon$$

IPO in $t=2$:

$$\pi_{ipo}^{Hr}(2) = p(E[W^{\sigma r}(2)], I) - 2I - 2B - \epsilon = \begin{cases} k \cdot (3I + B) - 3I - 2B - \epsilon & \text{for the simple p.} \\ (k - 1) \cdot (3I + 2B) - \epsilon & \text{for the complex p.} \end{cases}$$

In the case of a lock-up, the highest possible price per share in $t=1$ and $t=2$ is $p(E[W^\sigma(1)], I) = k \cdot (2I + B) - I$ (see(9)). The VC's profit using a lock-up:

$$\pi_{lock}^H = p(E[W^\sigma(1)], I) - I - B(2 - a) - \epsilon = k \cdot (2I + B) - 2I - 2B + Ba - \epsilon$$

The liquidation and the lock-up generate lower profits than a complete IPO in $t=1$. The choice between a complete IPO in $t=1$ and a complete IPO in $t=2$ depends on the kind of project.

For simple projects we get the following difference in profits:

$$\pi_{ipo}^H(1) - \pi_{ipo}^{H\theta}(2) = (k - 1) \cdot (2I + B) - \epsilon - k \cdot (3I + B) + 3I + 2B + \epsilon = -kI + I + B > 0$$

If $k < \bar{k}$, IPO in $t=1$ is more profitable than IPO in $t=2$ for simple projects.

For complex projects we get:

$$\pi_{ipo}^H(1) - \pi_{ipo}^{H\lambda}(2) = (k - 1) \cdot (2I + B) - \epsilon - (k - 1) \cdot (3I + 2B) + \epsilon = (k - 1) \cdot (-I - B) < 0$$

For complex projects, IPO in $t=2$ is more profitable than IPO in $t=1$.

Appendix D

Type L: Mimicking the H-firm vs. Liquidation Under Asymmetry

The VC with a type H chooses the price per share $p^* = k \cdot (2I + B) - I$ in $t=1$ and the number of shares he sells at the IPO $a^* = \frac{B}{k(2I + B) - I + B}$. If a type L went public in $t=1$ accepting the same conditions and trying to mimic an H-firm, the VC would get the

price p^* per share for a^* shares. In $t=2$, the NIs would recognize the type of firm. Hence, the price per share would be $p_{lock}^L = 0$ in $t=2$. In the sum, the VC would have spent I on the monetary investment, $B(2 - a^*)$ on the effort costs and ϵ on the IPO expenses. The profit of the VC would reach:

$$\pi_{lock}^L = a^* \cdot p^* - I - B(2 - a^*) - \epsilon = -I - B - \epsilon < \pi_{liq}^L(1).$$

A lock-up would be less profitable than a liquidation for an L-type firm.

Appendix E

Simple Project: The Choice of the Optimal Lock-up Strategy

The profit $\pi_{lock}^{H\theta}$ amounts under p^* and a^* to

$$(k-1) \cdot (2I+B) - B - \epsilon + B \cdot \frac{B}{k \cdot (2I+B) - I + B}.$$

Let us notice that, after a lock-up in $t=1$, the maximum possible share price for an H-firm in $t=2$ (see (9)) is an increasing function of the first period's share price p :

$\frac{d \frac{p \cdot k \cdot (2I+B)}{p+I}}{d p} = \frac{k \cdot (2I+B) \cdot (p+I) - p \cdot k \cdot (2I+B)}{(p+I)^2} = \frac{I \cdot k \cdot (2I+B)}{(p+I)^2} > 0$. Hence if the VC chooses a lower share price in $t=1$, the share price in $t=2$ is lower than p^* , as well.

Let us assume that the share price the VC chooses is \tilde{p} where $\tilde{p} < k \cdot (2I+B) - I$. Hence, the maximization problem of the VC is:

$$\begin{aligned} \max_a \pi_{lock}^{H\theta} &= \max_a a \cdot \tilde{p} + (1-a) \cdot \frac{\tilde{p} \cdot k \cdot (2I+B)}{\tilde{p}+I} - I - 2B + Ba - \epsilon = \\ &= \max_a a \cdot \left(\tilde{p} - \frac{\tilde{p} \cdot k \cdot (2I+B)}{\tilde{p}+I} + B \right) \end{aligned}$$

s.t.

$$0 \leq a < 1$$

Let us distinguish three cases:

(i) If $\tilde{p} - \frac{\tilde{p} \cdot k \cdot (2I+B)}{\tilde{p}+I} + B < 0$, then the VC's profit is maximized for $a = 0$. The profit then reaches $\frac{\tilde{p} \cdot k \cdot (2I+B)}{\tilde{p}+I} - I - 2B - \epsilon$, which is less than $k \cdot (2I+B) - 2I - 2B - \epsilon$, which is less than the profit under p^* and a^* .

(ii) If $\tilde{p} - \frac{\tilde{p} \cdot k \cdot (2I+B)}{\tilde{p}+I} + B = 0$, then the profit is $\frac{\tilde{p} \cdot k \cdot (2I+B)}{\tilde{p}+I} - I - 2B - \epsilon$, which is less than under p^* and a^* .

(iii) If $\tilde{p} - \frac{\tilde{p} \cdot k \cdot (2I + B)}{\tilde{p} + I} + B > 0$, the profit is maximized for $a = \frac{B}{(\tilde{p} + B)}$ if the VC wants to separate and for $a=1$ if the VC prefers pooling. In the latter case, we have no lock-up but a complete IPO. In the former case,

$$\begin{aligned} \max_p \pi_{lock}^{H\theta} &= \max_p \frac{B}{p+B} \cdot p + \left(1 - \frac{B}{p+B}\right) \cdot \frac{p \cdot k \cdot (2I+B)}{p+I} - I - 2B + B \frac{B}{p+B} - \epsilon = \\ &= \max_p \left(1 - \frac{B}{p+B}\right) \cdot \frac{p \cdot k \cdot (2I+B)}{p+I} \end{aligned}$$

s.t.

$$\begin{aligned} 0 < p < k \cdot (2I + B) - I \\ p - \frac{p \cdot k \cdot (2I + B)}{p + I} + B > 0 \end{aligned}$$

The profit is an increasing function of the share price since $\frac{d\pi_{lock}^{H\theta}}{dp} > 0$. Therefore, the VC chooses the highest possible price given the constraint: $p < k \cdot (2I + B) - I$. His profit is then lower than under p^* and a^* .

Appendix F

Simple Project: Lock-up vs. Complete IPO Under Asymmetry

The VC's profit when financing a simple project:

$$\begin{aligned} \text{Complete IPO in } t=1: \quad \pi_{ipo}^{H\theta}(1) &= \frac{k \cdot (2I+B)}{3} - 2I - B - \epsilon \\ \text{Complete IPO in } t=2: \quad \pi_{ipo}^{H\theta}(2) &= (k-1) \cdot (2I+B) + k \cdot I - I - B - \epsilon \\ \text{Lock-up with } p^* \text{ and } a^*: \quad \pi_{lock}^{H\theta} &= (k-1) \cdot (2I+B) - B - \epsilon + B \cdot \frac{B}{k \cdot (2I+B) - I + B} \end{aligned}$$

Lock-up with p^* and a^* is preferred to an IPO in $t=2$:

$$\pi_{lock}^{H\theta} - \pi_{ipo}^{H\theta}(2) = B \cdot \frac{B}{k \cdot (2I+B) - I + B} - k \cdot I + I > 0$$

(see the condition in Proposition 4).

Lock-up with p^* and a^* is preferred to a complete IPO in $t=1$:

$$\pi_{lock}^{H\theta} - \pi_{ipo}^{H\theta}(1) = \frac{2}{3}k \cdot (2I+B) - B + B \cdot \frac{B}{k \cdot (2I+B) - I + B} > 0.$$

Proof:

$$\text{Obviously, } 3BI > k \cdot (2I+B)^2 \cdot \underbrace{(1-2k)}_{\text{negative}}$$

After rearranging terms we get the desired inequality.

$$3BI > k \cdot (2I+B) \cdot [(2I+B) \cdot (1-2k)]$$

$$\begin{aligned}
3BI &> 2Ik(2I+B)+3kB(2I+B)-2kB(2I+B)-2k^2(2I+B)^2 \\
3BI - 3kB(2I+B) &> 2k(2I+B) \cdot (I - B - k \cdot (2I+B)) \\
\frac{3B(k \cdot (2I+B) - I - (B-B))}{k \cdot (2I+B) + B - I} &< 2k(2I+B) \\
2k(2I+B) - 3B\left(1 - \frac{B}{k \cdot (2I+B) + B - I}\right) &> 0 \\
\underbrace{\frac{2}{3}k \cdot (2I+B) - B + B \cdot \frac{B}{k \cdot (2I+B) - I + B}}_{\text{difference in the profits}} &> 0
\end{aligned}$$

Appendix G

Comparative Statics

For a simple project, the fraction sold by the VC in $t=1$ is a^* and the share price is p^* in equilibrium. For a complex project that is sold in $t=2$, the price per share is denoted by p_2 .

$$\begin{aligned}
\frac{d a^*}{d B} &= \frac{2kI - I}{(B - I + kB + 2kI)^2} > 0; & \frac{d a^*}{d I} &= \frac{-B(2k-1)}{(B - I + kB + 2kI)^2} < 0; & \frac{d a^*}{d k} &= \frac{-B(2I+B)}{(B - I + kB + 2kI)^2} < 0. \\
\frac{d p^*}{d B} &= k > 0; & \frac{d p^*}{d I} &= 2k - 1 > 0; & \frac{d p^*}{d k} &= 2I + B > 0. \\
\frac{d p_2}{d B} &= 2k > 0; & \frac{d p_2}{d I} &= 3k - 1 > 0; & \frac{d p_2}{d k} &= 3I + 2B > 0.
\end{aligned}$$