# Divestment, Entrepreneurial Incentives and the Decision to Go Public

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#### ABSTRACT

This paper develops a theory of the life cycle of the firm based on incentive constraints. The optimal sale of the firm is restricted by entrepreneurial moral hazard and a lack of commitment regarding future divestment. This leads to a dynamic inefficiency that causes the entrepreneur to delay and to stagger the sale of the firm. The analysis provides a common explanation for a range of empirical phenomena related to initial public offerings (IPO's), such as the waiting time until firms go public, lock-up periods, operating underperformance of IPO's and post-IPO divestment. The equilibrium divestment process is shown to be (constrained) inefficient: entrepreneurs sell too late and too much of the firm. Recommendations for financial regulation that restore efficiency are derived.

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### **ABSTRACT**

This paper develops a theory of the life cycle of the firm based on incentive constraints. The optimal sale of the firm is restricted by entrepreneurial moral hazard and a lack of commitment regarding future divestment. This leads to a dynamic inefficiency that causes the entrepreneur to delay and to stagger the sale of the firm. The analysis provides a common explanation for a range of empirical phenomena related to initial public offerings (IPO's), such as the waiting time until firms go public, lock-up periods, operating underperformance of IPO's and post-IPO divestment. The equilibrium divestment process is shown to be (constrained) inefficient: entrepreneurs sell too late and too much of the firm. Recommendations for financial regulation that restore efficiency are derived.

Keywords: going public, incentive constraints, life cycle, stock market efficiency

The rich evidence on initial public offerings (IPO's) poses interesting and controversial questions, such as: What determines the decision to go public? Why do firms wait until they seek listing? Why do entrepreneurs only sell a part of their stake at the IPO and unwind further positions later on? Why does the performance of firms decline after the IPO?

While IPO-prospectuses typically stress the need to obtain financing for new projects as the main reason for going public (Roell, 1996), there is broad evidence that IPO's are in fact used by the initial owners to sell out the firm. For example, Mikkelson et. al. (1997) report for the U.S. that 10 years after the IPO the initial owners' stake in the firm is only 18%, compared to 68% before the IPO. This large reduction is hardly surprising since founders of firms have a strong motive to sell the firm in order to reduce their exposure to firm-specific risk and to gain liquidity. One would therefore furthermore expect that firms are taken public soon after the foundation and once being public, the initial owners sell most of their stake immediately. However, Pagano et. al. (1995) report an average age of 33 years for firms that went public in Italy (similar numbers are obtained by Rydqvist and Högholm, 1995, for Sweden). In addition, Mikkelson et. al. (1997) find that owners divest a large part of the firm subsequent to the IPO (Rydqvist and Högholm, 1995, even report that the initial owner hardly divests at the IPO at all). This raises the question of why do owners postpone the sale of firm instead of selling as soon as possible. Further puzzling evidence comes from the operating performance of IPO's. Across countries there seems to be a strong tendency for the operating performance of firms to drop after the IPO (Jain and Kini, 1994, Pagano et. al., 1995, Mikkelson et. al., 1997). Possible explanations are based on window-dressing of earnings and the entrepreneur's desire to time the IPO such to let it coincide with a peak in the performance. However, rational investors should anticipate such motives and adjust their evaluation of the firm's prospects accordingly, rendering any motive for the entrepreneur to trick investors obsolete.

In this paper we provide a common explanation for the above issues (and other puzzling evidence related to IPO's) based on entrepreneurial moral hazard. We consider an entrepreneur who wants to divest her firm but faces the problem that a reduction in her stake in the firm also lowers her incentives to exert effort. Since with efficient markets the entrepreneur has to fully bear the arising efficiency losses, this will constrain her in selling the firm unless also the control of the firm is changed. The latter is rather uncommon for young and small firms (for which entrepreneurship is most common) since it results in a loss of entrepreneurial human capital. The entrepreneur's incentive to divest is furthermore limited because an entrepreneur has typically several opportunities to sell the firm in the course of time. While the efficiency losses of selling at the IPO may be fully internalized, subsequent divestment by the entrepreneur (such as for example through seasoned offerings) reduces her effort further and poses a negative externality on the existing shareholders. Therefore, if the entrepreneur cannot commit herself to retain her stake after the

IPO (especially younger firms and those not affiliated with a mature parent firm or backed by venture capitalists may effectively lack the ability to make such commitments), investors will further reduce their valuation of the firm at the IPO in order to be compensated for the additional efficiency losses.

We show that, as a consequence, the entrepreneur will stagger her divestment over time, i.e., she does not exclusively sell at the IPO but uses the secondary market and/or seasoned offerings for additional divestment. The entrepreneur's lack of commitment has furthermore implications for the decision to go public. In early stages of the life of the firm, when the proceeds from investment and effort are still far off, the entrepreneur's valuation of the firm is low (because she has to stay undiversified and illiquid for a long time) and hence the potential benefits from selling the firm are high. However, this also means that the entrepreneur has a high incentive to divest subsequently to the IPO. Investors anticipate that and will lower their valuation of the firm accordingly. We show that the second effect dominates the first, which causes the gains from going public to be low initially. Instead of going public immediately after the foundation of the firm, the entrepreneur may therefore wait with the listing.

The empirical implications of our theory are largely supported by the evidence. First, our model can endogenously produce a plausible life cycle of the firm: entrepreneurs wait until they take firms public (or may not go public at all), divest a certain stake at the IPO and divest thereafter in the secondary market (or through seasoned offerings) but do not completely exit unless there is also a change in control. The analysis also rationalizes the use of venture capital before the going public as a way to reduce commitment problems. Furthermore, our theory can explain the operating underperformance of IPO's as a result of lowered entrepreneurial incentives, which stem from a reduction in the entrepreneur's stake in the firm at the IPO. Another prediction of our model is that there should be a positive relationship between the degree of ownership by the entrepreneur and the operating performance. There is some evidence for that.<sup>1</sup> We also derive some appealing predictions regarding the relationship between the costs of going public and the age of public firms and between the timing of going public and the amount of divestment at the IPO.

We show that the entrepreneur's divestment plan is (constrained)-inefficient (i.e., inefficient beyond the moral hazard problem): the entrepreneur divests too much and too late and as a consequence too few firms go public. This in contrast to single-trade models, in which the stock market has been shown to operate efficiently (Kocherlakota, 1998, and Magill and Quinzii, 1999 and 2001). This inefficiency provides a rationale for IPO-lockup periods because they can act as a commitment device that can improve the entrepreneur's divestment plan. We furthermore derive a proposal for financial regulation that completely

<sup>&</sup>lt;sup>1</sup>Jain and Kini (1994) find indeed such a relation for U.S. IPO's, and Holthausen and Larcker (1996) for reverse leveraged buy-outs. Moreover, Downes and Heinkel (1982) find that the share of equity retained at the IPO is related to the valuation of the firm (on the other hand, Pagano et. al., 1995, and Mikkelson et. al., 1997, find no relationship between ownership and performance).

restores efficiency without reducing flexibility. This is achieved by requiring the entrepreneur to divest through existing shareholders, which forces the entrepreneur to internalize the adverse effects of divestment on the existing shareholders. Besides, our analysis also makes a strong case for disclosure and trading size restrictions for insiders: both disclosure of trades and limitations on the size of trades are needed to make a stock market operative in the absence of the ability of entrepreneurs to commit themselves to future divestment.

There has been surprisingly little theoretic research into the go public of firms. Most closely related to our analysis is probably the signalling theory by Leland and Pyle (1977). In this theory the equity retained by the entrepreneur serves as a signal for the quality of the firm and does therefore also predict a relationship between the IPO retention and firm performance thereafter. There is furthermore a strand of the literature that emphasizes the informational aspects of the stock market and relates it to the timing of going public (Ellingson and Rydqvist, 1997, Chemmanur and Fulghieri, 1999, Titman and Subrahmanyam, 1999, and Maksimovic and Pichler, 2001). In Chemmanur and Fulghieri (1999) the entrepreneur faces a trade-off between lower financing costs (compared to venture capitalist financing) and higher informational costs when going public. Ellingson and Rydqvist (1997) present a model in which trading at the stock market produces information that helps to reduce adverse selection. The initial owner may therefore prefer to first sell a part of the stake at the IPO and sell further shares later under more favorable informational conditions. Change of control has also been emphasized as a motive for going public (Zingales, 1995, Mello and Parsons, 1998). This papers argue that the initial owner maximizes the proceeds from selling the firm by differentiating between investors, which usually leads to selling the firm in different stages. For example, in Mello and Parsons (1998) the initial owner uses the IPO to sell shares to small and passive investors, while the marketing of controlling blocks occurs separately.<sup>2</sup> The analysis in this paper differs from the above contributions in that we explain the timing of the going public with a trade-off between entrepreneurs' incentives to sell their firms as soon as possible and distortions arising from an inefficient divestment plan.

The remainder of the paper is organized as follows. The next section outlines the static divestment problem under moral hazard. Section II extends to a dynamic framework and studies secondary market trading. Section III derives the implications for the life cycle of the firm and the decision to go public. Section IV and V contain empirical and policy implications, respectively. The final section concludes.

<sup>&</sup>lt;sup>2</sup>There is also a large literature on underpricing which generally emphasizes adverse selection (Chemmanur, 1993, Grinblatt and Wang, 1989, and Welch, 1989). A notable exemption are Stoughton and Zechner (1998). In their model oversubscription of the IPO is used by the owner to differentiate between investors in order to reduce monitoring.

## I. The Stock Market and Entrepreneurial Incentives

Consider an entrepreneur (denoted E) who has founded a firm at time t=0 that yields a pay-off f at t=1. The pay-off depends on the effort e exercised by the entrepreneur, specifically we assume that f = e. Effort is chosen at time of production (t = 1) and gives disutility of  $c(e) = e^2/2$ . The entrepreneur maximizes wealth and discounts time with  $\beta$  ( $\beta$  < 1). There are a large number of investors that maximize wealth but do not discount time. This is a simple way to model an incentive for the entrepreneur to sell the firm to investors (the incentive is motivated by the fact that an entrepreneur is exposed to firm-specific risk and often cash-constrained while investors are typically well diversified and liquid).<sup>3</sup> Ideally, the entrepreneur would like to sell the whole firm at t=0. We assume that the entrepreneur's firm-specific abilities and further costs rule out a complete exit (including a transfer of managerial responsibilities and control). The entrepreneur can, however, sell cash-flow rights (shares) to the investors at the beginning of the period. In doing so, E faces the following problem. If she sells a part of the firm, she participates less in the firm's pay-off and has less incentives to exercise effort. Although the investors cannot observe effort, it is assumed that they can observe how much of the firm is sold by the entrepreneur (this assumption will be discussed below). Consequently, investors lower their valuation of the firm when the entrepreneur reduces her stake in anticipation of a lower effort choice. This poses a restriction to E's divestment. In fact, E faces a trade-off between the gains from divestment (given by the discount factor  $1-\beta$ ) and distortions arising from the fact that her effort choice has to be incentive compatible.

More specifically, if all parameters of the model are common knowledge and the share  $\theta$  of the firm that is retained by the entrepreneur is observable, investors can fully infer the effort the entrepreneur will exercise. Investors are assumed to be competitive. The price p of the firm will then be exactly  $f(\tilde{e})$ , where  $\tilde{e}$  is the entrepreneur's effort choice given  $\theta$ . The two-stage maximization problem faced by the entrepreneur is

$$\max_{0 \le \theta \le 1} \{ \phi(\theta, \widetilde{e}) = (1 - \theta) p(\widetilde{e}) + \beta [\theta f(\widetilde{e}) - \frac{\widetilde{e}^2}{2}] \}, s.t.$$
(i)  $\widetilde{e} = \arg\max_{e \ge 0} \{ \theta f(e) - \frac{e^2}{2} \},$ 
(ii)  $p(\widetilde{e}) = f(\widetilde{e})$  (iii)  $f(\widetilde{e}) = \widetilde{e}$  (1)

The first part of the objective function  $\phi$  are the proceeds from sale of the firm, the second part is the net benefit from production discounted at  $\beta$ . Condition (i) requires the effort choice to be incentive compatible, (ii) states that investors correctly anticipate the production of the firm and (iii) is the assumed production function.

<sup>&</sup>lt;sup>3</sup>The assumption of a constant  $\beta$  is for simplification only. Generally, diversification and liquidity motives would imply that  $\beta$  increases when the entrepreneur sells a part of the firm. The consequences of this are discussed at then end of the next section.

**Proposition 1** (optimal divestment) The entrepreneur retains a share of  $\theta = 1/(2 - \beta)$  in the firm.

**Proof.** With  $f(\tilde{e}) = \tilde{e}$  (condition (iii)) it follows from (i) that  $\tilde{e} = \theta$ . From (ii) we have  $p(\tilde{e}) = f(\tilde{e}) = \theta$ . The FOC for the objective function is then  $1 - 2\theta + \beta[2\theta - \theta] = 0$ . Solving for  $\theta$  yields  $\theta = 1/(2 - \beta)$ .

As argued above, the moral hazard problem prevents complete divestment. An interesting question is whether the choice of  $\theta$  is efficient, in the sense that a social planner who has to respect the limited availability of assets (shares) and the unobservability of effort cannot improve the allocation.<sup>4</sup> The answer is yes. The intuition is that the negative externalities, which arise because a sale of shares reduces the incentives for the entrepreneur and lowers the pay-off for all shareholders, are fully internalized through a reduction in the price of the firm. In fact, Magill and Quinzii (1999, 2001) (and also Kocherlakota, 1998) have shown in a more general framework with uncertainty that a stock market solves the trade-off between the gains from trade and distorting incentives in an optimal way.

This analysis rests on two crucial assumptions. First, investors can observe trades. Since the firm's price depends on the share of the firm sold by the entrepreneur, investors have to know this share when making their trades (in the model all trades takes place simultaneously). This is not a cause for concern for IPO's, where the amount of shares offered is fixed or governed by rules. For the secondary market, one can argue that disclosure requirements for insiders coupled with quantitative trading restrictions make trades de facto observable. However, this is generally only ex-post disclosure.<sup>5</sup> This means that an investor that buys shares cannot observe whether the entrepreneur simultaneously sells to other investors and is therefore not able to price the shares correctly. The second crucial assumption is that trade only takes place once. Again, this is reasonable for IPO's but does not fit with secondary market trading. If there are multiple trading opportunities, investors face the danger that the entrepreneur divests subsequent to their investment. In fact, if the entrepreneur in our model were unexpectedly allowed to retrade, she would divest further.

**Proposition 2** (retrade) Given that the entrepreneur has an initial share of  $\theta_0$  ( $0 < \theta_0 \le 1$ ) in the firm, she retains a share of  $\theta = \theta_0/(1-2\beta)$  ( $< \theta_0$  for  $\beta < 1$ ) after trade.

**Proof.** analogous to Proposition 1 (the entrepreneur maximizes now  $(\theta_0 - \theta)p(\widetilde{e}) + \beta[\theta f(\widetilde{e}) - \widetilde{e}^2/2]$  s.t. conditions (i)-(iii) in (1)).

The intuition for this result is that a reduction in the entrepreneur's stake in the firm only affects the price of the shares the entrepreneur is currently selling  $(\theta_0 - \theta)$  but not the ones she has already sold  $(1 - \theta_0)$ . Thus her costs of divestment decrease and she divests

<sup>&</sup>lt;sup>4</sup>This is the concept of constrained pareto-efficiency dating back to Diamond (1967)

<sup>&</sup>lt;sup>5</sup>For a discussion of the observability of trades see Kocherlakota (1998) and Magill and Quinzii (1999).

more. According to Proposition 2, infinitely many (unexpected) trading opportunities would drive the share the entrepreneur retains to zero (since she only retains a fraction of her initial stake in the firm in every trading period).

## II. The Dynamic Divestment Problem

In practice investors do of course know that entrepreneurs can divest at later stages (for example through seasoned offerings). They will therefore lower their valuation for shares of the firm. In this section we study how this will affect the amount and the timing of divestment by the entrepreneur and whether the outcome remains efficient. We do that by extending the model for continuous time trading.

Firm's pay-off arises now after a period of length n. During this period the firm can be traded continuously. Effort is chosen after trading. The entrepreneur discounts with the continuous time discount factor  $\delta$  ( $\delta > 0$ ), investors continue to have no time preference. The firm is priced competitively by the investors for any trade during the period according to E's anticipated divestment path given her share in the firm  $\theta(t)$  prior trading (thus the price of shares in a trade does not depend on the amount of the firm divested at that point of time). We require  $\theta(t)$  to be differentiable.<sup>6</sup> Since the divestment path will be correctly anticipated by the investors, the price of the firm in equilibrium is constant over time. The entrepreneur's maximization problem is given by

$$\max_{0 \le \theta(t) \le 1} \{ \phi(\{\theta'(t)\}_{0 \le t \le n}, \theta(n), \widetilde{e}) \}$$

$$= \int_{0}^{n} -e^{-\delta t} \theta'(t) p(\widetilde{e}) dt + e^{-\delta n} [\theta(n) f(\widetilde{e}) - \frac{\widetilde{e}^{2}}{2}] \}, s.t.$$

$$(i) \ \widetilde{e} = \arg\max_{e \ge 0} \{ \theta(n) f(\widetilde{e}) - \frac{\widetilde{e}^{2}}{2} \} \}$$

$$(ii) \ \forall \ 0 \le k \le n : \ \{ \theta(t) \}_{k \le t \le n} =$$

$$\arg\max_{\{\theta(t)\}_{k \le t \le n}} \{ \phi_{t \ge k} (\{ \theta'(t) \}_{0 \le t \le n}, \theta(n), \widetilde{e}) \ with$$

$$\phi_{t \ge k} := \int_{k}^{n} -e^{-\delta t} \theta'(t) p(\widetilde{e}) dt + e^{-\delta n} [\theta(n) f(\widetilde{e}) - \frac{\widetilde{e}^{2}}{2}] \}$$

$$(iii) \ p(\widetilde{e}) = f(\widetilde{e}) \ (iv) \ f(\widetilde{e}) = \widetilde{e} \ (v) \ \theta(0) = 1$$

$$(2)$$

The objective function  $\phi$  is the continuous time generalization of the objective function from problem (1) with the initial condition  $\theta(0) = 1$ . Condition (i), (iii) and (iv) are as in the static optimization problem. Condition (ii) rules out any time-inconsistent divestment paths by requiring that the chosen path is optimal at any point in time. Time inconsistency

<sup>&</sup>lt;sup>6</sup>Note that this does not restrict the amount that can be sold by the entrepreneur since there is a continuum of trading opportunities. However, it implies that the entrepreneur cannot divest large amounts without being observed (in the next section we study the IPO and allow also for discrete divestment). This is motivated by disclosure requirements for large shareholders that are in place in many countries. For example, in the U.S. owners of more than 10% of a stock have to disclose their trades (ex-post). Coupled with market liquidity restrictions, this effectively prevents the large shareholder from divesting large stakes without being noticed.

arises because the price of a share in a trade depends on divestment decisions that take place after the trade. Given her time preference, E would like to commit herself to a divestment path with a certain divestment at t=0 but no divestment thereafter. However, once E has divested, she has an incentive to divest further (as Proposition 2 states). Investors will anticipate that and price the firm according to a time consistent divestment path.

**Proposition 3** (divestment path) The entrepreneur divests a constant amount  $\theta'(t) = -\delta/(1+\delta n)$  at any point in time and remains a share of  $\theta(n) = 1/(1+\delta n)$  after trading. **Proof.** See Appendix.

The reason why E divests in pieces and not all at once arises from her motive for retrade (Proposition 2). After she has sold an amount of the firm, she has an incentive to sell more of the firm. This is the case because when she retains less of the firm, her costs of divestment in terms of a reduction in the proceeds from future sales are reduced. In that way, any divestment creates a new incentive for divestment and leads to a divestment path. Rationality requires that investors anticipate subsequent divestment and lower their valuation of the firm. The restriction to small amounts of divestment at any point of time coupled with the fact that E fully internalizes the effect of her divestment on the proceeds from future sales, prevent a complete sale of the firm during the trading period. Thus, production of the firm will be positive and investors are prepared to pay for the firm (which, in turn, makes it worthwhile for E to sell).

The speed of divestment  $|\theta'(t)| = \delta/(1+\delta n)$  increases with the  $\delta$ , the costs of deferring divestment. The length of the trading period n only influences divestment in so far as it changes the factor at which the final pay-off is discounted  $(e^{-\delta n})$ . If one sets the discount factor to  $\delta := \delta^*/n$  (which makes the present value of the final pay-off independent of the length of the period), the share retained by E is  $\theta(n) = 1/(1+\delta^*)$  and independent of the length of the trading period.

The existence of a divestment path is potentially relevant for the equity premium. Kocherlakota (1998) and Magill and Quinzii (1999, 2001) have studied the equity premium in a general equilibrium model with a single trading period and have found that the introduction of moral hazard reduces the ability of the model to produce high premiums. Matters are likely to change in dynamic setting with moral hazard. This is because the presence of a divestment path leads to a reduction of prices in *anticipation* of a drop in performance in the future (due to future divestment). This may lead to lower price-earnings multiples since they are computed from current earnings.

We have argued in the previous section that the divestment chosen by the entrepreneur leads to constrained-efficiency. In the dynamic setting this is not the case anymore, i.e., a social planner that has to respect the unobservability of effort and the availability of assets (shares) can improve the allocation by setting a different divestment path.

**Proposition 4** (dynamic inefficiency) For n > 0: (i) the solution to (2) is constrained-inefficient; (ii) the efficient allocation is that divestment only takes place at t = 0 with a share of  $\theta^* = 1/(2 - e^{-\delta n})$  being retained; (iii) compared to the second best solution, E divests excessively, i.e.,  $\theta(n) < \theta^*$ .

**Proof.** See Appendix.

The non-optimality of the decentralized solution stems from the fact that the entrepreneur cannot commit herself to future trading. As a consequence she faces a lower valuation of her firm than what would be justified by her current stake in the firm. This restrains her from divesting the optimal amount immediately. Her motive for retrade, however, gives her an incentive to divest after the IPO. This causes an inefficient timing of the sales of the firm because she has a preference for time vis-à-vis the investors. Moreover, since for any t > 0 she does not fully internalize the agency costs of a reduction in her stake in the firm (she does not take into account the shares that are already sold), she divests in total more than the optimal amount.

We have simplified the analysis significantly be assuming a constant discount factor  $\beta$ . One can reasonably argue that a reduction in  $\theta$  reduces both the need for obtaining liquidity (because the entrepreneur has already partly cashed in) as well as the costs of a unit of firm-specific risk and hence  $\partial\theta/\partial\beta<0$ . In the static framework (Proposition 1), this does not cause any consequences as long as  $\beta\leq 1$ . For the divestment path, there are two effects. First, since as  $\theta$  decreases also  $\beta$  decreases, the potential benefits from further divestment are reduced, which makes the divestment path more convex. Second, as  $\beta$  decreases, also the dynamic inefficiency is decreased since investors realize that the entrepreneur now has less incentives to sell subsequently. This may increase the incentives to sell and make  $\theta(t)$  more concave. The net effects will depend on the exact specification of  $\beta(\theta)$ .

## III. The Decision to Go Public and the Life Cycle of the Firm

In the preceding section we have studied the dynamic aspects of stock market trading under moral hazard. Since trade as an entrepreneurial firm is only one of the possible stages in the life cycle of firm, the benefits at this stage interact with decisions made by the entrepreneur at other stages. In order to study this interaction, we embed the previous analysis in the life cycle of the firm.

The life cycle of a firm is typically characterized by a phase of full private financing after the setup. Then, a venture capitalist may obtain a stake in the firm. At some point of time, the entrepreneur may decide to take the firm public with an IPO, to be followed by secondary market trading. Eventually, as a last stage in the transformation from a purely

entrepreneurial firm to a professionally managed corporation, the firm may get acquired, for example by a corporate raider. In the following, we focus on how the decision to go public arises as a consequence of the dynamic inefficiency faced by the entrepreneur when selling the firm (venture capitalism and change of control are discussed at the end of the section)

We first extend the model from the previous section to two divestment stages: IPO and secondary market trading. The entrepreneur can first sell shares at the IPO and can then trade during a period of length n (the trading phase is identical to the model from the previous section). The difference between secondary market trading and an IPO is that in the latter a large number of shares are sold at one point of time. Moreover, the number of shares being sold at an IPO is generally known to investors and will affect the price of a share at the IPO (which compares to instantaneous disclosure for second market trading).

**Corollary 5** The entrepreneur sells  $1 - \theta(0) = 1 - (1 + \delta n)/(e^{-\delta n} + 2\delta n)$  at the IPO and retains a share of  $\theta(n) = 1/(e^{-\delta n} + 2\delta n)$  after trading.

**Proof.** See Appendix.

Corollary 5 uses a generalization of Proposition 3, namely that the entrepreneur divests a constant fraction of her initial stake  $\theta(0)$  (which is now different from 1) during the trading period. At the time of IPO the entrepreneur simply chooses the optimal initial stake  $\theta(0)$  for the divestment path. The utility arising from this divestment pattern can be used to compute the gains from gain public (the alternative of staying private being that the whole firm has to be kept until production takes place).

**Proposition 6** (the gains from going public) The gains from going public b(n) follow an hump-shape on  $[0, \infty)$  and are zero for n = 0 and  $n \to \infty$ .

**Proof.** See Appendix. ■

The intuition behind Proposition 6 is as follows. The gains from going public are low in early stages of the life when production is far off (n large). This is because the entrepreneur has then a strong desire to cash in. Investors anticipate this and since the entrepreneur cannot commit herself to keeping a stake in the firm, they will pay a low price for shares of the firm. This effect on the firm's price outweighs the potentially high benefits from selling the firm and makes the gains from going public initially low. As time passes by (n decreases) the time inconsistency problem is reduced because the entrepreneur has a greater interest in keeping a stake in the firm. Investors will value the firm higher and the gains from going public increase initially. However, as the time of production approaches, also the relative valuation of the firm by the entrepreneur vis- $\hat{a}$ -vis investors increases (since production is discounted less). This effect eventually outweighs the reduction in the time inconsistency problem and the gains from going public fall. At then end of the trading period, the gains from going public are zero.

On the contrary, if there is no time inconsistency the benefits from going public are always decreasing with the age of the firm (n increases) (the only effect of n on the gains from going public is then through a change in the discount factor  $e^{-\delta n}$ ). The entrepreneur would then choose to go public immediately after the foundation of the firm or remain private forever if the costs of going public exceed the initial gains from going public. In the absence of commitment, however, the entrepreneur may postpone listing until the benefits b(n) exceed the costs. If the gains at their maximum are still lower than the costs of going public, the firm will remain private forever.

The dynamic inefficiency of E's divestment furthermore gives rise to a plausible role for venture-capitalism and a transfer of control during the life cycle. In contrast to small shareholders, venture capitalists are usually active investors. They often have a seat in the board and can for example make future increases in their stake in the firm contingent on the performance of the firm. Consequently, venture capitalists have some influence over the entrepreneur and are at least partly able to force the entrepreneur to internalize the effect of a reduction in her stake on the value of the venture capitalist's stake.

Venture capitalism can be modelled as follows. Assume for simplicity that the entrepreneur can fully commit herself vis-á-vis the venture capitalists. There is then no dynamic inefficiency from divestment (through the venture capitalist). Assume furthermore that venture capitalists require a premium for the large exposure to firm specific risk and the illiquidity of their stake (compared to investors that can fully diversify and have a market to trade their assets). Their valuation of the firm will therefore be lower than for investors but still larger than for the entrepreneur. The entrepreneur may then sell a stake to a venture capitalist in early stages of the firm when the dynamic inefficiency is high (they effectively reduce the discount factor for the entrepreneur). Since the valuation of the firm by investors is still higher than for the venture capitalists, the entrepreneur may still take the firm public in later stages (when dynamic inefficiency is reduced). To the contrary, in the absence of the dynamic inefficiency venture capitalism and going public cannot jointly occur during the life cycle. This is because the entrepreneur goes either public immediately (with no need for venture capital then) or remains private forever (possibly with venture capital financing).

We assumed that efficiency losses (due to the loss of the firm-specific abilities of the entrepreneur and/or agency costs under the new form of ownership and control) make a transfer of control initially undesirable. If the firm goes public the productive opportunities of the firm are less and less exploited when the entrepreneur reduces her stake (since effort is reduced). At a certain point this may make it worthwhile to sell the whole firm to an investor even though there are efficiency losses involved in the transfer of control. In our view of the firm, this represents the final stage in the life cycle (only to be followed by liquidation of the firm). Again, if there were no time-inconsistency a change in control would either take place right after the setup of the firm or never. This is because in

absence of frequent divestment throughout the life of the firm, the firm gains value for the entrepreneur vis-á-vis investors, hence the sale of the firm should only occur right after the foundation.

## IV. Empirical Relevance

Our theory is build around the idea that the going public decision is driven by the entrepreneur's desire to divest the firm. There is indeed broad evidence that owners reduce their holdings substantially at the IPO and thereafter. For example, Mikkelson et. al., 1997, report for U.S. IPO's that the stake of insiders declines from 68% to 18% during the first 10 years after the IPO (however, this prediction is common to all models stressing the sale of the firm as the motive for the going public decision, e.g., Zingales, 1995). Since a reduction in ownership increases incentive problems, our model moreover implies that the operating performance of firms drops after the going public. There is overwhelming evidence for this phenomenon (see for example Jain and Kini, 1994, Holthausen and Larcker, 1996, and Mikkelson et. al., 1997). Alternatively, a drop in operating performance can also emerge if owners time the IPO to let it coincide with a peak in the performance of the firm (Ritter, 1984) or if window-dressing is used. More specifically, however, our theory furthermore predicts a relationship between the amount of equity retained by the entrepreneur and the operating performance of the firm. In their study of U.S. IPO's, Jain and Kini (1994) report that there is a significant positive relation between post-IPO operating performance and the amount of divestment by entrepreneurs. Holthausen and Larcker (1996) find similar results in their study of reverse leveraged buy-outs (RLBO's). With a more general focus, Mikkelson and Partch (1985) and Wruck (1989) find a positive relation between ownership concentration and performance of public firm. On the contrary, Mikkelson et. al. (1995) and Pagano et. al. (1995) do not find a relation between ownership structure and post-IPO operating performance.<sup>8</sup>

A further prediction of our model is that due the dynamic inconsistency, the entrepreneur does not solely divest at the IPO. Many studies report that the share divested by the original owners at the IPO is not very large but that after some years the entrepreneur has divested a substantial amount. In the sample of Mikkelson et. al., 1997, 24% is divested at IPO and 26% in the next ten years, other studies report that the entrepreneur sells very few at the IPO (for example Rydqvist, 1995). Furthermore, the presence of moral hazard in our model implies that the owner does not completely divest (at least not as long as the firm operates independently). This hypothesis is not obtained by other theories stressing

 $<sup>^{7}</sup>$ Similar results are obtained by Rydqvist and Högholm (1995) and Brennan and Franks (1997) for Sweden and the U.K., respectively.

<sup>&</sup>lt;sup>8</sup>A relation between equity retention and firm performance is also predicted by Leland and Pyle (1977). In their adverse selection model the entrepreneur has superior information about the firms prospects. The share retained serves as a signal for quality.

the sale of the firm as an important determinant of the going public decision. It also runs counter to standard portfolio diversification considerations. Although we are not aware of direct evidence for this hypothesis, most studies find that the original owner still remains a significant stake in the firm even after many years (18% in Mikkelson's study).

It is generally appealing that our simple model produces a plausible life cycle of the firm with the stages venture capitalism, going public (with an IPO and secondary market trading) and eventually acquisition by another firm. We are not aware of other papers that can reproduce such a life cycle of the firm with a single model. The plausibility of the life cycle arising from the model also indicates that the dynamic inefficiency in divestment is indeed relevant for the life cycle. This is because, as we argued in the previous section, all stages would collapse to the time immediately after the foundation in the absence of a lack of commitment (and could furthermore no co-exist).

Our model furthermore predicts a relationship between the costs of going public and the age of public firms.

**Proposition 7** (waiting time) An increase in the costs of going public increases the waiting time until going public (provided the firm goes public).

**Proof.** See Appendix. ■

U.S. stock markets are considered to provide cheap access to capital compared to European markets, one would therefore expect U.S. firms going public earlier than European firms. Quite strikingly, Pagano et. al. (1995) report an average age of 33 years for firms that went public in Italy (similar numbers are obtained by Rydqvist, 1995, for Sweden) compared to 11 years for the U.S. (Gompers, 1996).

The dynamic inefficiency in divestment has also an implication for the relationship between the timing of going public and the entrepreneur's divestment.

**Proposition 8** (divestment behavior) Firms that go public later divest either moderately or aggressively in contrast to firms that go public early.

**Proof.** See Appendix.  $\blacksquare$ 

Proposition 8 underlies that both firms with low and high  $\delta$  have low benefits from going public. This is not surprising for firms with low  $\delta$  because then the potential gains from going public are low. However, also high- $\delta$  firms have low benefits since the entrepreneurs of such firms have a strong motive to divest, hence they face a higher time inconsistency problem. The arising efficiency losses outweigh the higher potential benefits from going public that come with a high  $\delta$  (for the same reason are the gains from going public low in an early stage in the life of the firm). Thus firms that need the stock exchange most get listed at late stages in their live (or even remain private)! We are not aware of any direct evidence for (or against) the above relationship but it could possibly account for the observed diversity in firms that remain private (as noted in Pagano et. al., 1995).

In the next section we show that a *lock-up period* in which insiders agree to refrain from divesting during a specified time after an IPO can actually reduce the time inconsistency problem. We would thus expect that firms for which the inconsistency problem is less pronounced to make less use of lock-up periods. This should be the case for firms with venture capital. Since venture capitalists have an interest in keeping the value of their stake, they will (as argued in V) use their power (for example board participation) to avoid actions by the entrepreneur that are detrimental to the firm's performance, i.e., excessive divestment. Correspondingly, Brav and Gompers (2000) find that firms that are not backed by a venture capitalist have on average longer lock-up periods (however, Brav and Gompers consider adverse selection as the explanation for lock-up periods).

Lastly, our model implies that the valuation (as measured, for example, by P/E-ratios) should be higher for firms with low  $\delta$ . This is because these firms have lower divestment and their performance is expected to drop less relative to firms with higher  $\delta$ . Since firms with lower  $\delta$  divest also a lower amount at the IPO (Corollary 5), we should see a positive relationship between the amount of equity retained after IPO and the valuation of the firm at IPO. Downes and Heinkel test this prediction for U.S. IPO's (which also arises from the Leland-Pyle-model) and find clear evidence of such a relationship.

## V. Policy Implications

The preceding section has shown that the evidence broadly supports our model, which is driven by the inability of the stock market to provide an efficient transfer of ownership. In the following we examine whether regulations can reduce this inefficiency.

Our analysis makes obviously a strong case for requiring disclosure and limiting the size of trades by insiders. Such regulations are commonplace at major exchanges. If there is no disclosure, the entrepreneur could completely divest without the investors noticing it. The disciplining effect of a price drop in the shares on future divestment is then absent. Knowing this, investors will not trade with the entrepreneur. Disclosure, however, is not sufficient to avoid market breakdown. Regulation has also to ensure that the amount which can be divested by the entrepreneur at one point of time is small. The intuition for this is given by Proposition 2, which states that the entrepreneur would like to divest a fraction of her stake at every trading opportunity. For continuously traded firms, the entrepreneur could then completely exit during the trading period. This again rules out any divestment in equilibrium.

Since the dynamic inefficiency arises from a lack of commitment with respect to future divestment, any device that allows the entrepreneur to commit herself can enhance efficiency. For example, it is common practice that insiders agree at the IPO on a so-called lock-up period (usually 180 days), during which they are barred from trading shares. A prominent argument for lock-up periods is that any unfavorable information being with-

held by the insiders at the IPO will be disclosed during that period. A lock-up period can then limit the adverse affects of insider information (for example Brav and Gompers, 2000). However, a lock-up period also increases efficiency by reducing time inconsistency.

**Proposition 9** (lock-up period) Committing not to trade during a period k > 0 after IPO increases the utility of the entrepreneur.

**Proof.** See Appendix.

In our model, an increase in the duration of the lock-up period is always efficiency enhancing because it enables the entrepreneur to choose a more efficient timing of divestment. This will not necessarily be the case if there are losses due to reduced flexibility arising from the commitment. For example, the entrepreneur may want to divest in order to adjust to new information. In general, lock-up periods will pose a trade-off between flexibility and efficiency. This makes lock-up periods and other commitment devices that simply rule out trading during a certain period an inappropriate tool to deal entirely with the time inconsistency problem.

Is it possible to enhance the efficiency of stock market trading without losing flexibility? This would require that the entrepreneur can divest at any time but also has to ensure that she internalizes the effect of divestment on existing shareholders. In the following we propose a mechanism that fully restores (constrained) efficiency without reducing flexibility. The underlying idea is simple: the entrepreneur can divest at any time but has to do so through existing shareholders. Consider, for example, the case of a single existing shareholder. If the entrepreneur sells shares to this shareholder, the shareholder will price the reduction in the value of the shares already owned into the transaction. Since the entrepreneur has no other means of divesting, the shareholder cannot be made worse off and any efficiency loss through a reduced stake will be internalized. The story becomes more complicated if there are several shareholders. This is because a single shareholder will fail to internalize the effect a transaction has on other shareholders. Therefore, the entrepreneur has to make a joint offer to all shareholders in order to achieve a complete internalization. Consider the following mechanism. An entrepreneur who wants to sell at t proposes an amount  $\theta(t) - \theta^*(t)$  she wishes to sell (where  $\theta(t)$  is her current stake) and a price p to the shareholders. If all shareholders agree the offer is accepted and every shareholder buys a fraction of  $\theta(t) - \theta^*(t)$  according to his share in the outside equity  $(1-\theta(t))$  at price p. Otherwise, the offer is rejected and no divestment takes place.

**Proposition 10** (divestment offer) If an offer  $(\theta(t) - \theta^*(t), p^*)$  is accepted, it does not change the total wealth of the investors.

**Proof.** See Appendix.

Proposition 10 states that the entrepreneur fully internalizes any effect of her divestment on the wealth of the investors. Moreover, since the share transactions take place in proportion to every investors' stake in the firm, there is no divergence between the investors: either the offer is accepted or turned down unanimously. As in the previous analysis, the entrepreneur does not have to share any surplus with the investors (since investors do not behave strategically). Consequently, every divestment that can increase the utility of the entrepreneur without reducing the wealth of the investors will take place; the divestment will therefore be constrained pareto-efficient. Another way to put this is the following. Since investors know that they will be fully compensated for any divestment taking place in the future, they will price the shares according to the entrepreneur's current stake in the firm. Hence there is no time-inconsistency problem. The advantage of this mechanism in contrast to a lock-up period is that it does not reduce the entrepreneur's flexibility. For example, the entrepreneur could react to an unexpected change in her discount rate (e.g., a liquidity shock) by selling shares through an offer. 10

## VI. Summary

This paper presented a new view of the life cycle of the firm and the decision to go public; our analysis suggested that both can be explained by a dynamic inefficiency arising from a lack of commitment with respect to future divestment by the entrepreneur. Our model is consistent with a wide range of empirical facts that otherwise can only be explained by a combination of models. We furthermore derived implications that allow to test for more evidence for the theory.

Our results are useful for evaluating and designing financial regulation. They suggest that the fact that entrepreneurs spread their divestment over time is not an efficient outcome. The analysis provides therefore a theoretic justification for lock-up periods. We have furthermore derived a mechanism that can fully restore efficiency without reducing flexibility. With respect to trading restrictions for insiders, we have identified requirements that ensure that entrepreneurs who are subject to moral hazard can divest in the absence of commitment.

<sup>&</sup>lt;sup>9</sup>Requiring unanimity voting among a large number of shareholders may be unreasonable in practice. However, a less restrictive rule such as majority voting will give the same outcome since there is no divergence among investors.

<sup>&</sup>lt;sup>10</sup>The proposed mechanism is in fact quite similar to regulations in mainly European countries that apply for registered increases of capital. Such regulations require that capital increases have to be accepted by the shareholder's meeting and include the right for every shareholder to buy a number of new shares at a special price. Like our mechanism, the amount of shares which can be bought is proportional to the number of shares a shareholder already holds, which ensures that any effect caused by the capital increase on shareholders (such as dilution) is internalized.

## **Appendix**

**Proof of Proposition 3.** Setting  $f(\tilde{e}) = \tilde{e}$  in (i), taking derivative with respect to  $\tilde{e}$  and setting to zero gives  $\theta(n) - \tilde{e} = 0 \Rightarrow \tilde{e} = \theta(n)$ . From (iii) and (iv) we have then  $p(\tilde{e}) = \theta(n)$ . Inserting in the objective function yields  $\phi = \int_0^n -e^{-\delta t}\theta'(t)\theta(n)dt + e^{-\delta n}\theta(n)^2/2$ . The FOC for (ii) at t = k is

$$\frac{d\phi_{t\geq k}}{d\theta(k)} = \frac{\partial\phi(\theta'(k))}{\partial\theta(k)} + \int_{k}^{n} \frac{\partial\phi(\theta'(t))}{\partial\theta'(t)} \frac{d\theta'(t)}{d\theta'(k)} dt + \frac{\partial\phi_{t\geq k}(\theta(n))}{\partial\theta(n)} \frac{\partial\theta(n)}{\partial\theta(k)} + \frac{\partial\phi_{t\geq k}(\theta(n))}{\partial\theta(n)} \int_{k}^{n} \frac{\partial\phi(\theta(n))}{\partial\theta'(t)} \frac{d\theta'(t)}{d\theta'(k)} dt = 0$$

By the envelope theorem we have

$$\int_{k}^{n} \frac{\partial \phi(\theta'(t))}{\partial \theta'(t)} \frac{d\theta'(t)}{d\theta'(k)} dt + \frac{\partial \phi_{t \ge k}(\theta(n))}{\partial \theta(n)} \int_{k}^{n} \frac{\partial \phi(\theta(n))}{\partial \theta'(t)} \frac{d\theta'(t)}{d\theta'(k)} dt = 0$$

and that the FOC can be evaluated for  $\partial \theta(n)/\partial \theta(k) = 1$ . The FOC therefore simplifies to

$$\frac{d\phi_{t\geq k}}{d\theta(k)} = \frac{\partial\phi(\theta'(k))}{\partial\theta(k)} + \frac{\partial\phi_{t\geq k}(\theta(n))}{\partial\theta(n)} = 0$$

Inserting the objective function gives  $\frac{d\phi_{t\geq k}}{d\theta(k)} = -e^{-\delta k}\theta(n) + \int_k^n -e^{-\delta t}\theta'(t)dt + e^{-\delta n}\theta(n) = 0$ . Differentiating with respect to k yields  $\delta e^{-dk}\theta(n) + e^{-dk}\theta'(t) = 0 \Rightarrow \theta'(t) = -\delta\theta(n)$  for  $0 \leq t < n$ . For t = n define  $\overline{\theta}_n := \lim_{t \to n} \theta(t)$ ,  $\theta(n)$  is then  $\arg\max_{\theta(n)} \{(\overline{\theta}_n - \theta(n))\theta(n) + \theta(n)^2/2\}$  (maximization of utility in the last period) which gives  $\theta(n) = \overline{\theta}_n$ , hence there is no (discrete) end-of-period divestment. Computation of  $\theta(n)$ ,  $\theta'(t)$ ,  $\theta(t)$ :  $\theta(n) = 1 + \int_0^n \theta'(t)dt = 1 + \int_0^n -\delta\theta(n)dt = 1 - \delta n\theta(n) \Rightarrow \underline{\theta(n)} = 1/(1+\delta n) \Rightarrow \underline{\theta'(t)} = -\delta/(1+\delta n)$ . The divestment path is then  $\theta(t) = 1 + \int_0^t \theta'(s)ds = 1 - \delta t/(1+\delta n)$ .

**Proof of Proposition 4.** (i) If we restrict us to allocations in which investors get zero profits, a constrained efficient solution is given by a feasible divestment path  $\{\theta(t)\}_{0 \le t \le 1}$  that maximizes the objective function  $\phi$  in (2) subject to condition (iii). Feasibility requires that the incentive constraint (i) is fulfilled and that conditions (iv) and (v) are met. The solution to (2) (Proposition 3) shows that E divests for all  $0 \le t \le 1$ . The social planner could improve the allocation by shifting divestment to time t = 0 and leave everything else, i.e. total divestment and price unchanged. This will increase the utility of E because she receives the proceeds from divestment earlier but leaves the position of the investors unchanged; (ii) From (i) it is clear that a constrained-efficient allocation requires all trades taking place at t = 0, i.e.  $\theta^*(0) = \theta^*(t) = \theta^*(n)$  for all  $0 \le t \le 1$  (we skip the requirement that  $\theta(t)$  is differentiable to ensure existence of the constrained-efficient solution). Constrained efficient divestment has then to satisfy

$$\theta^* = \arg\max_{0 \leq \theta^* \leq 1} (1 - \theta^*) f(\widetilde{e}) + e^{-\delta n} [\theta^* f(\widetilde{e}) - \widetilde{e}^2/2]$$

s.t. to conditions (i) and (iv) in problem (2). Using  $\tilde{e} = \theta^*$  (optimal effort according to condition (i)) and  $f(\tilde{e}) = \tilde{e}$  (condition (iv)), the FOC to the problem is  $1 - 2\theta^* + e^{-\delta n} = 0 \Rightarrow \theta^* = 1/(2 - e^{-\delta n})$ ; (iii)  $\theta(n) < \theta^* \Leftrightarrow 1/(1 + \delta n) < 1/(2 - e^{-\delta n}) \Leftrightarrow e^{-\delta n} + \delta n > 1$ . Proof that  $f(x) = e^{-x} + x > 1$  for x > 0:  $f'(x) = -e^{-x} + 1$  and  $f'(x) = 0 \Rightarrow x = 0$ ;  $f''(x) = e^{-x}$  and  $f''(0) = 1 > 0 \Rightarrow x = 0$  is minimum, since f(0) = 1 it follows that  $e^{-x} + x > 1$  for  $x \neq 0$ .

**Proof of Corollary 5.** The entrepreneur's objective function is

$$(1 - \theta(0))p(\widetilde{e}) + \int_0^n -e^{-\delta t}\theta'(t)p(\widetilde{e})dt + e^{-\delta n}[\theta(n)f(\widetilde{e}) - \frac{\widetilde{e}^2}{2}]\},$$

where  $1 - \theta(0)$  is the share sold at the IPO and  $\{\theta'(t)\}_{0 < t \le n}$  her divestment path in the secondary market. Final share  $\theta(n)$  given that  $\theta(0)$  is retained at IPO: from Proposition 3 we have that  $\theta'(t) = -\delta\theta(n)$ , with  $\theta(n) = \theta(0) + \int_0^n \theta'(t)dt$  it follows that  $\theta(n) = \theta(0)/(1 + \delta n)$ . Choice of  $\theta(0)$ : the objective function  $\phi(\theta(0))$  can be simplified to

$$\phi(\theta(0)) = (1 - \theta(0))p(\tilde{e}) + \int_0^n -e^{-\delta t}\theta'(t)\theta(n)dt + e^{-\delta n}\theta(n)^2/2$$

$$= \frac{(1 - \theta(0))\theta(0)}{1 + \delta n} + \int_0^n e^{-\delta t}\delta \frac{\theta(0)^2}{(1 + \delta n)^2}dt + e^{-\delta n}\frac{\theta(0)^2}{2(1 + \delta n)^2}$$

$$= \frac{(1 - \theta(0))\theta(0)}{1 + \delta n} + \frac{\theta(0)^2}{(1 + \delta n)^2}[\delta(-\frac{e^{-\delta n}}{\delta} + \frac{1}{\delta}) + \frac{e^{-\delta n}}{2}]$$

$$= \frac{(1 + \delta n)(1 - \theta(0))\theta(0) + \theta(0)^2(1 - e^{-\delta n}/2)}{(1 + \delta n)^2},$$

where  $p(\tilde{e}) = \theta(0)$ ,  $\theta'(t) = -\delta\theta(n)$  and  $\theta(n) = \theta(0)/(1+\delta n)$  have been used. The FOC with respect to  $\theta(0)$  is:  $(1+\delta n)(1-2\theta(0))+2\theta(0)(1-e^{-\delta n}/2)=0$ . Solving for  $\theta(0)$  gives  $\theta(0)=(1+\delta n)/(e^{-\delta n}+2\delta n)$ . From  $\theta(n)=\theta(0)/(1+\delta n)$  it follows that  $\theta(n)=1/(e^{-\delta n}+2\delta n)$ .

**Proof of Proposition 6.** Utility from staying private is:  $u_P(n) = e^{-n\delta}(f(\tilde{e}) - \tilde{e}^2/2) = e^{-nd}/2$ . Utility from going public: inserting  $\theta(0), \theta(n), \theta'(t)$  (from Corollary 5) in the objective function gives:  $u_{GP}(n) = \theta(n)/2 = 1/(2(e^{-n\delta} + 2\delta n))$  (see Corollary 5)  $\Rightarrow b(n) = u_{GP}(n) - u_P(n) = (1 - 2e^{-2\delta n} - 2\delta n e^{-\delta n})/(2(e^{-n\delta} + 2\delta n))$ . Hump-shape: to show that b(n) has unique extremum that is a maximum on  $[0, \infty)$ . Without loss of generality set  $\delta = 1$  in  $b(n) \Rightarrow FOC$  of b(n) wrt. to n is  $b'(n^*) = e^{-n^*} - 2 + e^{-n^*}(e^{-n^*} + 2n^*)^2 = 0$ . Combining b''(n) with  $b'(n^*) = 0 \Rightarrow b''(n^*) < 0 \Rightarrow n^*$  is maximum and unique. Existence of  $n^*$  follows from  $\lim_{n\to\infty} b(n) = b(0) = 0$  and  $b(1) \geq 0$ .

**Proof of Proposition 7.** Follows directly from the hump-shape of b(n) (Proposition 6).

**Proof of Proposition 8.** The gains from going public b as a function of  $\delta$  exhibit a hump-shape for  $\delta \in [0, \infty)$  (this follows from Proposition 6 noting that  $b(\delta, n)$  can be written as  $b(\delta n)$ )  $\Rightarrow$  firms with high and low  $\delta$  have low  $b(\delta)$  and thus go public later.

**Proof of Proposition 9.** Let  $\theta(0), \theta'(t), \theta(n)$  the equilibrium divestment before commitment. Consider the divestment  $\theta_L(0), \theta'_L(t), \theta_L(n)$  in the presence of a lock-up

period of length k with  $\theta_L(t) = \theta(k)$  for  $0 \le t \le k$  and  $\theta'_L(t) = \theta'(t)$  for t > k. This divestment is feasible since  $\theta_L(t) = \theta(t)$  during the trading period (t > k). From  $\theta_L(n) = \theta(n) \Rightarrow e_L^* = e^*$ ,  $p(e_L^*) = p(e^*) \Rightarrow$  total proceeds  $(1 - \theta(n))p(e^*)$  from selling the firm do not change  $\Rightarrow$  present value of the proceeds increases  $(\delta > 0)$  since a part of the proceeds are received earlier.

**Proof of Proposition 10.** We assume that an investor accepts an offer if it does not reduce his wealth (consistent with the maintained assumption that investors are price takers we rule out strategic behavior). Consider investor k who holds a share  $\theta_k$  in the firm, k's wealth if the offer rejected is  $w_k(t) = \theta_k(t)\theta(n)$  and if the offer is accepted  $w_k^*(t) = \theta_k(t)\theta^*(n)(1-\theta^*(t))/(1-\theta(t)) - p\theta_k(t)(\theta(t)-\theta^*(t))/(1-\theta(t))$ , where  $\theta(n)$  and  $\theta^*(n)$  are according to the time-consistent divestment path with initial condition  $\theta(t)$  and  $\theta^*(t)$ , respectively;  $w_k^*(t) \geq w_k(t) \Leftrightarrow p \leq [(1-\theta^*(t))\theta^*(n)-(1-\theta(t))\theta(n)]/(\theta(t)-\theta^*(t)) \Rightarrow p$  independent of  $k \Rightarrow$  offer accepted by all investors if  $p^* \leq p \Rightarrow$  E will offer exactly p; from  $p^* = p \Rightarrow w_k^*(t) = w_k(t)$  for arbitrary investor  $k \Rightarrow$  total wealth of all investors is also constant.  $\blacksquare$ 

### References

- Brav, Alon, and Paul A. Gompers, 2000, Insider Trading Subsequent to Initial Public Offerings: Evidence from Expiration of Lock-Up Provisions, *mimeo Duke University*.
- Brennan, Michael, and Julian Franks, 1997, Underpricing, Ownership and Control in Initial Public Offerings of Equity Securities in the UK, *Journal of Financial Economics* 45, 391–413.
- Chemmanur, Thomas J., 1993, The Pricing of Initial Public Offerings: A Dynamic Model with Information Production, *Journal of Finance* 48, 285–304.
- Chemmanur, Thomas J., and Paolo Fulghieri, 1999, A Theory of the Going-Public Decision, Review of Financial Studies 12, 249–279.
- Diamond, Peter A., 1967, The Role of the Stock Market in a General Equilibrium Model with Technological Uncertainty, *American Economic Review* 57, 759–776.
- Downes, David H., and Robert Heinkel, 1982, Signaling and the Valuation of Unseasoned New Issues, *Journal of Finance* 37, 1–10.
- Ellingson, Tore, and Kristian Rydqvist, 1997, The Stock Market as a Screening Device and the Decision to Go Public, mimeo Stockholm School of Economics.
- Gompers, Paul, 1996, Grandstanding in the Venture Capital Industry, *Journal of Financial Economics* 43, 133–156.
- Grinblatt, Mark, and Chuan Yang Hwang, 1989, Signalling and the Pricing of New Issues, Journal of Finance 44, 293–420.
- Holthausen, Robert W., and David F. Larcker, 1996, The Financial Performance of Reverse Leveraged Buyouts, *Journal of Financial Economics* 32, 293–332.
- Jain, Bharat A., and Omesh Kini, 1994, The Post-Issue Operating Performance of IPO-Firms, *Journal of Finance* 49, 1699–1726.
- Kocherlakota, Narayana R., 1998, The Effects of Moral Hazard on Asset Prices When Financial Markets are Complete, *Journal of Monetary Economics* 41, 39–56.
- Leland, Hayne E., and David H. Pyle, 1977, Informational Assymmetries, Financial Structure, and Financial Intermediation, *Journal of Finance* 32, 371–387.
- Magill, Michael, and Martine Quinzii, 2001, Capital Market Equilibrium with Moral Hazard, forthcoming in Journal of Mathematical Economics.

- Magill, Michael, and Martine Quinzii, 1998, Incentives and Risk Sharing in a Stock Market Equilibrium, *University of Southern California working paper*.
- Maksimovic, Vojislav, and Pegaret Pichler, 2001, Technological Innovation and Initial Public Offerings, *Review of Financial Studies* 14, 459–494.
- Mello, Antonio S., and John E. Parsons, 1998, Going Public and the Ownership Structure of the Firm, *Journal of Financial Economics* 49, 79–109.
- Mikkelson, Wayne H., and M. Megan Partch, 1985, Stock Price Effects and Costs of Secondary Distributions, *Journal of Financial Economics* 14, 263–290.
- Mikkelson, Wayne H., M. Megan Partch, and Kshitij Shah, 1997, Ownership and Operating Performance of Companies That Go Public, *Journal of Financial Economics* 44, 281–307.
- Pagano, Marco, Fabio Panetta, and Luigi Zingales, 1995, Why Do Companies Go Public? An Empirical Analysis, *NBER working paper no.5367*.
- Ritter, Jay R., 1984, The 'Hot Issue' Market of 1980, Journal of Business 57, 215–240.
- Roell, Ailsa, 1996, The Decision to Go Public: An Overview, *European Economic Review* 40, 1071–1081.
- Rydqvist, Kristian, and Kenneth Hogholm, 1995, Going Public in the 1980s: Evidence from Sweden, European Financial Management 1, 287–315.
- Titman, Sheridan, and Avanidhara Subrahmanyam, 1999, The Going Public Decision and the Development of Financial Markets, *Journal of Finance*.
- Welch, Ivo, 1989, Seasoned Offerings, Imitation Costs, and the Underpricing of Initial Public Offerings, *Journal of Finance* pp. 421–449.
- Wruck, Karen, 1989, Equity Ownership Concentration and Firm Value: Evidence from Private Equity Financings, *Journal of Financial Economics* 23, 3–28.
- Zingales, Luigi, 1995, Insider Ownership and the Decision to Go Public, *Review of Economic Studies* 62, 425–448.